Distributed Consensus in Multivehicle Cooperative Control: Theory and Applications

Workshop 5: Cooperative Control of Multiple Autonomous Vehicles
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Research Interests

Control Systems & Robotics

- Autonomous Control of Robotic Vehicles
  - e.g., guidance, navigation, and control of unmanned air/ground vehicles
- Cooperative Control of Multiple Autonomous Vehicles
  - e.g., swarms of multiple unmanned air/ground vehicles, multi-robot coordination, distributed algorithms, spacecraft formation flying

Platforms in the **CO**operative **VE**hicle **N**etworks (COVEN) Laboratory at Utah State University

Potential Applications for Autonomous Vehicles

Civil and Commercial:

- Automated Mining
- Monitoring environment
- Monitoring disaster areas
- Communications relays
- Law enforcement
- Precision agriculture

Military:

- Special Operations: Situational Awareness
- Intelligence, surveillance, and reconnaissance
- Communication node
- Battle damage assessment

Homeland Security:

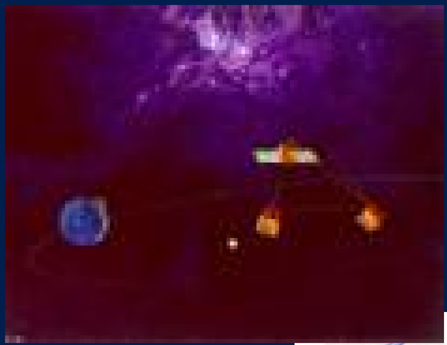
- Border patrol
- Surveillance
- Rural/Urban search and rescue


Epson

Cooperative/Coordinated Control

- **Motivation:**
  While single vehicles performing solo missions will yield some benefits, greater benefits will come from the cooperation of teams of vehicles.

- **Common Theme:**
  Coordinate the movement of multiple vehicles in a certain way to accomplish an objective.
  - e.g. many small, inexpensive vehicles acting together can achieve more than one monolithic vehicle.
    - e.g., networked computers
    - Shifts cost and complexity from hardware platform to software and algorithms.



- **Multi-vehicle Applications:**
  Space-based interferometers, future combat systems, surveillance and reconnaissance, hazardous material handling, distributed reconfigurable sensor networks …

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Cooperative Control Categorization

- Formation Control

  - Approaches: leader-follower, behavioral, virtual structure/leader, artificial potential function, graph-rigidity

  - Applications: mobile robots, unmanned air vehicles, autonomous underwater vehicles, satellites, spacecraft, automated highways

- Task Assignment, cooperative transport, cooperative role assignment, air traffic control, cooperative timing

  - Cooperative search, reconnaissance, surveillance (military, homeland security, border patrol, etc.)

  - Cooperative monitoring of forest fires, oil spills, wildlife, etc.

  - Rural search and rescue.

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Cooperative Control: Inherent Challenges

- Complexity:
  - Systems of systems.
- Communication:
  - Limited bandwidth and connectivity.
  - What? When? To whom?
- Arbitration:
  - Team vs. Individual goals.
- Computational resources:
  - Will always be limited

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Cooperative Control: Centralized vs Distributed Schemes

- **Centralized Schemes**

  Assumptions: availability of global team knowledge, centralized planning and coordination, fully connected network

  Practical Issues: sparse & intermittent interaction topologies (limited communication/sensing range, environmental factors)

- **Distributed Schemes**

  Features: Local neighbor-to-neighbor interaction, evolve in a parallel manner

  Strengths: reduced communication/sensing requirement; improved scalability, flexibility, reliability, and robustness

Distributed Consensus Algorithms

- Basic Idea

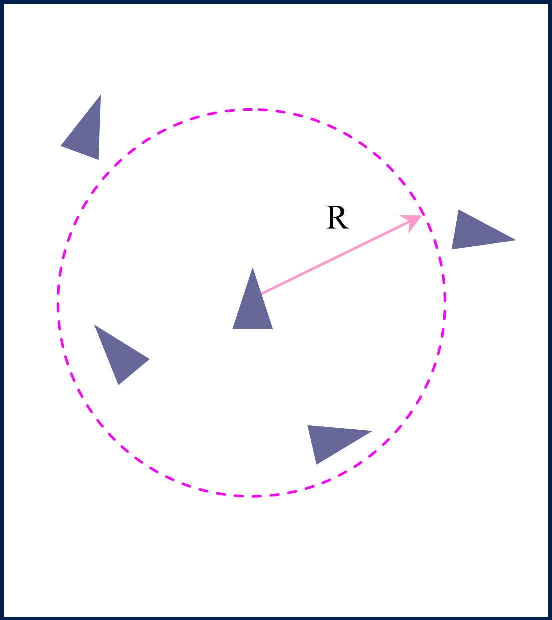
  Each vehicle updates its information state based on the information states of its local (possibly time-varying) neighbors in such a way that the final information state of each vehicle converges to a common value.

- Extensions

  Relative state deviations, incorporation of other group behaviors (e.g., collision avoidance)

- Feature

  Only local neighbor-to-neighbor interaction required



Vicsek's Model

Boids: http://www.red3d.com/cwr/boids/

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Consensus Algorithms – Literature Review

- Historical Perspective

  biology, physics, computer science, economics, load balancing in industry, complex networks

- Theoretic Aspects

  algebraic graph theory, nonlinear tools, random network, optimality and synthesis, communication delay, asynchronous communication, …

- Applications

  rendezvous, formation control, flocking, attitude synchronization, sensor fusion, …

Wei Ren, Randal W. Beard, *Distributed Consensus in Multi-vehicle Cooperative Control*, Communications and Control Engineering Series, Springer-Verlag, London, 2008 (ISBN: 978-1-84800-014-8)

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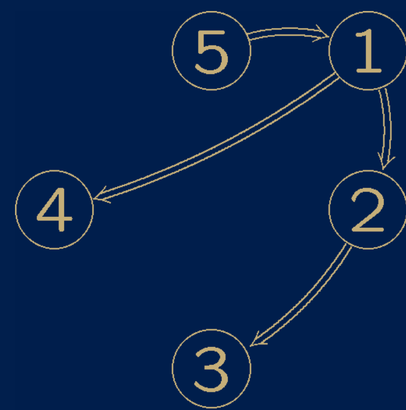
Modeling of Vehicle Interactions

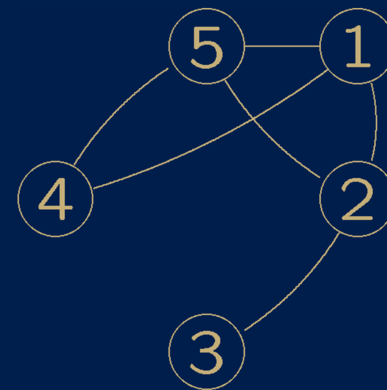
Graph $\mathcal{G}$: $(\mathcal{V}, \mathcal{E})$
Nodes: $\mathcal{V} = \{1, \ldots, n\}$
Edges: $\mathcal{E} = \{(i, j)\}$
i is a neighbor of j
if $(i, j) \in \mathcal{E}$

A directed path
$(i_1, i_2), (i_2, i_3), \ldots$

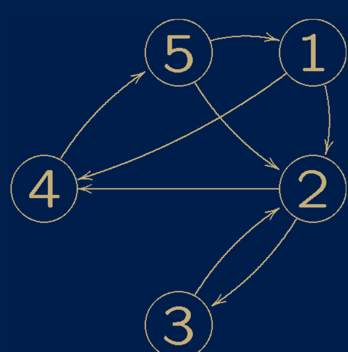
$2 \to 3 \to 4$



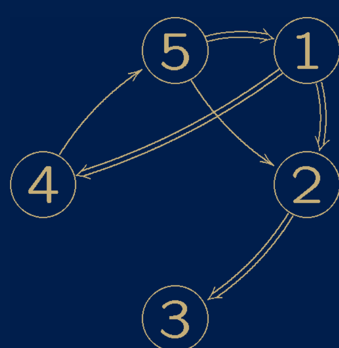
A directed spanning tree



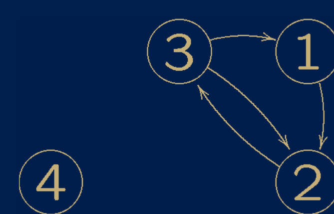
A undirected graph
that is connected



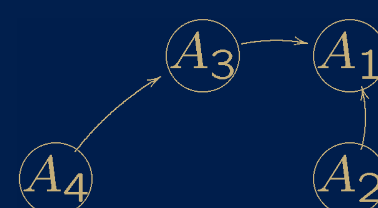
A directed graph that
is strongly connected



A graph that has a spanning
tree but not strongly connected



(i) Separated groups



(ii) Multiple leaders

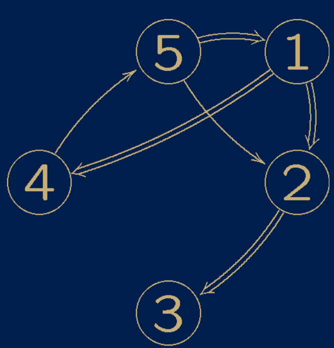
Modeling of Vehicle Interactions (cont.)

Adjacency Matrix:

Let $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ be the adjacency matrix associated with $\mathcal{G}$, where $a_{ij} = 1$ if $(j,i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise.

(Nonsymmetric) Laplacian Matrix:

Let $\mathcal{L} = [\ell_{ij}] \in \mathbb{R}^{n\times n}$ be the nonsymmetric Laplacian matrix associated with $\mathcal{G}$, where $\ell_{ij} = -a_{ij}$, $i \neq j$, $\ell_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$.



$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

adjacency matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 2 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Laplacian matrix

Outline

- Part 1: Consensus for Single-integrator Kinematics – Theory and Applications

- Part 2: Consensus for Double-integrator Dynamics – Theory and Applications
- Part 3: Consensus for Rigid Body Attitude Dynamics – Theory and Applications
- Part 4: Synchronization of Networked Euler-Lagrange Systems – Theory and Applications

Consensus Algorithm for 1st-order Kinematics

Single-integrator Kinematics: $\dot{\xi}_i = u_i$, $i = 1, \ldots, n$, where $\xi_i \in \mathbb{R}^m$ is the state and $u_i \in \mathbb{R}^m$ is the control input.

Algorithm:

$$u_i = -\sum_{j \in \mathcal{N}_i(t)} (\xi_i - \xi_j),$$

where $\mathcal{N}_i(t)$ denotes the time-varying neighbor set of vehicle i.

Consensus is *reached* if for all $\xi_i(0)$, $\xi_i(t) \to \xi_j(t)$ as $t \to \infty$.

The closed-loop system can be written in matrix form as

$$\dot{\xi}(t) = -[\mathcal{L}(t) \otimes I_m]\xi(t),$$

where $\xi = [\xi_1^T, \ldots, \xi_n^T]^T$, $\mathcal{L}$ is the Laplacian matrix, $\otimes$ denotes the Kronecker product, and I_m denotes the $m \times m$ identity matrix

Convergence Result (Fixed Graph)

Result: For a fixed graph, consensus is reached if and only if the directed graph has a directed spanning tree.

The Laplacian matrix has a simple zero eigenvalue and all the others have positive real parts.

Directed graph has a directed spanning tree.

Consensus is reached.

