Optimal Design of Formations for Wireless Networks of Mobile and Static Agents

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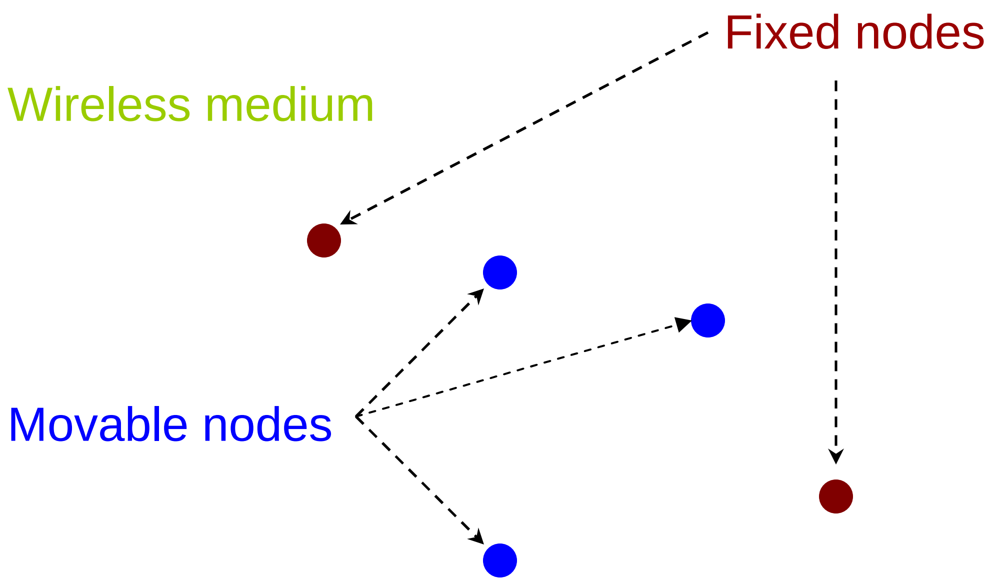
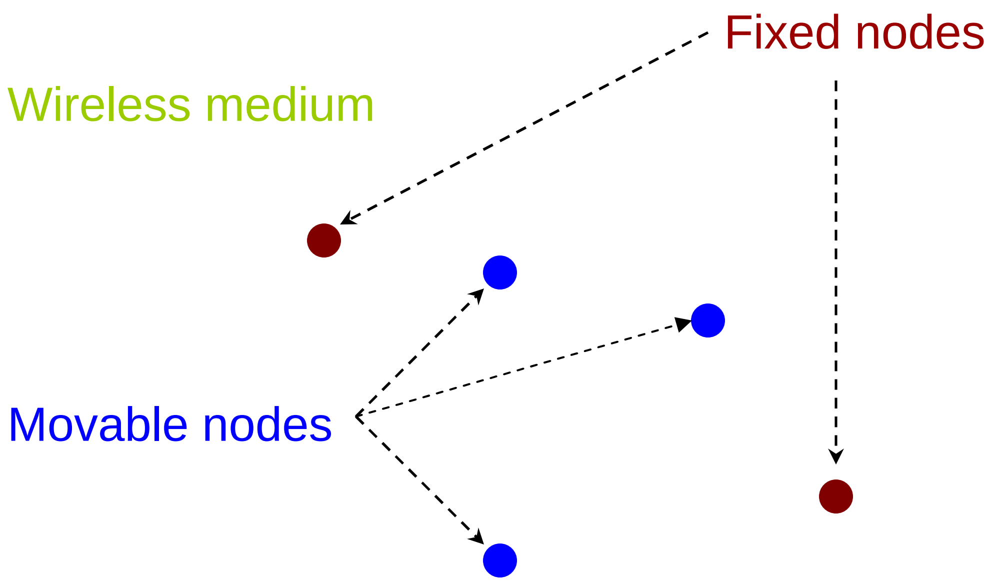
Outline

- Introduction to the problem
- Modeling assumptions (Path loss and interference/channel assignment)
- Description of our optimization paradigm
- Examples of design problems
- Basic optimality properties
- Example of a distributed implementation
- Numerical results
- Conclusions

Introduction to the Problem



Introduction to the Problem



Given network-centric constraints and a cost function, we want to optimize with respect to the following variables:

1. Positions of the movable nodes

2. At each node: allocation of transmission power for communication with other nodes.

Prior work on optimal placement

Coverage: Cortes, Martinez and Bullo (2004, 2005)

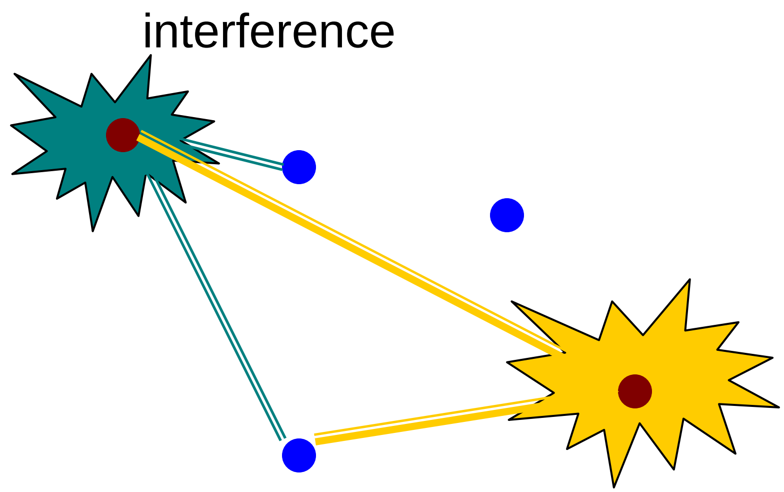
Minimum power sensor networks Xing et all (2007)

Optimal placement in the absence of interference, Boyd (2004, book)

From the CS community:

Minimal relay placement in sensor networks, Wang et all (2005) also Hou 2005

Modeling assumptions

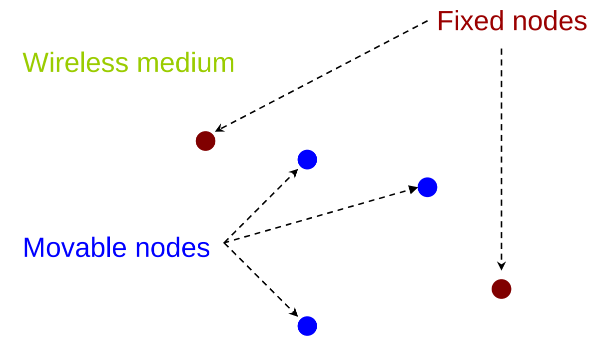


Assumption 1: Each node has a distinct reception channel. More than one source node can transmit to the same destination node via channel multiplexing (CDMA).

Assumption 2: There is no inter-channel interference, but distinct source nodes will interfere when transmitting to the same destination node.

Description of the optimization paradigm

Optimization variables:



$\mathbf{P}_{i \to k}$ power (dBmW) received at node *k* from node *i*

$(\mathbf{x}_i, \mathbf{y}_i)$ positions for the movable nodes

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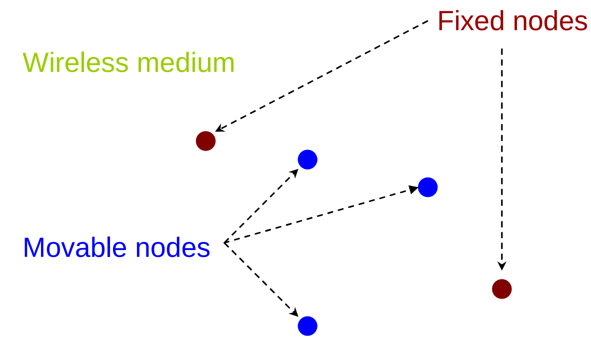

$\mathbf{P}_{i\rightarrow k}$ power (dBmW) received at node *k* from node *i*

$(\mathbf{x}_i, \mathbf{y}_i)$ positions for the movable nodes

Consider the following class of functions:

$$\mathcal{P}(\mathbf{F}) = \sum_{k=1}^{\Gamma} \varsigma_k 10^{\sum_{l=1}^{\Xi \quad \Phi} \xi_{l,k} \mathbf{z}_l + \sum_{i=1,j=1}^{\Delta+\Omega} (\beta_{i,j,k} \mathbf{P}_{i\rightarrow j} + \tau_{i,j,k} \mathbf{e}_{i,j})}$$

where,

ς_k and $\tau_{i,j,k}$ are positive real constants

$\beta_{i,j,k}$ and $\xi_{l,k}$ are real constants

$\mathbf{z}_l$ auxiliary variables

$\mathbf{d}_{i,j}$ Euclidean distance between nodes *i* and *j*

Description of the optimization paradigm

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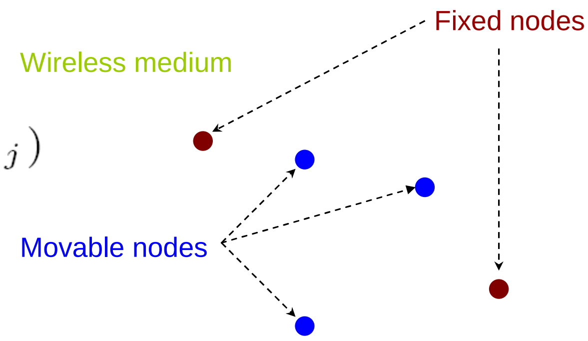


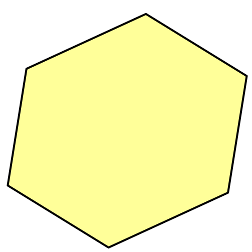
$$\mathcal{P}\left(\mathbf{F}\right) = \sum_{k=1}^{\Gamma} \varsigma_k 10^{\sum_{l=1}^{\Xi} {}^{\Phi}\varsigma_{l,k} \mathbf{z}_l + \sum_{i=1,j=1}^{\Delta+\Omega} (\beta_{i,j,k} \mathbf{P}_{i \to j} + \tau_{i,j,k} \mathbf{e}_{i,j})}$$

$$\mathbf{F}^* = \arg\min_{F} \mathcal{U}(\mathbf{F})$$

Subject to:

$$\mathcal{P}_k\left(\mathbf{F}\right) \leq 1,\ \ k \in \{1, \ldots, \Phi\}$$

$$\zeta_{i,k} \mathbf{x}_i + \vartheta_{i,k} \mathbf{y}_i \leq 1,\ \ (i,k) \in \mathbb{S} \times \{1, \ldots, \Gamma\}$$



Polyhedral convex set placement constraints

Design examples that fit our framework

Consider the following class of functions:

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SINR constraints:

$$\mathbf{S}_{i\rightarrow j} \stackrel{def}{=} \frac{\eta_{i,i} 10^{0.1\mathbf{P}_{i\rightarrow j}}}{\sum_{k\in\{1,\ldots,\Delta+\Omega\}-\{i,j\}} \eta_{k,i} 10^{0.1\mathbf{P}_{k\rightarrow j}} + \sigma_N^2}$$

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$$\prod_{(i,j)\in\{1,\ldots,\Delta+\Omega\}^2} \left(\kappa \mathbf{S}_{i\rightarrow j}\right)^{\frac{\varrho_{i,j,k}}{\Upsilon}} \geq 2^{\lambda_k}$$

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Using high SIR formula (see Chiang book):

$$\mathbf{R}_{i \to j} = \frac{1}{\Upsilon} \log_{10}\left(\kappa \mathbf{S}_{i \to j}\right)$$

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Networked control necessary and sufficient conditions for stabilizability
Tatikonda (2003), Yuksel (in press)

Omniscience and secret key generation in the presence of an overlay node
Wyner et all (2002) , Csiszar and Narayan (2004)

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$$\sum_{(i,j)\in\{1,\ldots,\Delta+\Omega\}^2} \varrho_{i,j,k} \mathbf{R}_{i\to j} \geq \lambda_k$$

Similarly, we can deal with path outage probability constraints

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V. Gupta and N. C. Martins, “Optimal tracking control across erasure communication links, in the presence of preview,” to appear in the IJRNC

V. Gupta and N. C. Martins, J. S. Baras “Optimal Output Feedback Control Using TwoRemote Sensors over Erasure Channels,” to appear in the IEEE TAC

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Subject to:

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Power constraints (dBmW):

$$\mathbf{P}_i^{total} \leq \Psi$$

$$\mathbf{P}_i^{total} = 10\log_{10}\left[\sum_{k\in\{1,\ldots,\Delta+\Omega\}-\{i\}} \phi 10^{0.1\mathbf{P}_{i\to k} + \alpha\mathbf{d}_{i,k}}\right]$$

Under exponential power path loss (Ack: A. Swami and B. Sadler at ARL)

About exponential power loss ….