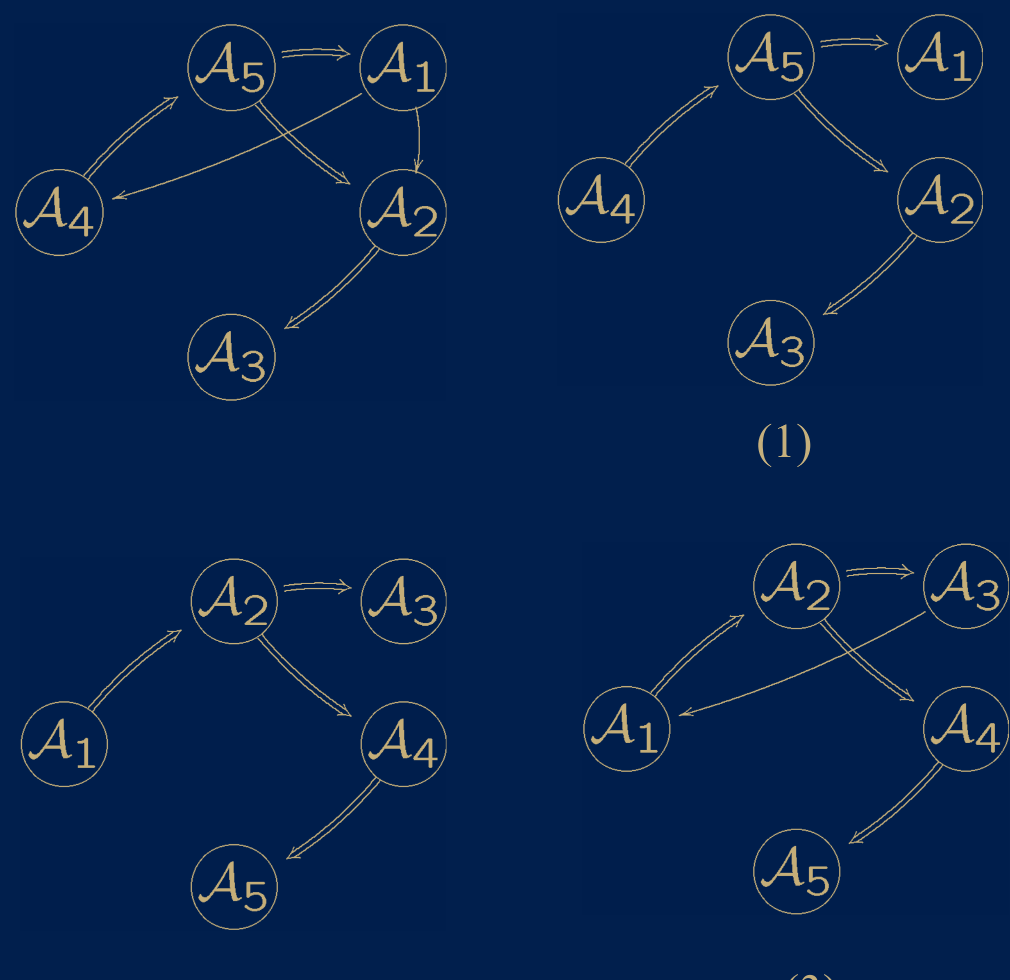
Sketch of the Proof

An inductive approach

• Step 1: find a spanning tree that is a subset of the graph

• Step 2: show that consensus can be achieved with the spanning tree (renumber each agent)

• Step 3: show that if consensus can be achieved for a graph, then adding more links will still guarantee consensus



(1)

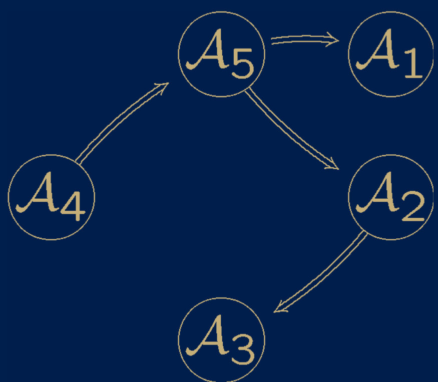
(2)

(3)

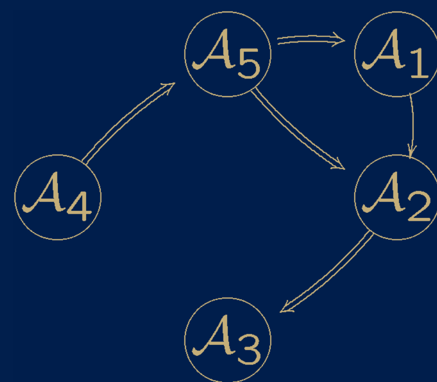
Consensus Equilibrium (Fixed Topology)

When directed graph $\mathcal{G}$ has a directed spanning tree, $\xi_i(t) \to \sum_{j=1}^{n}(c_j \xi_j(0))$ as $t \to \infty$, where $\sum_{j=1}^{n} c_j = 1$ and $c_j \geq 0$.

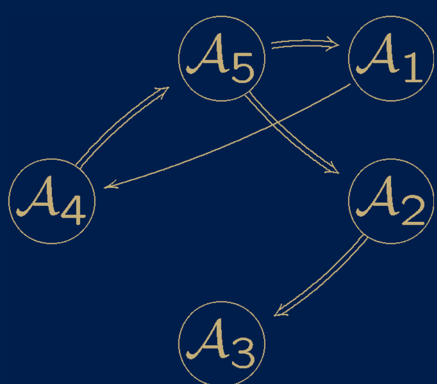
The initial condition of a node contributes to the equilibrium value if and only if the node has a directed path to all the other nodes.



$\xi_i(\infty) \to \xi_4(0)$



$\xi_i(\infty) \to \xi_4(0)$



$\xi_i(\infty) \to c_4\xi_4(0) + c_5\xi_5(0) + c_1\xi_1(0)$, where $c_4 + c_5 + c_1 = 1$ and $c_4, c_5, c_1 > 0$.

Convergence Result (Switching Graphs)

Result: Let $t_1, t_2, \cdots$ be an infinite time sequence at which the directed graph switches and $\tau_i = t_{i+1} - t_i$ is lower bounded, $i = 0, 1, \cdots$. Consensus is reached using the first-order algorithm if there exists an infinite sequence of uniformly bounded, non-overlapping time intervals $[t_{i_j}, t_{i_j+l_j})$, $j = 1, 2, \cdots$, starting at $t_{i_1} = t_0$, with the property that each interval $[t_{i_j+l_j}, t_{i_{j+1}})$ is uniformly bounded and the union of the directed graphs across each such interval has a directed spanning tree. Furthermore, if the union of the directed graphs does not have a directed spanning tree after some finite time, consenus cannot be achieved.

Sketch of the Proof

Solution: $\xi(t) = [\Phi(t, t_0) \otimes I_m]\xi(0)$
$= [\Phi(t, t_j)\Phi(t_j, t_{j-1}) \cdots \Phi(t_1, t_0) \otimes I_m]\xi(0)$,
where Φ is the transition matrix corresponding to $-\mathcal{L}(t)$.

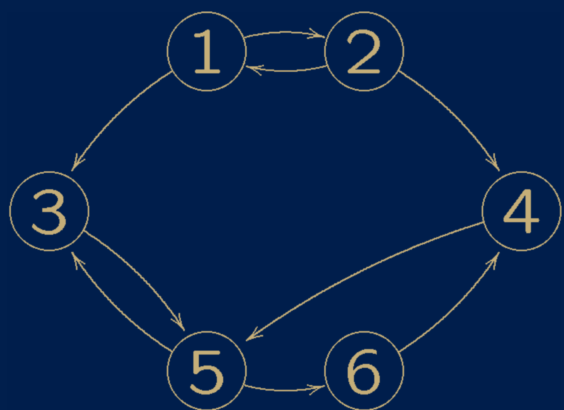
Definitions:

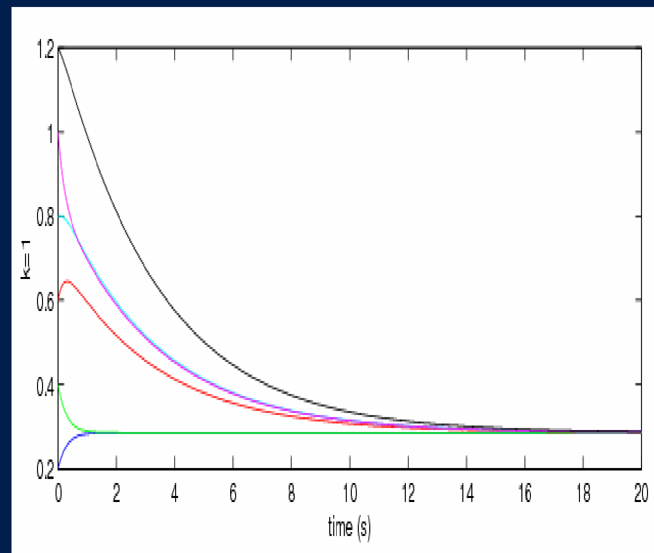
Stochastic Matrix P: nonnegative entries, row sum equal to 1
SIA (stochastic indecomposable & aperiodic): $\lim_{k\to\infty} P^k = \mathbf{1}\nu^T$, where $\mathbf{1} = [1, \cdots, 1]^T$ and $\nu \geq 0$.

Lemma [Wolfowitz63] Let $\mathcal{S} = \{S_j | j = 1, \cdots\}$ be a set of SIA matrices and N_t be the number of different types of all $n \times n$ SIA matrices. If there exists a constant $0 \leq d < 1$ satisfying $\lambda(W) \leq d$, where $W = S_{k_1} S_{k_2} \cdots S_{k_{N_t+1}}$ and $\lambda(W) = 1 - \min_{i_1, i_2} \sum_j \min(w_{i_1 j}, w_{i_2 j})$, then for each infinite sequence $S_{i_1}, S_{i_2}, \cdots$, $\lim_{j\to\infty} S_{i_j} S_{i_{j-1}} \cdots S_{i_1} = \mathbf{1}\nu^T$.

Examples - Consensus and Directed Spanning Trees

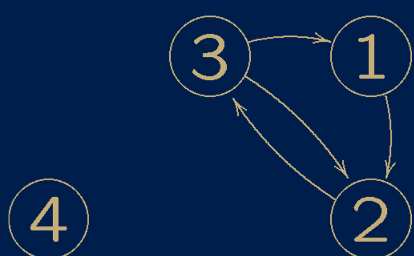
Consensus can be achieved:



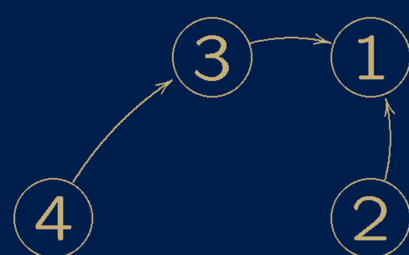


Equilibrium determined by vehicles 1 and 2

Cases when consensus cannot be achieved:

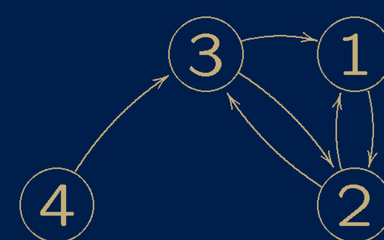


(i) Separated groups



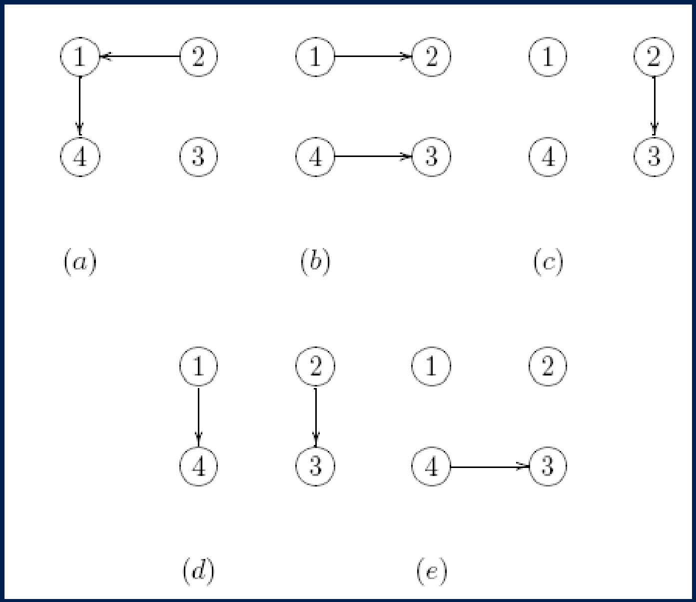
(ii) Multiple leaders

Consensus can be achieved:

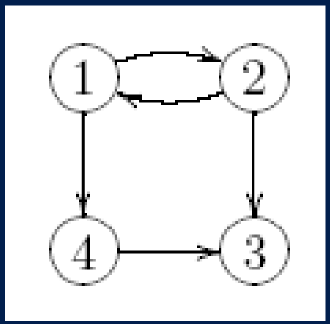


Union of (i) and (ii)

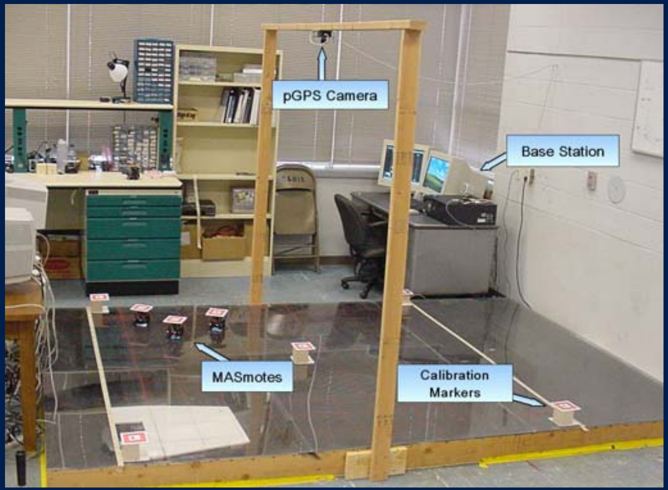
Rendezvous - Experiments



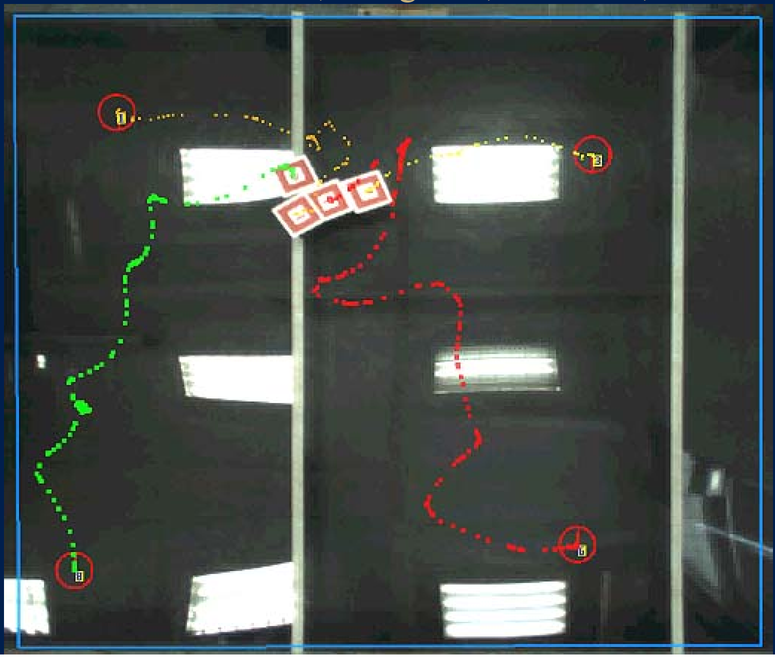
Switching Topologies



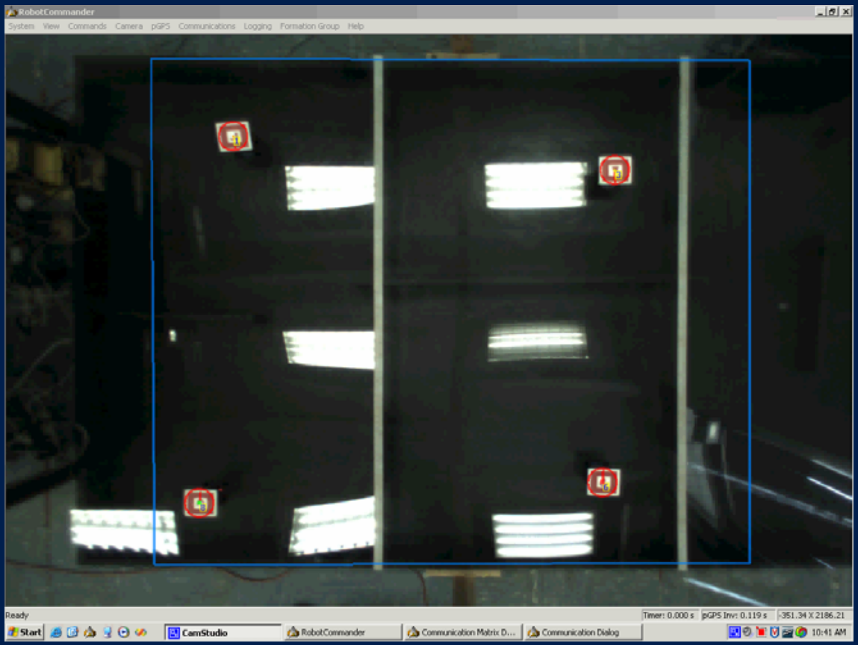
Union Topology



Experiment was performed in CSOIS (joint work with Chao, Bougeous, Sorensen, and Chen)



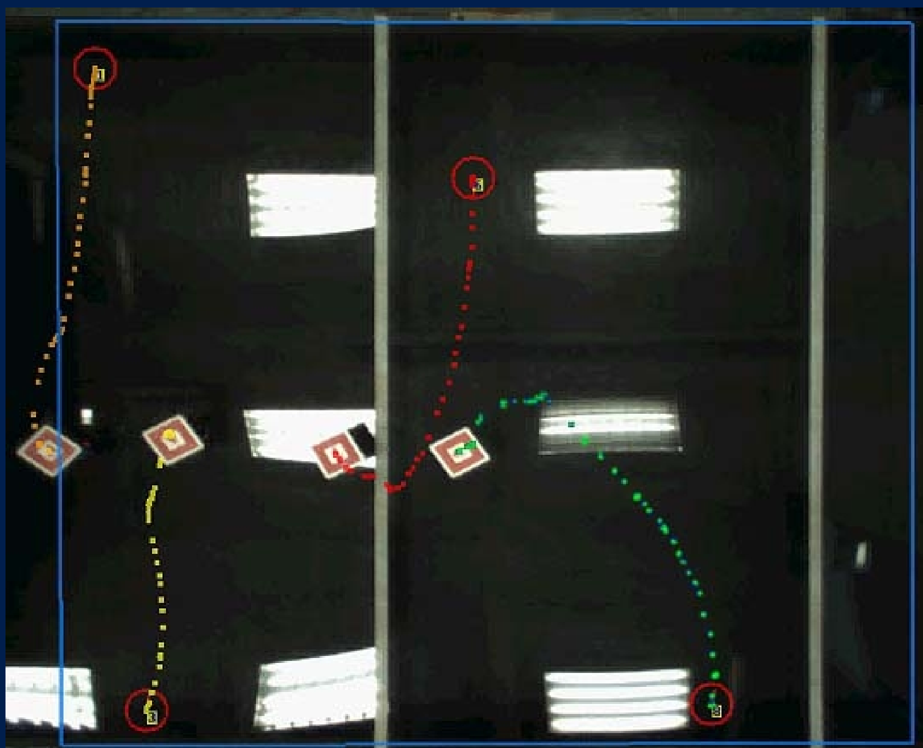
Rendezvous Demos - Switching Topologies



Switching Topologies

Union Topology

Axial Alignment



Consensus with a Virtual Leader

Leader State: ξ^r satisfying $\dot{\xi}^r = f(t, \xi^r)$

Algorithm for Followers:

$$u_i = \frac{1}{|\mathcal{N}_i(t)| + a_{iL}(t)} \sum_{j \in \mathcal{N}_i(t)} [\widehat{\dot{\xi}}_j - \gamma(\xi_i - \xi_j)] + \frac{1}{|\mathcal{N}_i(t)| + a_{iL}(t)} a_{iL}(t)[f(t, \xi^r) - \gamma(\xi_i - \xi^r)], \quad i = 1, \ldots, n,$$

where $\gamma > 0$, a_{iL} specifies whether a vehicle has access to the leader's state, and $\widehat{\dot{\xi}}_j$ is an estimate of $\dot{\xi}_j$.

Convergence Result:

1) The closed-loop system is input-to-state stable with $\xi_i - \xi^r$ being the state and $\widehat{\dot{\xi}}_j - \dot{\xi}_j$ being the input if the leader has a directed path to all the followers.

2) $\xi_i(t) \to \xi^r(t)$ as $t \to \infty$ if the leader has a directed path to all the followers and $\widehat{\dot{\xi}}_j \to \dot{\xi}_j$.

Sketch of the Proof

Define $\epsilon_j \triangleq \widehat{\dot{\xi}}_j - \dot{\xi}_j$ and $\delta \triangleq [\delta_1^T, \ldots, \delta_n^T]^T$ with $\delta_i \triangleq \sum_{j=1}^n a_{ij}\epsilon_j$.

The closed-loop system can be written as

$$\dot{\xi} - \mathbf{1} \otimes \dot{\xi}^r = -\gamma(\xi - \mathbf{1} \otimes \xi^r) + [W^{-1}(t) \otimes I_m]\delta,$$

where $W = [w_{ij}] \in \mathbb{R}^{n \times n}$ is given as $w_{ij} = -a_{ij}$, $i \neq j$, and $w_{ii} = a_{iL} + \sum_{j=1, j\neq i}^n a_{ij}$.

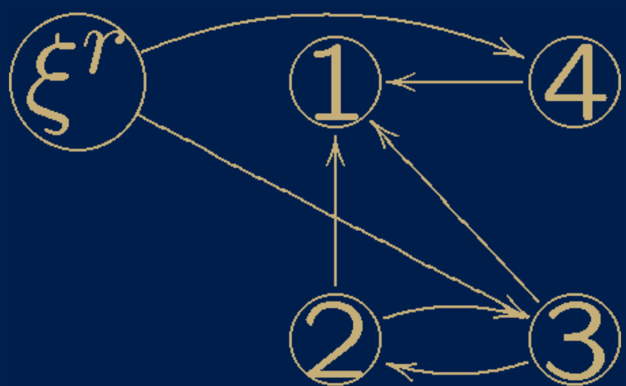
The closed-loop system is input-to-state stable with $\dot{\xi} - \mathbf{1} \otimes \dot{\xi}^r$ being the state and δ being the input.