Sub sea radio frequency networks

Immune to noise due to turbulence (good for mobility)

Low delay (Good for synchronization)

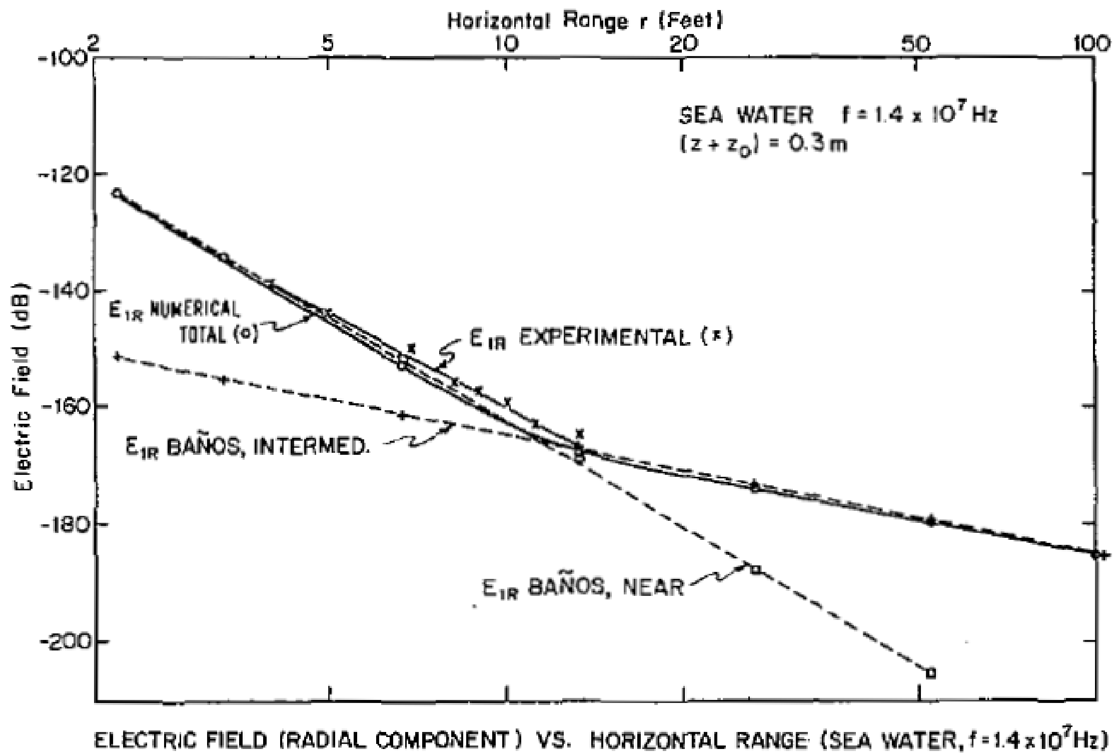

Fig. 9. Electric field versus horizontal range (radial component, 14 MHz).

M. Siegel and R. W. P. King, *"Electromagnetic Propagation Between Antennas Submerged in the Ocean,"* IEEE Transactions on Antennas and Propagation, Vol AP-21, No 4, July 1973

Design examples that fit our framework

About exponential power loss ….

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Radio Acoustic Modem Pair (surface unit package)

Design examples that fit our framework

About exponential power loss ….

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Technical Specification

Performance

Radio

- Range: seawater: up to 50 m
  - Through seabed: up to 1km
  - Seawater 20 m + 200 m in air

Acoustic

- Range: Up to 10 km

Antenna

Radio

- Magnetic coupled loop
- Options available for extended range

Acoustic

- Omni-directional acoustic transducer

Data

- 100 bps data rate
- RS232 data interface
- 19,200 baud
- Analogue interface for sensors without a microprocessor
- Spread spectrum modulation scheme

Power Requirements

- Power through external connector
- 24 Vdc supply
- Receive 36 mW (Typ. 1.5 mA @ 24 VDC)
- Transmit 36 W (Typical 1.5 A @ 24 VDC)
- standby mode with wakeup timer

Physical (typical)

- Submersible housing-115mm x 315 mm
- Radio Antenna – 500 mm loop diameter
- Acoustic Transducer – 105mmx75mm

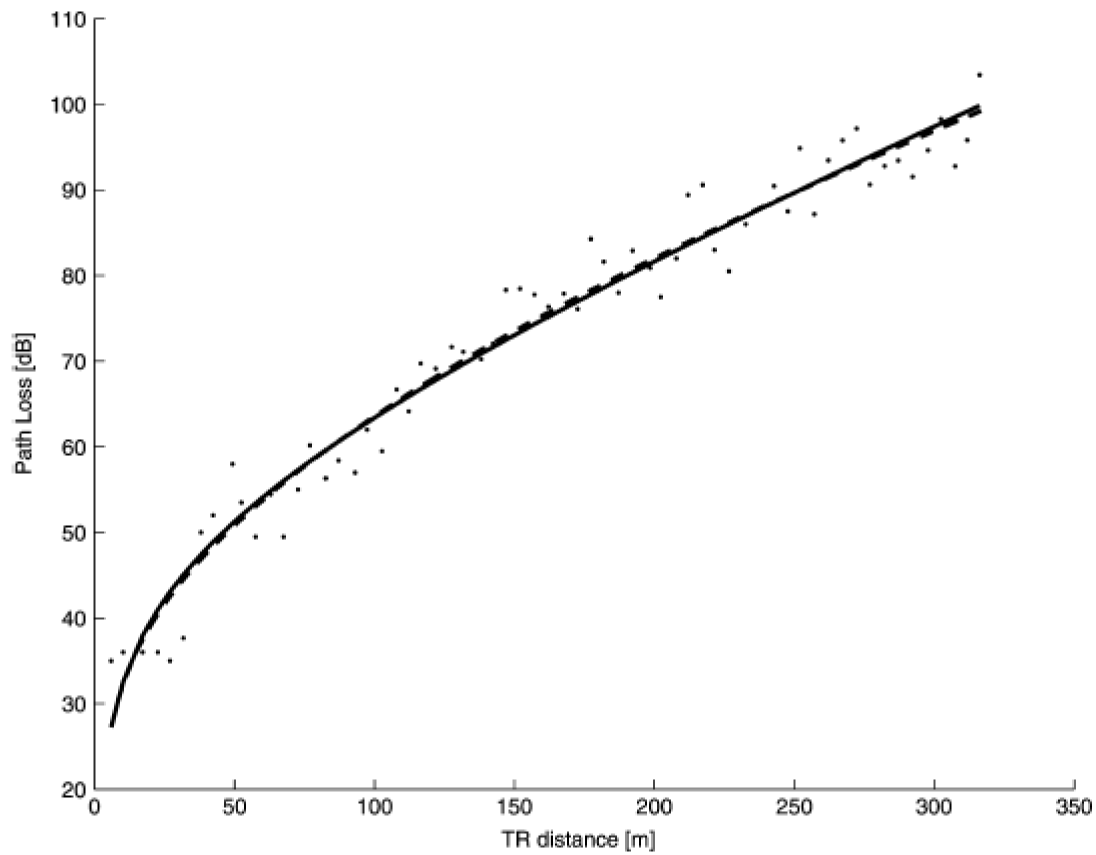
Environmental

- 1,000m depth (50 m depth option)
- Temp. operating -10 to + 35°C

Design examples that fit our framework

About exponential power loss ….

Urban areas



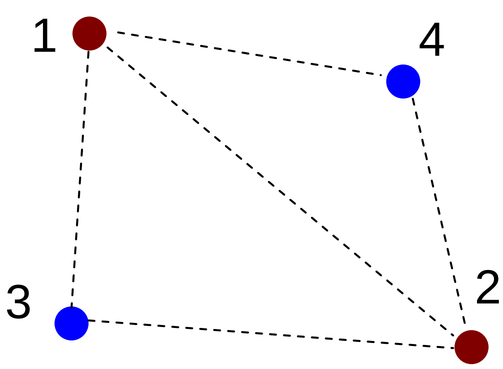
M. Franceschetti, J. Bruck and L. J. Schulman, *“A Random Walk Model of Wave Propagation,”* IEEE Transactions on Antennas and Propagation, Vol 52, No 5, May 2004

Design examples that fit our framework

- We are given the parameters ϕ and α that are needed in the total power formula

$$\mathbf{P}_i^{total} = 10\log_{10}\left[\sum_{k\in\{1,\ldots,\Delta+\Omega\}-\{i\}} \phi 10^{0.1\mathbf{P}_{i\to k}+\alpha\mathbf{e}_{i,k}}\right]$$

- We pre-specify a positive real constant Ψ representing the maximal power available at each node.
- We are given the positions of the fixed nodes (χ_1, γ_1) and (χ_2, γ_2).



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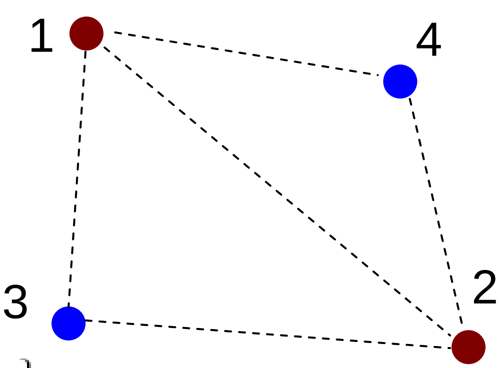

We want:

maximize $\min\{\mathbf{R}_{1\to 2}^{total}, \mathbf{R}_{2\to 1}^{total}\}$

where

$$\mathbf{R}_{1\to 2}^{total} = \mathbf{R}_{1\to 2} + \underbrace{\min\{\mathbf{R}_{1\to 3}, \mathbf{R}_{3\to 2}\}}_{(A)-\text{route via node 3}} + \underbrace{\min\{\mathbf{R}_{1\to 4}, \mathbf{R}_{4\to 2}\}}_{(B)-\text{route via node 4}}$$

$$\mathbf{R}_{2\to 1}^{total} = \mathbf{R}_{2\to 1} + \underbrace{\min\{\mathbf{R}_{2\to 3}, \mathbf{R}_{3\to 1}\}}_{(C)-\text{route via node 3}} + \underbrace{\min\{\mathbf{R}_{2\to 4}, \mathbf{R}_{4\to 1}\}}_{(D)-\text{route via node 4}}$$

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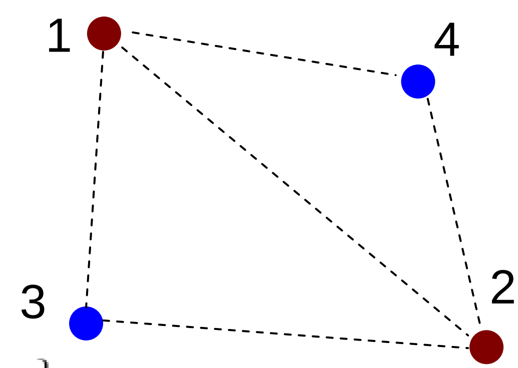