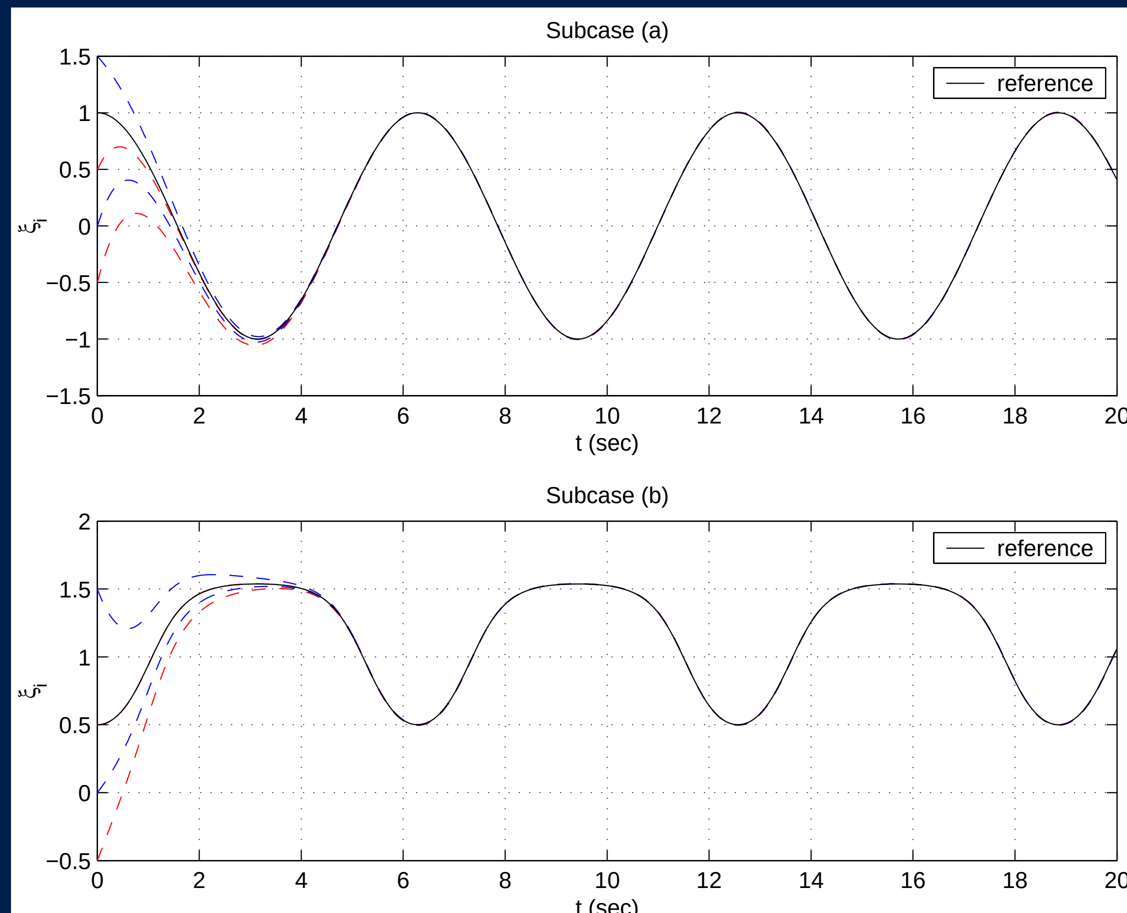
Examples – Consensus Tracking

Convergence result

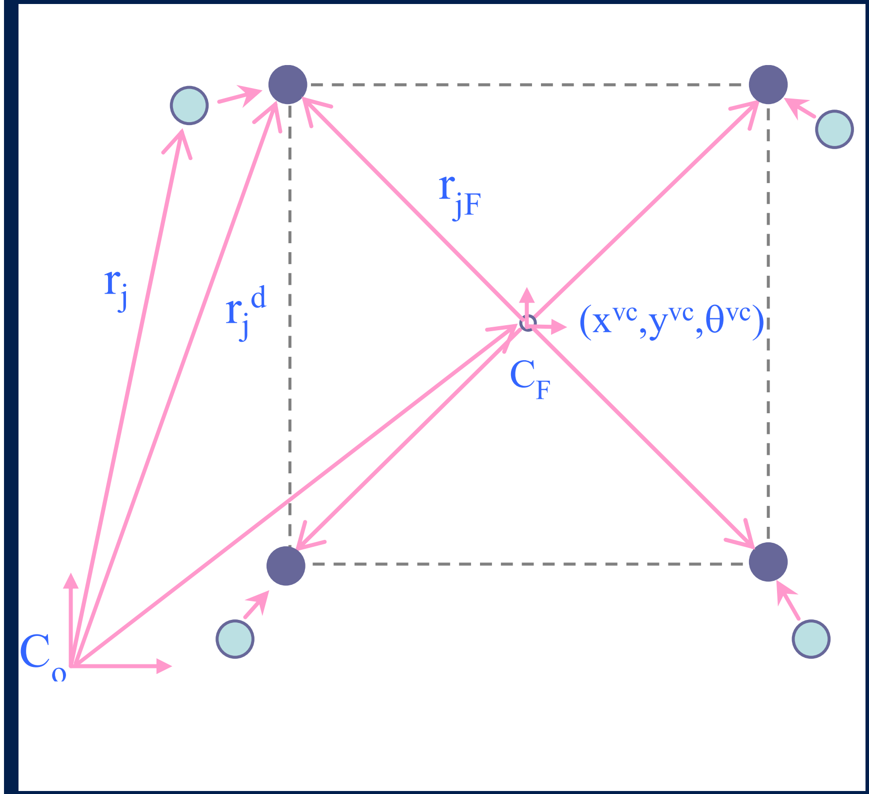
(a): $\xi^r = \cos(t)$

(b): $\dot{\xi}^r = \sin(t)\sin(2\xi^r)$, where $\xi^r(0) = 0.5$.





Example - Virtual Leader/Structure Based Formation Control (Centralized)



C_o: inertial frame

C_F: a virtual coordinate frame

Formation state: $\xi^{vc} = [x^{vc}, y^{vc}, \theta^{vc}]^T$

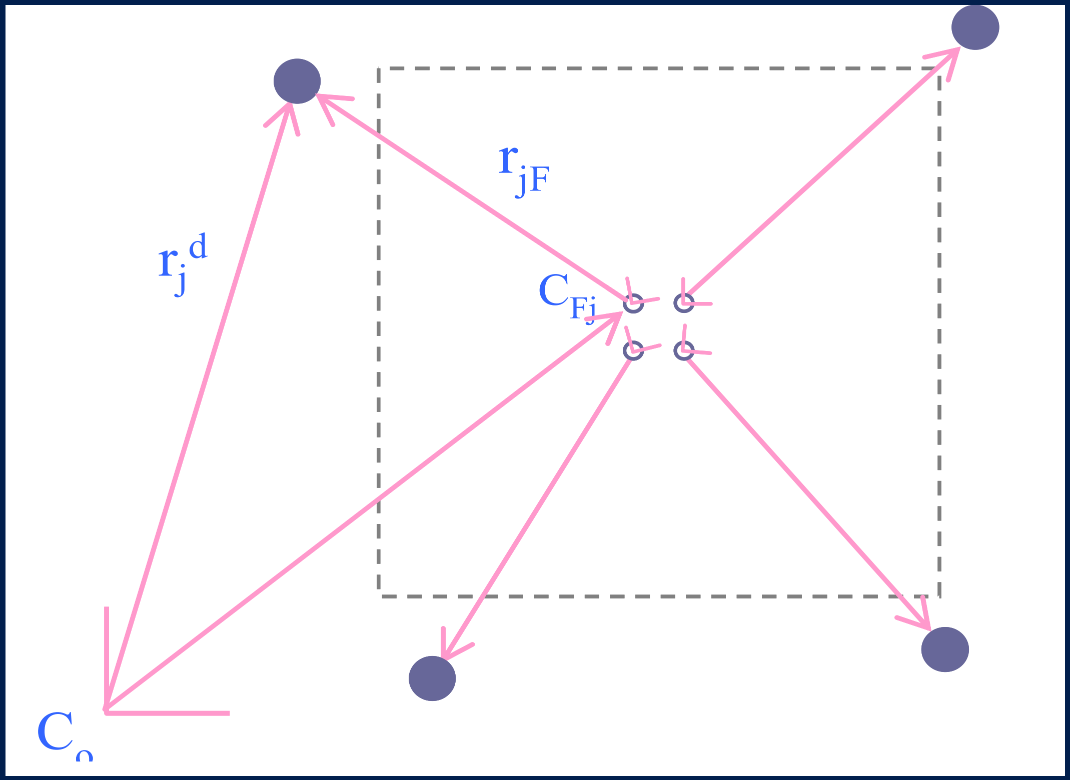
$r_j = [x_j, y_j]^T$: actual position

$r_j^d = [x_j^d, y_j^d]^T$: desired position

$r_{jF} = [x_{jF}, y_{jF}]^T$: desired deviation relative to C_F.

$$\begin{bmatrix} x_j^d(t) \\ y_j^d(t) \end{bmatrix} = \begin{bmatrix} x^{vc}(t) \\ y^{vc}(t) \end{bmatrix} + R(\theta^{vc}(t)) \begin{bmatrix} x_{jF}(t) \\ y_{jF}(t) \end{bmatrix}.$$

Example - Virtual Leader/Structure Based Formation Control (decentralized)



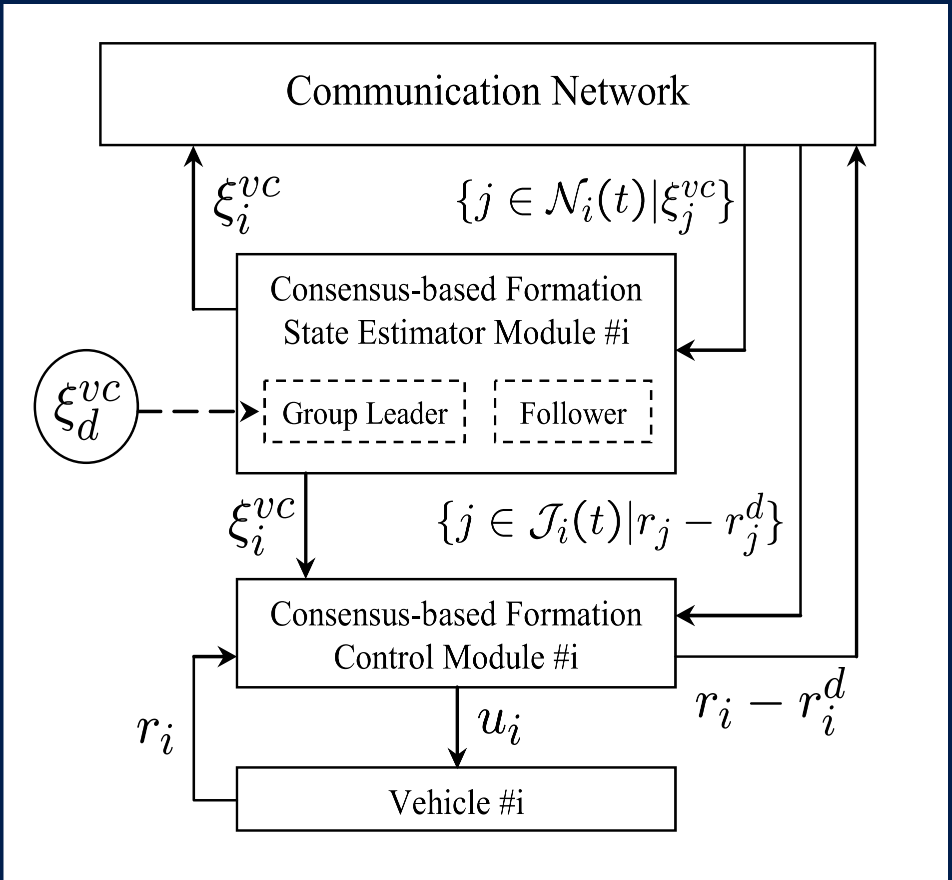
$$\begin{bmatrix} x_j^d(t) \\ y_j^d(t) \end{bmatrix} = \begin{bmatrix} x_j^{vc}(t) \\ y_j^{vc}(t) \end{bmatrix} + R(\theta_j^{vc}(t)) \begin{bmatrix} x_{jF}(t) \\ y_{jF}(t) \end{bmatrix}.$$

$\xi_i^{vc} = [x_i^{vc}, y_i^{vc}, \theta_i^{vc}]^T$: the i^{th} vehicle's understanding or estimation of the virtual coordinate frame.

When each vehicle has inconsistent understanding or knowledge of ξ^{vc}, the formation geometry cannot be maintained.

A Unified Scheme for Distributed Formation Control

Vehicle Dynamics: $\dot{r}_i = u_i, \quad i = 1, \ldots, n$ where $r_i = [x_i, y_i]^T$ is the position and $u_i = [u_{xi}, u_{yi}]^T$ is the control input to the i^{th} vehicle.



$\xi_i^{vc} = [x_i^{vc}, y_i^{vc}, \theta_i^{vc}]^T$: formation state estimate.
$\xi_d^{vc} = [x_d^{vc}, y_d^{vc}, \theta_d^{vc}]^T$: desired state for ξ_i^{vc}.

$\mathcal{N}_i(t)$ and $\mathcal{J}_i(t)$: neighbor sets

Formation State Estimation Level

$$\dot{\xi}_i^{vc} = \frac{\dot{\xi}_d^{vc} - \gamma(\xi_i^{vc} - \xi_d^{vc}) + \sum_{j \in \mathcal{N}_i}[\widehat{\dot{\xi}_j^{vc}} - \gamma(\xi_i^{vc} - \xi_j^{vc})]}{1 + |\mathcal{N}_i|}, \quad i \in \mathcal{L}$$

$$\dot{\xi}_i^{vc} = \frac{\sum_{j \in \mathcal{N}_i}[\widehat{\dot{\xi}_j^{vc}} - \gamma(\xi_i^{vc} - \xi_j^{vc})]}{|\mathcal{N}_i|}, \quad i \notin \mathcal{L},$$

where $\mathcal{L}$ denotes the set of group leaders that have knowledge of ξ_d^{vc}, and $\gamma > 0$.

Objective: $\xi_i^{vc} \to \xi_d^{vc}$

Note: Only the group leaders have direct access to ξ_d^{vc}, which may be time varying, and the number of the group leaders can be any number from 1 to n.

Vehicle Control Level

$$u_i = \dot{r}_i^d - \alpha_i(r_i - r_i^d) - \sum_{j \in \mathcal{J}_i} k_{ij}[(r_i - r_i^d) - (r_j - r_j^d)],$$

where $\alpha_i > 0$, $k_{ij} > 0$, and $r_i^d = [x_i^d, y_i^d]^T$ with

$$\begin{bmatrix} x_i^d \\ y_i^d \end{bmatrix} = \begin{bmatrix} x_i^{vc} \\ y_i^{vc} \end{bmatrix} + \begin{bmatrix} \cos(\theta_i^{vc}) & -\sin(\theta_i^{vc}) \\ \sin(\theta_i^{vc}) & \cos(\theta_i^{vc}) \end{bmatrix} \begin{bmatrix} x_{iF} \\ y_{iF} \end{bmatrix}.$$

Objective: $r_i \to r_i^d$

Note: The estimation topology defined by $\mathcal{J}_i$ may be different from the inter-vehicle interaction topology defined by $\mathcal{N}_i$.

Experimental Platform at USU

Communication: TCP/IP

Position and orientation measurement: encoder data

Local interaction (emulated)

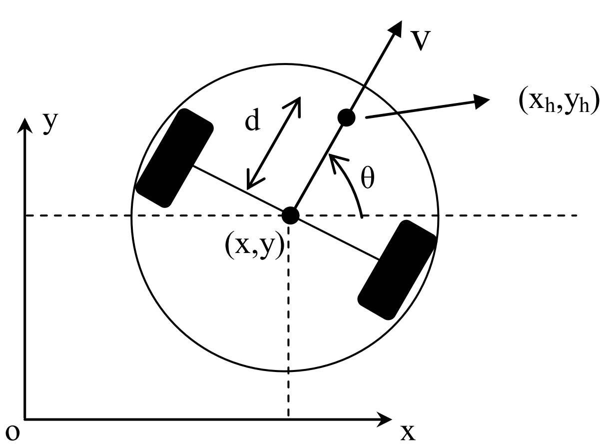
Mobile Robot Kinematic Model

Let (r_{xi}, r_{yi}), θ_i, and (v_i, ω_i) denote the Cartesian position, orientation, and linear and angular velocity of the i^{th} robot, respectively. The kinematic equations for the i^{th} robot are

$$\dot{r}_{xi} = v_i \cos(\theta_i), \quad \dot{r}_{yi} = v_i \sin(\theta_i), \quad \dot{\theta}_i = \omega_i.$$

Feedback linearization:

$x_i = r_{xi} + d_i \cos(\theta_i)$ and $y_i = r_{yi} + d_i \sin(\theta_i)$ Letting $\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & \sin(\theta_i) \\ -\frac{1}{d_i}\sin(\theta_i) & \frac{1}{d_i}\cos(\theta_i) \end{bmatrix} \begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix}$, gives $\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix}$.



Experimental Specification

Four AmigoBots are required to maintain a square formation and the virtual coordinate frame located at the center of the square follows a circle moving in a clockwise direction.

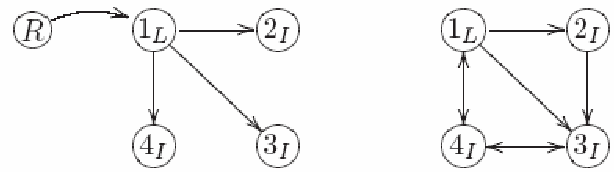
$\xi_d^{vc} = [x_d^{vc}, y_d^{vc}, \theta_d^{vc}]^T$ satisfies

$$\dot{x}_d^{vc} = v_d^{vc} \cos(\theta_d^{vc}), \quad \dot{y}_d^{vc} = v_d^{vc} \sin(\theta_d^{vc}), \quad \dot{\theta}_d^{vc} = \omega_d^{vc},$$

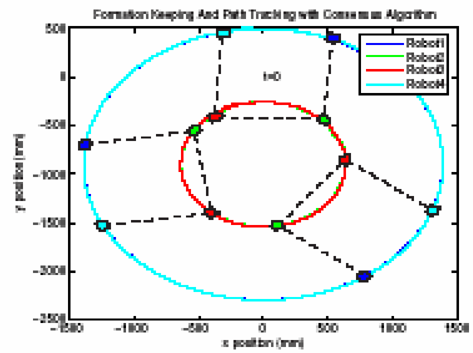
where $v_d^{vc} = \frac{9\pi}{500}$ m/sec, $\omega_d^{vc} = \frac{\pi}{50}$ rad/sec, $(x_d^{vc}(0), y_d^{vc}(0)) = (0, 0)$ m, and $\theta_d^{vc}(0) = 0$ rad.

Let $d_i = 0.15$ m, $x_{jF}^d = \ell_j \cos(\phi_j)$, and $y_{jF}^d = \ell_j \sin(\phi_j)$, where $\ell_j = 0.6$ m and $\phi_j = \pi - \frac{\pi}{4} j$ rad, $j = 1, \ldots, 4$.

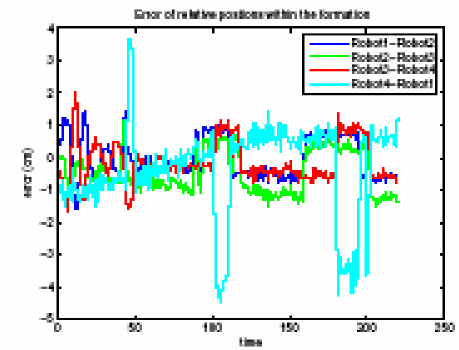
Formation Control with a Simple Group Leader



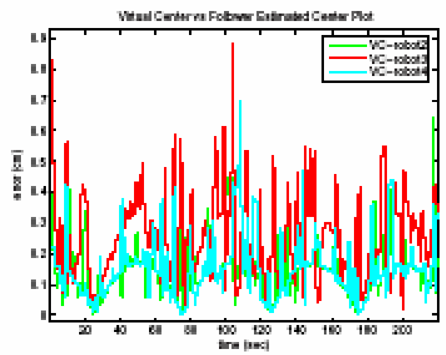
(a) Estimation Topology g_{ij}^{vc} (b) Inter-robot Communication Topology g_{ij}



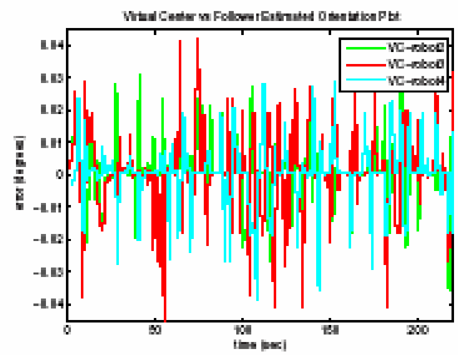
(a) Single Leader with Intelligent Followers Trajectory



(b) Relative Position Error within Formation



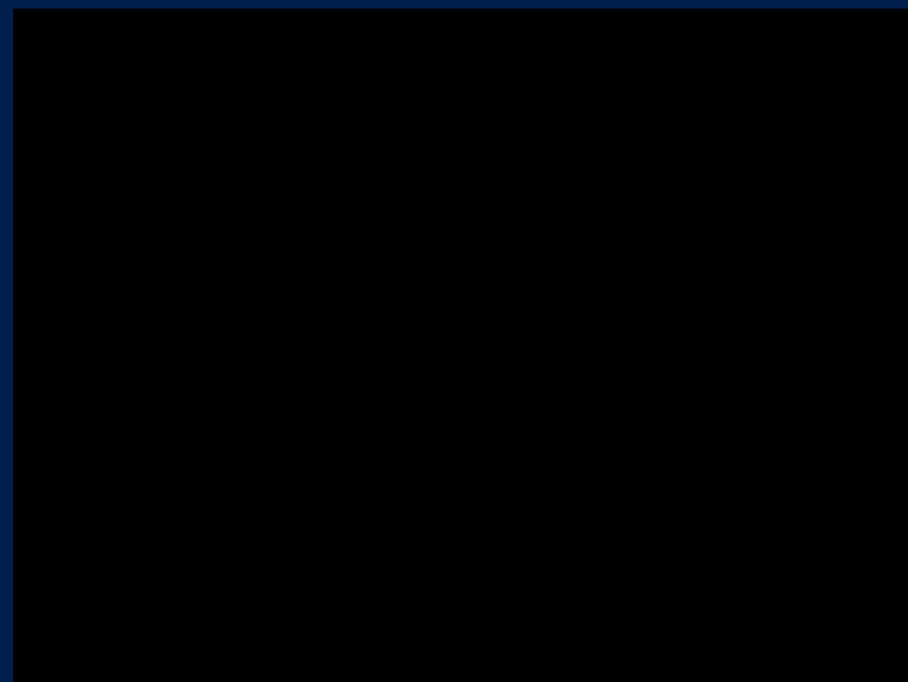
(c) Virtual Center Position Estimation Error



(d) Virtual Center Orientation Estimation Error

Experimental Demonstration (formation control)

Four robots maintaining a square shape



Three robots in line formation

Outline

- Part 1: Consensus for Single-integrator Kinematics – Theory and Applications
- Part 2: Consensus for Double-integrator Dynamics – Theory and Applications
- Part 3: Consensus for Rigid Body Attitude Dynamics – Theory and Applications
- Part 4: Synchronization of Networked Euler-Lagrange Systems – Theory and Applications

Consensus Algorithm for Double-integrator Dynamics

Double-integrator Dynamics: $\dot{\xi}_i = \zeta_i$, $\dot{\zeta}_i = u_i$.

Second-order Algorithm (relative damping):
$u_i = -\sum_{j \in \mathcal{N}_i(t)} [(\xi_i - \xi_j) + \gamma(\zeta_i - \zeta_j)]$ where $\gamma > 0$.

Consensus is *reached* if for all $\xi_i(0)$ and $\zeta_i(0)$, $\xi_i(t) \to \xi_j(t)$ and $\zeta_i(t) \to \zeta_j(t)$ as $t \to \infty$.

Second-order Algorithm (absolute damping):
$u_i = -\sum_{j \in \mathcal{N}_i(t)} (\xi_i - \xi_j) - \gamma \zeta_i$ where $\gamma > 0$.

Consensus is *reached* if for all $\xi_i(0)$ and $\zeta_i(0)$, $\xi_i(t) \to \xi_j(t)$ and $\zeta_i(t) \to 0$ as $t \to \infty$.

Convergence Result:
Consensus is reached if the interaction graph has a directed spanning tree and γ is sufficiently large.

Convergence Analysis (Relative Damping)

Matrix Form:

$$\begin{bmatrix} \dot{\xi} \\ \dot{\zeta} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} 0_{n\times n} & I_n \\ -L & -\gamma L \end{bmatrix}}_{\Gamma} \otimes I_m \right) \begin{bmatrix} \xi \\ \zeta \end{bmatrix},$$

where $\xi = [\xi_1^T, \ldots, \xi_n^T]^T$ and $\zeta = [\zeta_1^T, \ldots, \zeta_n^T]^T$.