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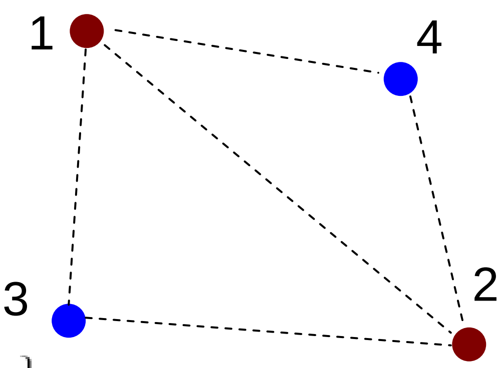
Subject to: $\mathbf{P}_i^{total} \leq \Psi,\ i \in \{1, \ldots, 4\}$



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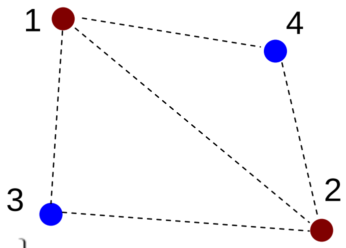
Subject to: $\mathbf{P}_i^{total} \leq \Psi,\ i \in \{1, \dots, 4\}$

Under exponential path loss and the following high SINR formula (Chiang book), we can cast the above problem in our framework:

$$\mathbf{R}_{i\to j} = \frac{1}{\Upsilon} \log_{10}\left(\kappa \mathbf{S}_{i\to j}\right)$$

Expressing the example in our framework



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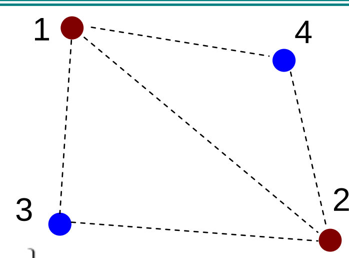
Expressing the example in our framework



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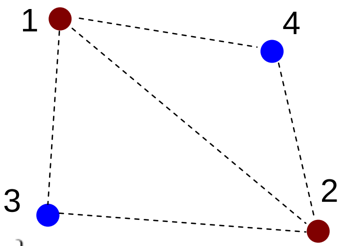
Expressing the example in our framework



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$$\mathbf{R}_{i\to j} = \frac{1}{\Upsilon} \log_{10}\left(\kappa \mathbf{S}_{i\to j}\right)$$

Consider the following class of functions:

$$\mathcal{P}\left(\mathbf{F}\right) = \sum_{k=1}^{\Gamma} \varsigma_k 10^{\sum_{l=1} \xi_{l,k} \mathbf{z}_l + \sum_{i=1,j=1} \beta_{i,j,k} \mathbf{P}_{i\to j} + \tau_{i,j,k} \mathbf{d}_{i,j}}$$

$$\mathbf{F}^* = \arg\min_{F} \mathcal{U}(\mathbf{F})$$

Subject to:

$$\mathcal{P}_k\left(\mathbf{F}\right) \leq 1, \quad k \in \{1, \ldots, \Phi\}$$

$$\min 10^{-\mathbf{z}_1}$$

$$\mathbf{z}_2 \geq \mathbf{z}_1 \text{ and } \mathbf{z}_3 \geq \mathbf{z}_1$$

$$\underbrace{\kappa \mathbf{S}_{1\to 2}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{1\to 2}}} \geq 10^{\mathbf{z}_2 - \mathbf{z}_4 - \mathbf{z}_5} \text{ and } \underbrace{\kappa \mathbf{S}_{2\to 1}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{2\to 1}}} \geq 10^{\mathbf{z}_3 - \mathbf{z}_6 - \mathbf{z}_7}$$

$$\underbrace{\kappa \mathbf{S}_{1\to 3}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{1\to 3}}} \geq 10^{\mathbf{z}_4} \text{ and } \underbrace{\kappa \mathbf{S}_{3\to 2}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{3\to 2}}} \geq 10^{\mathbf{z}_4}$$

$$\underbrace{\kappa \mathbf{S}_{1\to 4}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{1\to 4}}} \geq 10^{\mathbf{z}_5} \text{ and } \underbrace{\kappa \mathbf{S}_{4\to 2}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{4\to 2}}} \geq 10^{\mathbf{z}_5}$$

$$\underbrace{\kappa \mathbf{S}_{2\to 3}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{2\to 3}}} \geq 10^{\mathbf{z}_6} \text{ and } \underbrace{\kappa \mathbf{S}_{3\to 1}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{3\to 1}}} \geq 10^{\mathbf{z}_6}$$

$$\underbrace{\kappa \mathbf{S}_{2\to 4}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{2\to 4}}} \geq 10^{\mathbf{z}_7} \text{ and } \underbrace{\kappa \mathbf{S}_{4\to 1}^{\frac{1}{\Upsilon}}}_{10^{\mathbf{R}_{4\to 1}}} \geq 10^{\mathbf{z}_7}$$

$$10^{\mathbf{P}_i^{total} - \Psi} \leq 1, \quad i \in \{1, \ldots, 4\}$$

Basic optimality properties

Consider the following class of functions:

$$\mathcal{P}\left(\mathbf{F}\right) = \sum_{k=1}^{\Gamma} \varsigma_k 10^{\sum_{l=1} \xi_{l,k} \mathbf{z}_l + \sum_{i=1,j=1} \beta_{i,j,k} \mathbf{P}_{i \to j} + \tau_{i,j,k} \mathbf{d}_{i,j}}$$

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- Our optimization problem in its general form is convex.

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- Our optimization problem in its general form is convex.
- Using a linear programming approximation of the Euclidean distance, we can cast the resulting optimization as a Geometric program.
- Many instances of our general framework admit a distributed implementation based on primal/dual recursion.