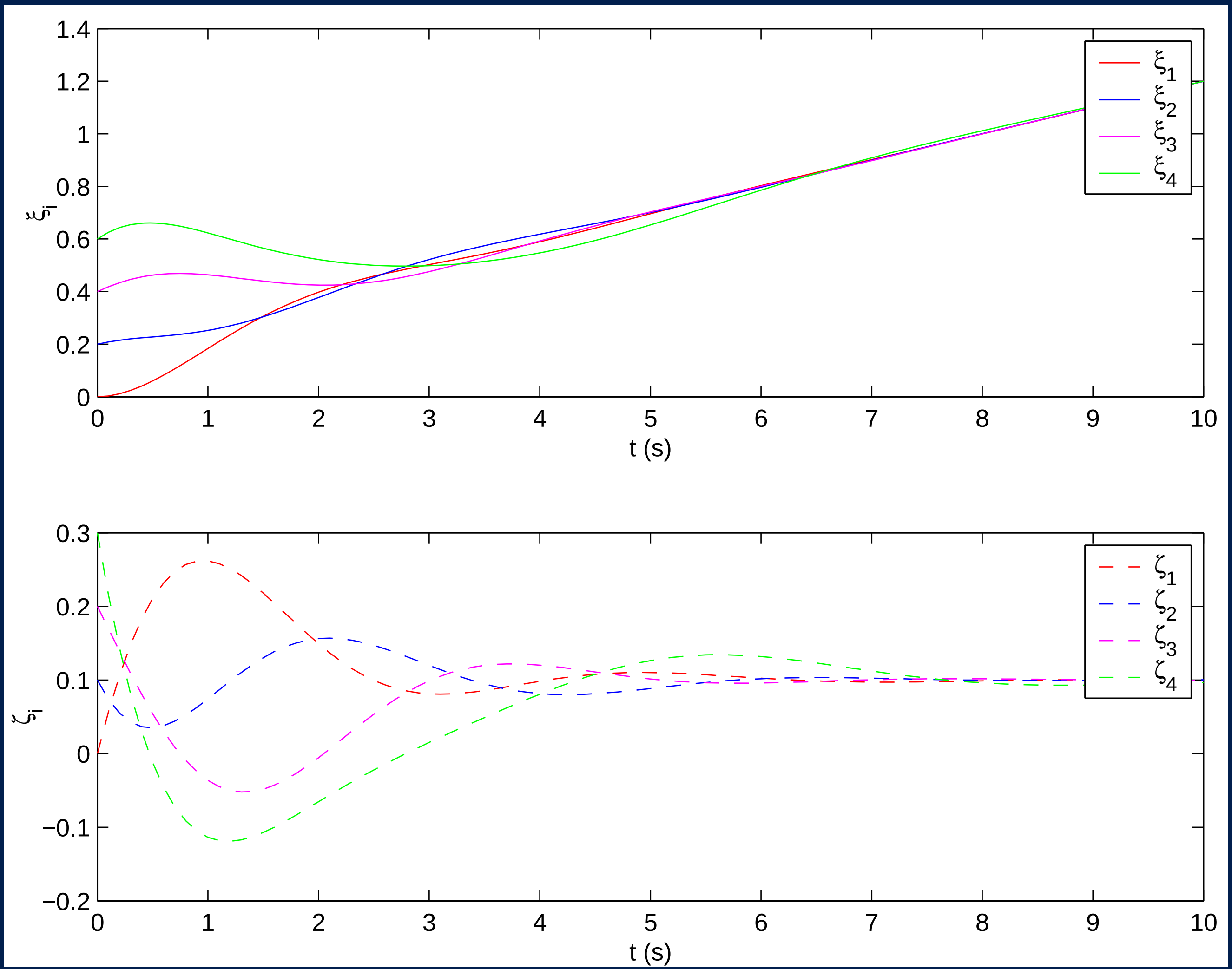
Eigenvalues of Γ:

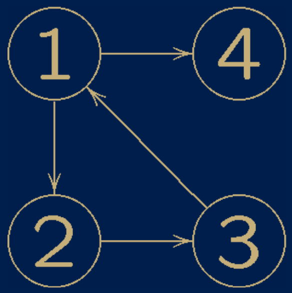
$$\lambda_{i+} = \frac{\gamma\mu_i + \sqrt{\gamma^2\mu_i^2 + 4\mu_i}}{2}, \quad \lambda_{i-} = \frac{\gamma\mu_i - \sqrt{\gamma^2\mu_i^2 + 4\mu_i}}{2},$$

where λ_{i+} and λ_{i-} are called eigenvalues of Γ that are associated with μ_i with μ_i being the ith eigenvalue of $-\mathcal{L}$.

Consensus is reached using the second-order algorithm with relative damping if and only if matrix Γ has exactly two zero eigenvalues and all the other eigenvalues have negative real parts.

Example – with Relative Damping





$$\gamma = 1$$

Switching Topologies – Directed Case

Let $\sigma : [0, \infty) \to \mathcal{P}$ be a piecewise constant switching signal with switching times $t_0, t_1, \cdots$, and $\mathcal{P}$ denotes a set indexing the class of all possible directed interaction graphs for the n vehicles that have a directed spanning tree.

Convergence Result:
Let $t_0, t_1, \cdots$ be the times when the interaction graph switches. Also let τ be the dwell time such that $t_{i+1} - t_i \geq \tau$, $\forall i = 0, 1, \cdots$. If the interaction graph has a directed spanning tree for each $t \in [t_i, t_{i+1})$, the condition for γ is satisfied for each $\sigma \in \mathcal{P}$, and the dwell time τ is sufficiently large, then consensus is reached exponentially.

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Sketch of the Proof

The closed-loop system can be written as

$$\begin{bmatrix} \dot{\xi} \\ \dot{\zeta} \end{bmatrix} = (\Gamma_\sigma \otimes I_m) \begin{bmatrix} \xi \\ \zeta \end{bmatrix},$$

where $\sigma : [0, \infty) \to \mathcal{P}$ is a piecewise constant switching signal with switching times $t_0, t_1, \cdots$, and $\mathcal{P}$ denotes a set indexing the class of all possible directed interaction topologies for the n vehicles that have a directed spanning tree.

Let $\xi_{ij} = \xi_i - \xi_j$ and $\zeta_{ij} = \zeta_i - \zeta_j$ be the consensus error variables. Defining the consensus error vector as $\tilde{\xi} = [\xi_{12}^T, \xi_{13}^T, \cdots, \xi_{1n}^T]^T$ and $\tilde{\zeta} = [\zeta_{12}^T, \zeta_{13}^T, \cdots, \zeta_{1n}^T]^T$, gives $\begin{bmatrix} \dot{\tilde{\xi}} \\ \dot{\tilde{\zeta}} \end{bmatrix} = (\Delta_\sigma \otimes I_m) \begin{bmatrix} \tilde{\xi} \\ \tilde{\zeta} \end{bmatrix}$.

Use a result for switched linear systems in [Morse96].

Application - Cooperative Monitoring with UAVs

Single UAV Experiment

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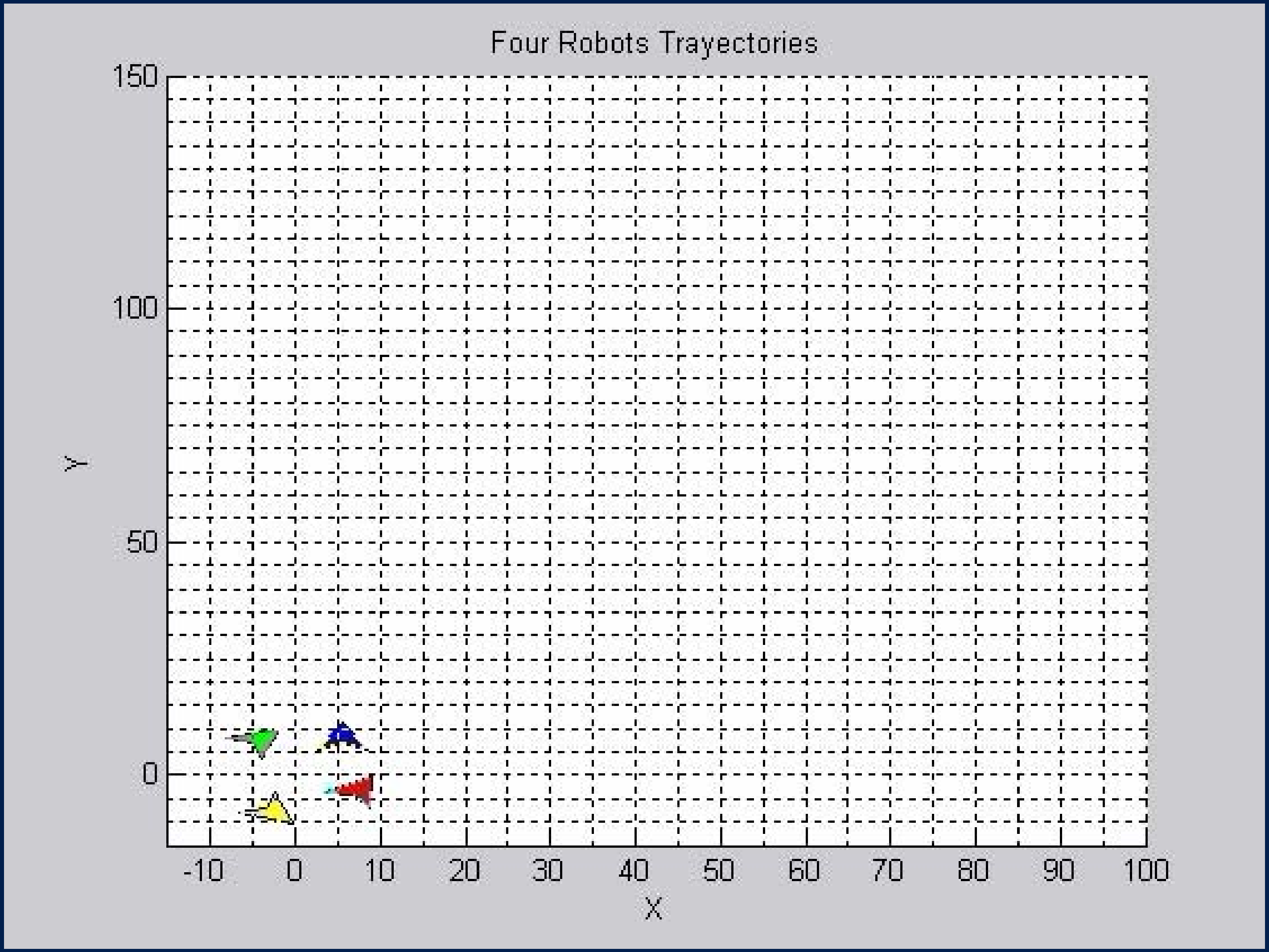
Application - Cooperative Monitoring with UAVs (cont.)



Coupled Second-order Harmonic Oscillators

Leaderless Case:

$u_i = -\alpha\xi_i - \sum_{j\in\mathcal{N}_i(t)}(\zeta_i - \zeta_j)$, where $\alpha > 0$.

Convergence Result:

Suppose that the interaction graph has a directed spanning tree. All ξ_i reach consensus on osicllatory motions, so do all ζ_i.

Leader-following Case:

Virtual leader, labeled as oscillator 0 with states ξ_0 and ζ_0 satisfying $\dot{\xi}_0 = \zeta_0$ and $\dot{\zeta}_0 = -\alpha\xi_0$.

Algorithm:

$u_i = -\alpha\xi_i - \sum_{j\in\mathcal{N}_i(t)}(\zeta_i - \zeta_j) - a_{i0}(t)(\zeta_i - \zeta_0)$, where a_{i0} denotes whether v_0 is available to oscillator i.

Convergence Result:

Suppose that the virtual leader has a directed path to all oscillators. Then $\xi_i(t) \to \xi_0(t)$ and $\zeta_i(t) \to \zeta_0(t)$ for large t.