Example of distributed implementation

Consider the following cost:

$$\mathcal{U}(\mathbf{Q}) = \sum_{i=1}^{\Delta+\Omega} \varphi_i 10^{0.1\mathbf{P}_i^{total}}$$

We want

$$\mathbf{Q}^* = \arg\min_{\mathbf{Q}} \mathcal{U}(\mathbf{Q})$$

Subject to:

$$\mathcal{R}(\mathbf{S}_{i\to j}) \geq \varrho_{i\to j}, \ (i,j) \in \bigcup_{i=1}^{\Delta+\Omega} \{i\} \times \mathbb{O}(i)$$

$$\mathbf{P}_i^{total} \leq \Psi_i, \ i \in \{1,\ldots,\Delta+\Omega\}$$

Δ - Fixed nodes

Ω - Movable nodes

$\{\mathbb{O}(i)\}_{i=1}^{\Delta+\Omega}$

Destination nodes

$$\mathbf{P}_i^{total} = 10\log_{10}\left[\sum_{k\in\{1,\ldots,\Delta+\Omega\}-\{i\}} \phi 10^{0.1\mathbf{P}_{i\to k}+\alpha\mathbf{e}_{i,k}}\right]$$

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We want

$$\mathbf{Q}^* = \arg\min_{\mathbf{Q}} \mathcal{U}(\mathbf{Q})$$

Subject to:

$\Delta+\Omega$

$$\mathcal{R}(\mathbf{S}_{i\rightarrow j}) \geq \varrho_{i\rightarrow j}$$

$$\mathbf{P}_i^{total} \leq \Psi_i,$$

Invertible

$$\mathcal{R}(\mathbf{S}_{i\rightarrow j}) = \frac{1}{\Upsilon} \log_{10}\left(\kappa \mathbf{S}_{i\rightarrow j}\right)$$

$$\mathcal{R}(\mathbf{S}_{i\rightarrow j}) = \frac{1}{\Upsilon} \log_{10}\left(1 + \kappa \mathbf{S}_{i\rightarrow j}\right)$$

Example of distributed implementation

Consider the following cost:

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$$\mathbf{S}_{i\to j} \geq \mathcal{R}^{-1}(\varrho_{i\to j}),\ (i,j) \in \bigcup_{i=1}^{\Delta+\Omega} \{i\} \times \mathbb{O}(i)$$

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Ω - Movable nodes

$\{\mathbb{O}(i)\}_{i=1}^{\Delta+\Omega}$

Destination nodes

Primal-dual recursion: basic properties

F. Kelly, A. Mauloo and D. Tan, *"Rate control for communication networks: shadow prices, proportional fairness and stability,"* Journal of Operational Research Society, vol 49, no 3, pp. 237-252, Mar 1998

G. B. Danzig, P. Wolfe, *"The decomposition algorithm for linear programming,"* Econometrica, 4, 1961

H. Everett, *"Generalized lagrange multiplier method for solving problems of optimum allocation of resources,"* Operations Research, 11(3):399417, 1963.

M. Chiang, S. H. Low, A. R. Calderbank, J. C. Doyle, *"Layering as optimization decomposition: questions and answers,"* Proc. IEEE MILCOM, Washington D.C., October 2006

F. Paganini, W. Wang, J. C. Doyle, S. H. Low, *"Congestion control for high performance, stability, and fairness in general networks,"* IEEE/ACM Transactions on Networking, Volume 13, Issue 1, Feb. 2005 Page(s):43 - 56

S. Low and D. Lapsley, *"Optimization flow control,"* IEEE/ACM Transactions on Networking, vol 7, no 6 pp. 861-874, Jun 1997

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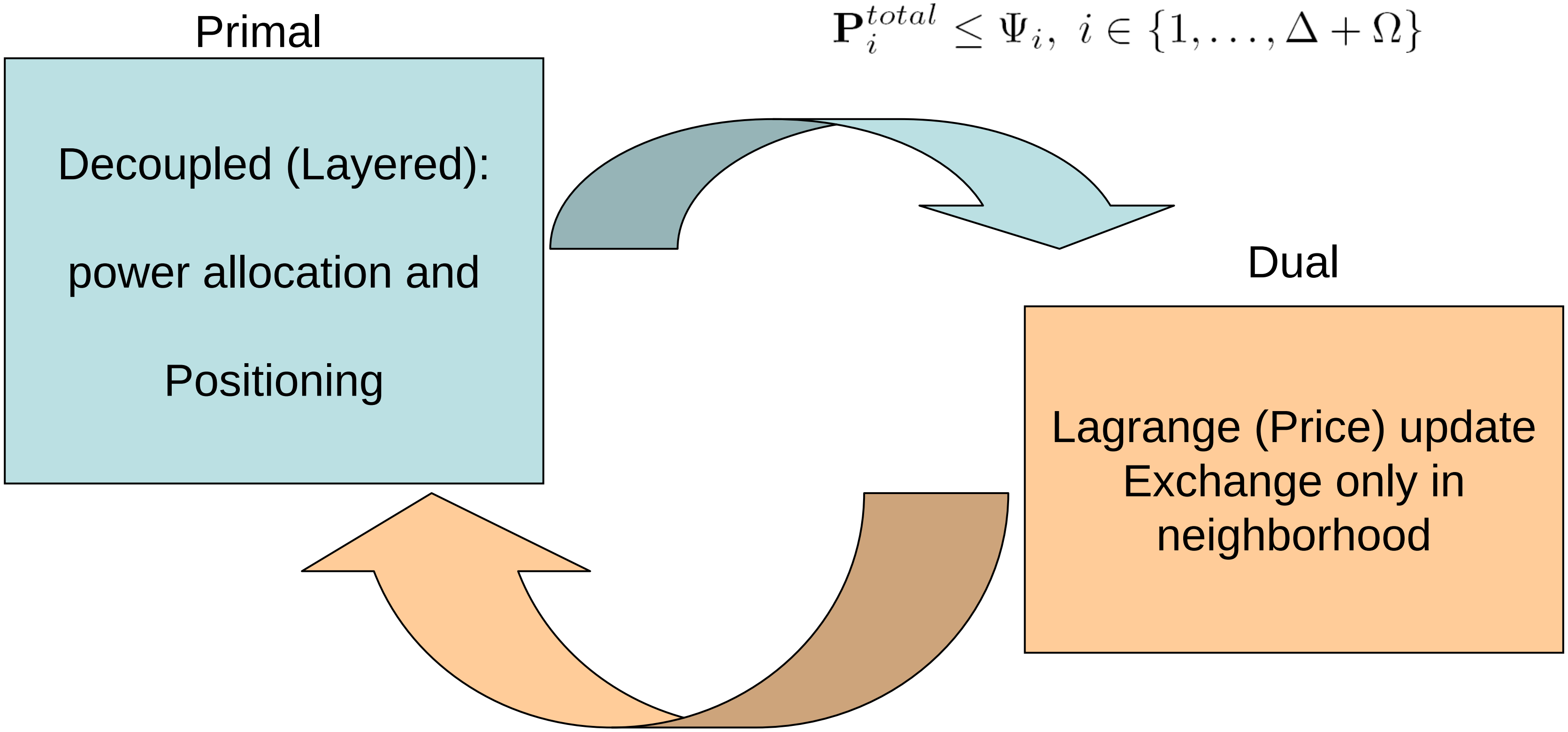
Subject to:

$$\mathbf{S}_{i\to j} \geq \mathcal{R}^{-1}(\varrho_{i\to j}),\ (i,j) \in \bigcup_{i=1}^{\Delta+\Omega} \{i\} \times \mathbb{O}(i)$$

$$\mathbf{P}_i^{total} \leq \Psi_i,\ i \in \{1,\ldots,\Delta+\Omega\}$$



Key facts for decomposition: Relaxations

$$\mathcal{U}(\mathbf{Q}) = \sum_{i=1}^{\Delta+\Omega} \varphi_i 10^{0.1\mathbf{P}_i^{total}}$$

We want

$$\mathbf{Q}^* = \arg\min_{\mathbf{Q}} \mathcal{U}(\mathbf{Q})$$

Subject to:

$$\mathbf{S}_{i\to j} \geq \mathcal{R}^{-1}(\varrho_{i\to j}),\ (i,j) \in \bigcup_{i=1}^{\Delta+\Omega} \{i\} \times \mathbb{O}(i)$$

$$\mathbf{P}_i^{total} \leq \Psi_i,\ i \in \{1, \ldots, \Delta+\Omega\}$$

Original problem

$$\tilde{\mathcal{U}}(\mathbf{V}) \overset{def}{=} \sum_{i=1}^{\Delta+\Omega} \sum_{j\in\mathbb{O}(i)} \varphi_i 10^{0.1\mathbf{v}_{i\to j}}$$

$$\mathbf{V} \overset{def}{=} \{\mathbf{v}_{i\to j}\}_{i=1, j\in\mathbb{O}(i)}^{i=\Delta+\Omega}$$

minimize $\tilde{\mathcal{U}}(\mathbf{V})$

subject to:

$$\mathbf{S}_{i\to j} \geq \mathcal{R}^{-1}(\varrho_{i\to j}),\ (i,j) \in \bigcup_{i=1}^{\Delta+\Omega} \{i\} \times \mathbb{O}(i)$$

$$\phi 10^{0.1\mathbf{P}_{i\to j} + \alpha \mathbf{e}_{i,j}} \leq 10^{0.1\mathbf{v}_{i\to j}},\ j \in \mathbb{O}(i)$$

$$\sum_{j\in\mathbb{O}(i)} 10^{0.1\mathbf{v}_{i\to j}} \leq 10^{0.1\Psi_i}$$

Equivalent problem

Primal-Dual decomposition

$$\tilde{\mathcal{U}}(\mathbf{V}) \stackrel{def}{=} \sum_{i=1}^{\Delta+\Omega} \sum_{j\in\mathbb{O}(i)} \varphi_i 10^{0.1\mathbf{v}_{i\to j}}$$

$$\mathbf{V} \stackrel{def}{=} \{\mathbf{v}_{i\to j}\}_{i=1,j\in\mathbb{O}(i)}^{i=\Delta+\Omega}$$

minimize $\tilde{\mathcal{U}}(\mathbf{V})$

subject to:

$$\mathbf{S}_{i\to j} \geq \mathcal{R}^{-1}(\varrho_{i\to j}),\ (i,j) \in \bigcup_{i=1}^{\Delta+\Omega} \{i\} \times \mathbb{O}(i)$$

$$\phi 10^{0.1\mathbf{P}_{i\to j}+\alpha\mathbf{e}_{i,j}} \leq 10^{0.1\mathbf{v}_{i\to j}},\ j \in \mathbb{O}(i)$$

$$\sum_{j\in\mathbb{O}(i)} 10^{0.1\mathbf{v}_{i\to j}} \leq 10^{0.1\Psi_i}$$

$$\max_{\mathbf{H}} \underbrace{\overset{\Diamond}{\min_{(\mathbf{Q},\mathbf{V})}} \mathcal{L}(\mathbf{Q},\mathbf{V},\mathbf{H})}_{\text{primal problem}}$$

Lagrange M. → $\mathbf{H} \stackrel{def}{=} \{\mathbf{h}_{i,j}\}_{i=1,j\in\mathbb{O}(i)}^{i=\Delta+\Omega}$

$$\mathbf{h}_{i,j} \geq 0,\ (i,j) \in \bigcup_{i=1}^{\Delta+\Omega} \{i\} \times \mathbb{O}(i)$$

$$\mathcal{L}(\mathbf{Q},\mathbf{V},\mathbf{H}) \stackrel{def}{=} \underbrace{\mathcal{L}_1\left(\{\mathbf{P}_{i\to j}\}_{i=1,j\in\mathbb{O}(i)}^{i=\Delta+\Omega},\mathbf{V},\mathbf{H}\right)}_{\text{power allocation}} + \underbrace{\mathcal{L}_2\left(\{\mathbf{x}_{i+\Delta},\mathbf{y}_{i+\Delta}\}_{i=1}^{\Omega},\mathbf{H}\right)}_{\text{node placement}}$$

$$\mathcal{L}_1\left(\{\mathbf{P}_{i\to j}\}_{i=1,j\in\mathbb{O}(i)}^{i=\Delta+\Omega},\mathbf{V},\mathbf{H}\right) \stackrel{def}{=} \tilde{\mathcal{U}}(\mathbf{V}) + \sum_{i=1}^{\Delta+\Omega} \sum_{j\in\mathbb{O}(i)} \mathbf{h}_{i,j}\left[\mathbf{P}_{i\to j} - \mathbf{v}_{i\to j} + 10\log\varphi\right]$$

$$\mathcal{L}_2\left(\{\mathbf{x}_{i+\Delta},\mathbf{y}_{i+\Delta}\}_{i=1}^{\Omega},\mathbf{H}\right) \stackrel{def}{=} \sum_{i=1}^{\Delta+\Omega} \sum_{j\in\mathbb{O}(i)} \mathbf{h}_{i,j} 10\alpha\mathbf{e}_{i,j}$$