Convergence Analysis (Leadless Case)

The closed-loop system can be written in matrix form as

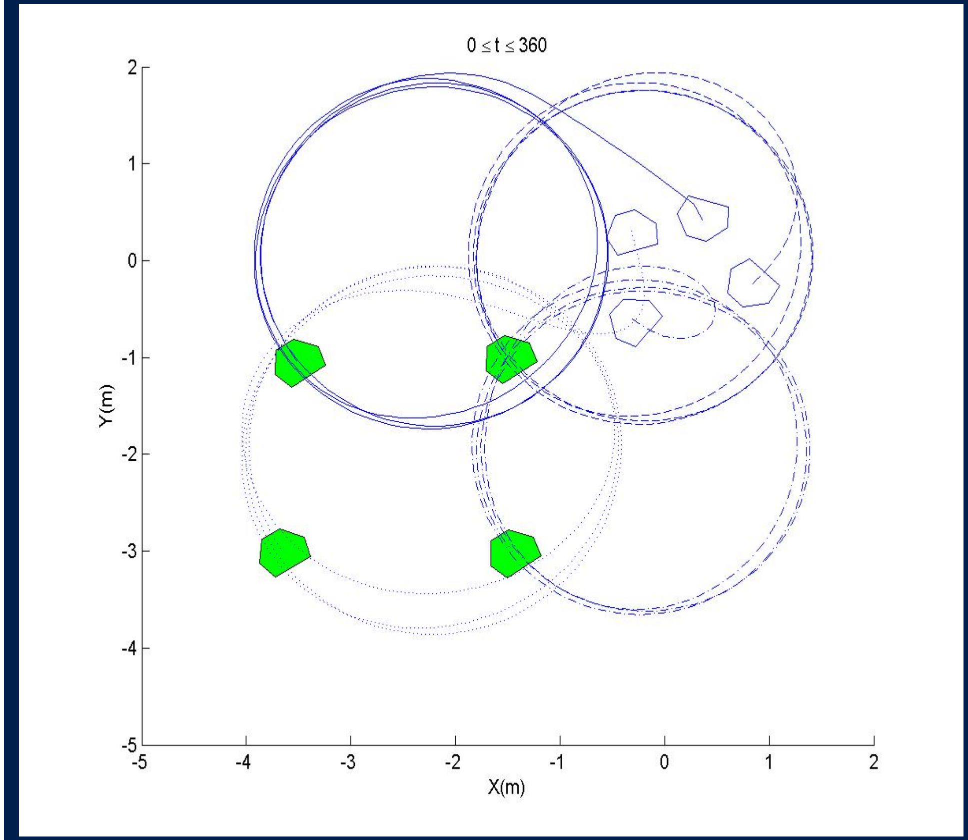
$$\begin{bmatrix} \dot{\xi} \\ \dot{\zeta} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} 0_{n\times n} & I_n \\ -\alpha I_n & -\mathcal{L} \end{bmatrix}}_{\mathcal{Q}} \otimes I_m \right) \begin{bmatrix} \xi \\ \zeta \end{bmatrix},$$

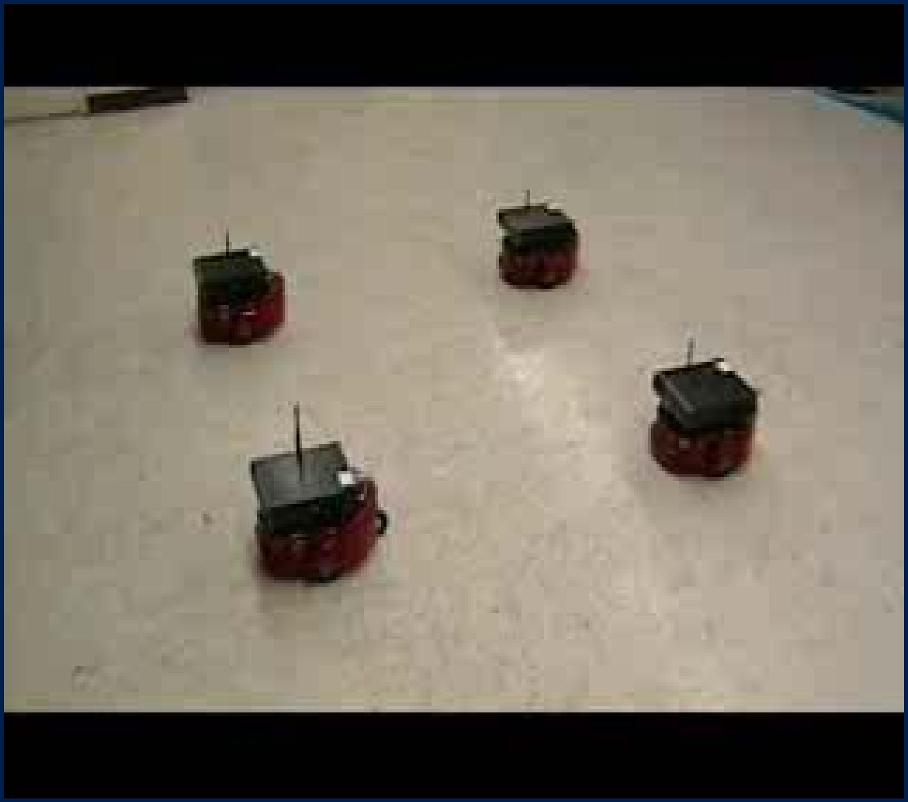
Let μ_i be the ith eigenvalue of $-\mathcal{L}$. The eigenvalues of $\mathcal{Q}$ are given by $\lambda_{i\pm} = \frac{\mu_i \pm \sqrt{\mu_i^2 - 4\alpha}}{2}$.

Directed graph $\mathcal{G}$ has a directed spanning tree implies $\mu_1 = 0$ and $\text{Re}(\mu_i) < 0$, $i = 2, \ldots, n$. Accordingly, $\lambda_{1\pm} = \pm\sqrt{\alpha}\iota$. In addition, $\text{Re}(\lambda_{i\pm}) < 0$, $i = 2, \ldots, n$.

The result then follows from $\begin{bmatrix} \xi(t) \\ \zeta(t) \end{bmatrix} = (e^{\mathcal{Q}t} \otimes I_m) \begin{bmatrix} \xi(0) \\ \zeta(0) \end{bmatrix}$.

Application - Synchronized Motion Coordination





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Consensus for Rigid Body Attitude Dynamics

Rigid body attitude dynamics:

$$\dot{\hat{q}}_i = -\frac{1}{2}\omega_i \times \hat{q}_i + \frac{1}{2}\bar{q}_i\omega_i, \qquad \dot{\bar{q}}_i = -\frac{1}{2}\omega_i \cdot \hat{q}_i$$
$$J_i\dot{\omega}_i = -\omega_i \times (J_i\omega_i) + \tau_i.$$

Control Torque:

$$\tau_i = -k_G\widehat{q^{d*}q_i} - d_G\omega_i - \sum_{j\in\mathcal{N}_i}[a_{ij}\widehat{q_j^*q_i} + b_{ij}(\omega_i - \omega_j)],$$

where $k_G, d_G, a_{ij}, b_{ij} > 0$ and q^d denotes the desired attitude for the team.

Convergence Result:

Suppose that the interaction graph is undirected. If $k_G > 2\sum_{j\in\mathcal{N}_i} a_{ij}$, then $q_i(t) \to q^d$ and $\omega_i(t) \to 0$. If $k_G = 0$ and the undirected graph is a tree, then $q_i(t) \to q_j(t)$ and $\omega_i(t) \to 0$.

Sketch of the Proof

Consider a Lyapunov function candidate:

$$V = k_G \sum_{i=1}^{n} \left\| q^{d*} q_i - \mathbf{q}_I \right\|^2 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} a_{ij} \left\| q_j^* q_i - \mathbf{q}_I \right\|^2 + \frac{1}{2} \sum_{i=1}^{n} (\omega_i^T J_i \omega_i),$$

where $\mathbf{q}_I = [0, 0, 0, 1]^T$.

The proof is then based on Lyapunov theory and LaSalle's invariance principle.

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Unavailability of Angular Velocity Measurements

Control Torque:

$\dot{\hat{x}}_i = \Gamma \hat{x}_i + \sum_{j\in\mathcal{N}_i} b_{ij}(q_i - q_j) + \kappa q_i$

$y_i = P\Gamma \hat{x}_i + P\sum_{j\in\mathcal{N}_i} b_{ij}(q_i - q_j) + \kappa P q_i$

$\tau_i = -k_G \widehat{q^{d*} q_i} - \sum_{j\in\mathcal{N}_i} a_{ij} \widehat{q_j^* q_i} - \widehat{q_i^* y_i},$

where $\Gamma \in \mathbb{R}^{4\times 4}$ is Hurwitz and κ is a positive scalar.

Convergence Result:

Suppose that the interaction graph is undirected.

If $k_G > 2\sum_{j\in\mathcal{N}_i} a_{ij}$, then $q_i(t) \to q^d$ and $\omega_i(t) \to 0$.

Sketch of the Proof

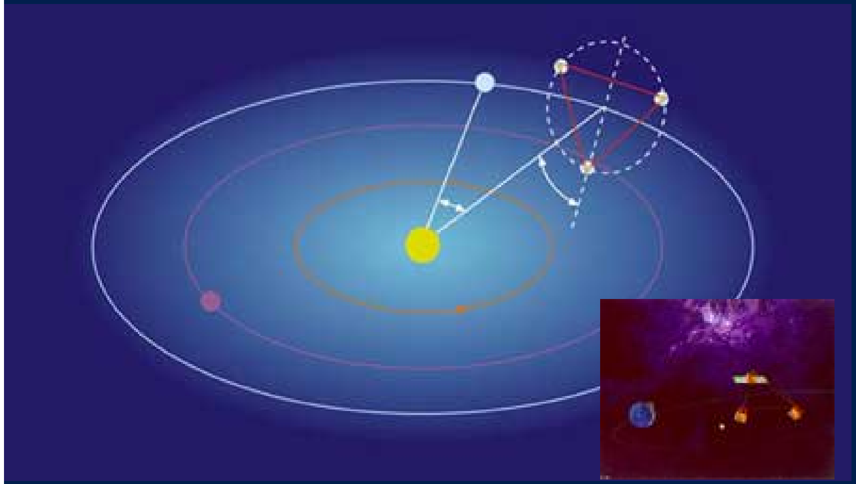
Consider the Lyapunov function candidate

$$V = k_G \sum_{i=1}^{n} \left\| q^{d*} q_i - \mathbf{q}_I \right\|^2 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} a_{ij} \left\| q^{d*} q_i - q^{d*} q_j \right\|^2$$
$$+ \frac{1}{2} \sum_{i=1}^{n} (\omega_i^T J_i \omega_i) + \dot{\hat{x}}^T (M \otimes I_4)^{-1} (I_n \otimes P) \dot{\hat{x}},$$

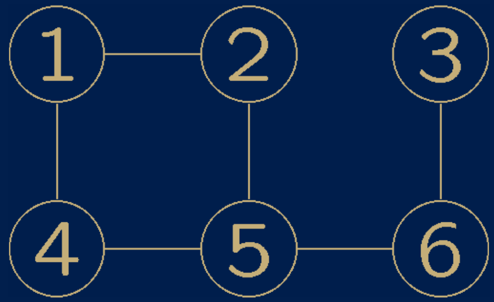
where $\hat{x} = [\hat{x}_1^T, \ldots, \hat{x}_n^T]^T$, $M = \mathcal{L}_B + \kappa I_n$ with $\mathcal{L}_B = [\ell_{ij}] \in \mathbb{R}^{m \times m}$ defined as $\ell_{ij} = -b_{ij}$ and $\ell_{ii} = \sum_{j \neq i} b_{ij}$.

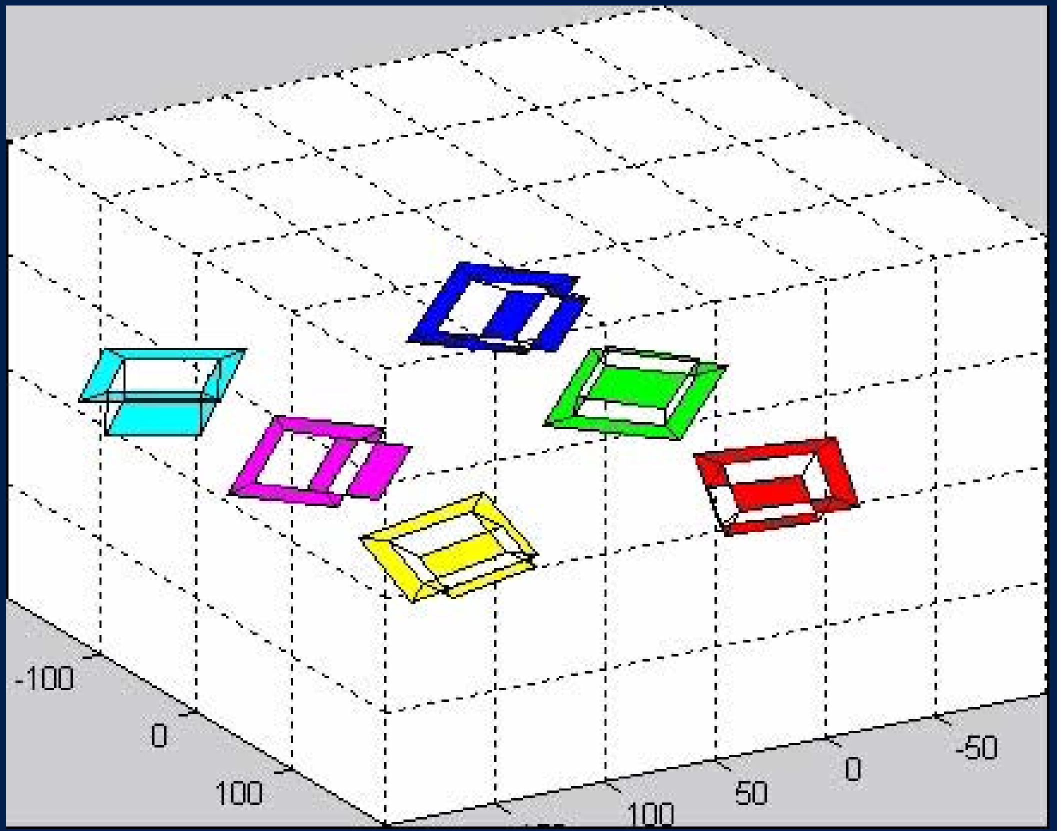
The proof is then based on Lyapunov theory and LaSalle's invariance principle.

Application - Spacecraft Attitude Synchronization



Synchronized spacecraft rotations





Experimental Demonstration (attitude control)

Reference Attitude Tracking

Let $q^r(t) \in \mathbb{R}^4$ and $\omega^r(t) \in \mathbb{R}^3$ denote, respectively, the (time-varying) reference attitude and angular velocity, which satisfy the rotational dynamics given by

$$\dot{\hat{q}}^r = -\frac{1}{2}\omega^r \times \hat{q}^r + \frac{1}{2}\overline{q}^r\omega^r, \qquad \dot{\overline{q}}^r = -\frac{1}{2}\omega^r \cdot \hat{q}^r$$
$$J^d\dot{\omega}^r = -\omega^d \times (J^r\omega^r) + \tau^r.$$

Objective:

$q_i(t) \to q^r(t)$ and $\omega_i(t) \to \omega^r(t)$.

Reference Attitude Tracking Control Law

Control Torque:

$$\tau_i = \omega_i \times (J_i\omega_i) + \frac{1}{|\mathcal{N}_i|+1}J_i(\dot{\omega}^r + \sum_{j\in\mathcal{N}_i}\dot{\omega}_j)$$
$$- \frac{1}{|\mathcal{N}_i|+1}\{k_{qi}\widehat{p_{\pi_i}} + K_{\omega i}[(\omega_i - \omega^r) + \sum_{j\in\mathcal{N}_i}(\omega_i - \omega_j)]\}, \; i \in i_L$$

$$\tau_i = \omega_i \times (J_i\omega_i) + \frac{1}{|\mathcal{N}_i|}J_i\sum_{j\in\mathcal{N}_i}\dot{\omega}_j - \frac{1}{|\mathcal{N}_i|}[k_{qi}\widehat{q_{\pi_i}} + \sum_{j\in\mathcal{N}_i}K_{\omega i}(\omega_i - \omega_j)], \; i \notin i_L$$

where i_L denotes the set of vehicles to which q^r, ω^r and $\dot{\omega}^r$ are available, $k_{qi} > 0$, $K_{\omega i} \in \mathbb{R}^{3\times 3} > 0$, $p_{\pi_i} = [\prod_{j\in\mathcal{N}_i}(q_j^* q_i)]q^{r*}q_i$, and $q_{\pi_i} = \prod_{j\in\mathcal{N}_i}(q_j^* q_i)$.

Convergence Result:

$q_i(t) \to q^r(t)$ and $\omega_i(t) \to \omega^r(t)$ as $t \to \infty$ if and only if the leader has a directed path to all followers in the team and the directed graph can be simplied.

Sketch of the Proof

Let $q_{n+1} \equiv q^r$ and $\omega_{n+1} \equiv \omega^r$. Also let $\mathcal{J}_i = \mathcal{N}_i$ if $i \notin i_L$ and $\mathcal{J}_i = \mathcal{N}_i \cup \{n+1\}$ if $i \in i_L$. The control torque can be rewritten as

$$\tau_i = \omega_i \times (J_i \omega_i) + \frac{1}{|\mathcal{J}_i|} J_i \sum_{j \in \mathcal{J}_i} \dot{\omega}_j - \frac{1}{|\mathcal{J}_i|}[k_{qi} \widehat{s_{\pi_i}} + K_{\omega i} \sum_{j \in \mathcal{J}_i} (\omega_i - \omega_j)],$$

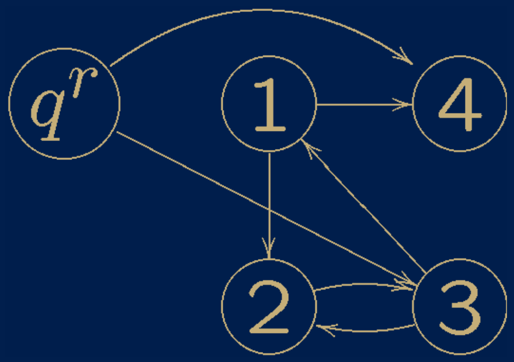
where $s_{\pi_i} = \prod_{j \in \mathcal{J}_i} (q_j^* q_i)$.

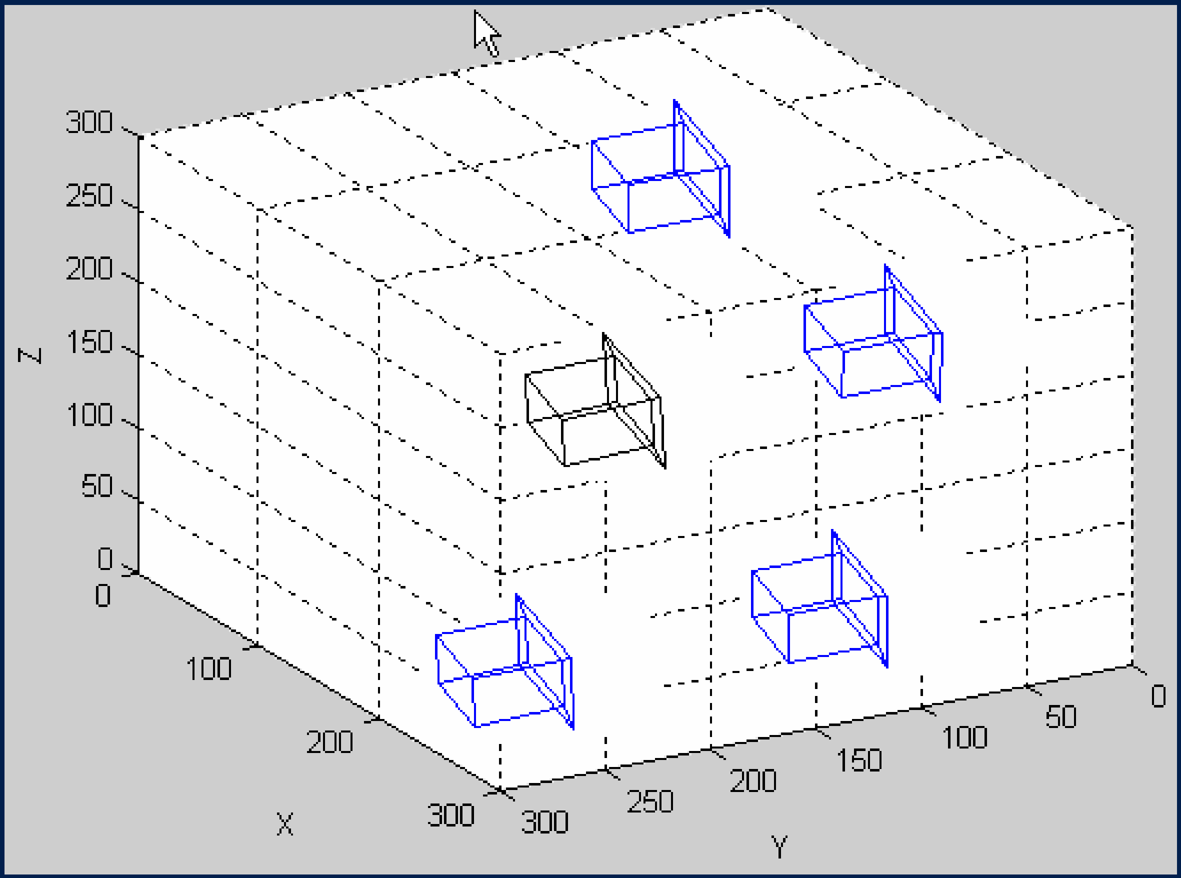
Combining the rotation dynamics and the control torque, gives

$$J_i \dot{\omega}_{\sigma_i} = -k_{qi} \widehat{s_{\pi_i}} - K_{\omega i} \omega_{\sigma_i}, \quad i = 1, \cdots, n,$$

where $\omega_{\sigma_i} = \sum_{j \in \mathcal{J}_i} (\omega_i - \omega_j)$. Note that the quaternion and angular velocity pair $(s_{\pi_i}, \omega_{\sigma_i})$ satisfies the quaternion kinematics. It follows that $\widehat{s_{\pi_i}} \to 0$ and $\omega_{\sigma_i} \to 0$, $i = 1, \cdots, n$. The result then follows the property of the nonsymmetric Laplacian matrix.

Application - Spacecraft Reference Attitude Tracking





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Synchronization of Networked Euler-Lagrange Systems

Euler-Lagrange systems are represented by

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g(q_i) = \tau_i, \quad i = 1,\dots,n,$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the symmetric positive-definite inertia matrix, $C_i(q_i,\dot{q}_i)\dot{q}_i \in \mathbb{R}^p$ is the vector of Coriolis and centrifugal torques, $g(q_i)$ is the vector of gravitational torques, and τ_i is the vector of torques produced by the actuators associated with the ith system.

Control Torque:

$\tau_i = g(q_i) - \sum_{j\in\mathcal{N}_i}[k_q(q_i - q_j) + k_{\dot{q}}(\dot{q}_i - \dot{q}_j)] - K_i\dot{q}_i$,
where $k_q, k_{\dot{q}} > 0$, and $K_i \in \mathbb{R}^{p\times p}$ is symmetric positive definite.

Convergence Result:

$q_i(t) \to q_j(t)$ and $\dot{q}_i(t) \to 0$ as $t \to \infty$ if the undirected interaction graph is connected.

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