Primal-dual recursion: Numerical aspects

(Initialization) Initialize $\mathbf{H}(0)$ as $\mathbf{h}_{i,j}(0)=0$

(Primal step)

$$(\mathbf{Q}^*(k+1),\mathbf{V}^*(k+1)) = \arg\overset{\diamond}{\min_{(\mathbf{Q},\mathbf{V})}}\mathcal{L}(\mathbf{Q},\mathbf{V},\mathbf{H}(k))$$

subject to:

$$\mathbf{S}_{i\to j}\geq\mathcal{R}^{-1}(\varrho_{i\to j}),\ (i,j)\in\bigcup_{i=1}^{\Delta+\Omega}\{i\}\times\mathbb{O}(i)$$

$$\begin{aligned}\phi 10^{0.1\mathbf{P}_{i\to j}+\alpha\mathbf{e}_{i,j}} &\leq 10^{0.1\mathbf{v}_{i\to j}},\ j\in\mathbb{O}(i)\\ \sum_{j\in\mathbb{O}(i)}10^{0.1\mathbf{v}_{i\to j}} &\leq 10^{0.1\Psi_i}\end{aligned}$$

$$\mathcal{L}(\mathbf{Q},\mathbf{V},\mathbf{H})\overset{def}{=}$$

$$\underbrace{\mathcal{L}_1\left(\{\mathbf{P}_{i\to j}\}_{i=1,j\in\mathbb{O}(i)}^{i=\Delta+\Omega},\mathbf{V},\mathbf{H}\right)}_{\text{power allocation}}+\underbrace{\mathcal{L}_2\left(\{\mathbf{x}_{i+\Delta},\mathbf{y}_{i+\Delta}\}_{i=1}^{\Omega},\mathbf{H}\right)}_{\text{node placement}}$$

$$\mathcal{L}_1\left(\{\mathbf{P}_{i\to j}\}_{i=1,j\in\mathbb{O}(i)}^{i=\Delta+\Omega},\mathbf{V},\mathbf{H}\right)\overset{def}{=}$$

$$\tilde{\mathcal{U}}(\mathbf{V})+\sum_{i=1}^{\Delta+\Omega}\sum_{j\in\mathbb{O}(i)}\mathbf{h}_{i,j}\left[\mathbf{P}_{i\to j}-\mathbf{v}_{i\to j}+10\log\varphi\right]$$

$$\mathcal{L}_2\left(\{\mathbf{x}_{i+\Delta},\mathbf{y}_{i+\Delta}\}_{i=1}^{\Omega},\mathbf{H}\right)\overset{def}{=}\sum_{i=1}^{\Delta+\Omega}\sum_{j\in\mathbb{O}(i)}\mathbf{h}_{i,j}10\alpha\mathbf{e}_{i,j}$$

(Price update)

$$\mathbf{h}_{i,j}(k+1)=[\varepsilon(k)\mathbf{w}^*_{i,j}(k)+\mathbf{h}_{i,j}(k)]^+,$$

$$(i,j)\in\bigcup_{i=1}^{\Delta+\Omega}\{i\}\times\mathbb{O}(i)$$

$$\mathbf{w}^*_{i,j}(k)\overset{def}{=}10\log\varphi+\mathbf{P}^*_{i\to j}(k)+10\alpha\mathbf{e}^*_{i,j}(k)-\mathbf{v}^*_{i\to j}(k)$$

(Initialization) Initialize $\mathbf{H}(0)$ as $\mathbf{h}_{i,j}(0) = 0$

(Primal step)

$$(\mathbf{Q}^*(k+1), \mathbf{V}^*(k+1)) = \arg \overset{\diamond}{\min_{(\mathbf{Q},\mathbf{V})}} \mathcal{L}(\mathbf{Q}, \mathbf{V}, \mathbf{H}(k))$$

subject to:

$$\mathbf{S}_{i\to j} \geq \mathcal{R}^{-1}(\varrho_{i\to j}),$$

$$\phi 10^{0.1\mathbf{P}_{i\to j} + \alpha \mathbf{e}_{i,j}} \leq$$

$$\sum_{j\in\mathbb{O}(i)} 10^{0.1\mathbf{v}_{i\to j}} \leq$$

(Price update)

$$\mathbf{h}_{i,j}(k+1) = [\varepsilon(k)\mathbf{w}^*_{i,j}$$

$$\mathcal{L}(\mathbf{Q},\mathbf{V},\mathbf{H}) \overset{def}{=}$$

$$\underbrace{\mathcal{L}_1\left(\{\mathbf{P}_{i\to j}\}_{i=1,j\in\mathbb{O}(i)}^{i=\Delta+\Omega}, \mathbf{V}, \mathbf{H}\right)}_{\text{power allocation}} + \underbrace{\mathcal{L}_2\left(\{\mathbf{x}_{i+\Delta}, \mathbf{y}_{i+\Delta}\}_{i=1}^{\Omega}, \mathbf{H}\right)}_{\text{node placement}}$$

$$+ 10\log\varphi]$$

$$\mathbf{h}_{i,j} 10\alpha\mathbf{e}_{i,j}$$

Solution to the placement problem

$$(\mathbf{x}_i, \mathbf{y}_i)(t+1) = (\mathbf{x}_i, \mathbf{y}_i)(t) + \varepsilon(t) \sum_{j\in\{\mathbb{O}(i)\bigcup\mathbb{I}(i)\}} \mathbf{h}_{ij}\overrightarrow{\theta_{ij}}(t)$$

$$\overrightarrow{\theta_{ij}}(t) \overset{def}{=} \begin{cases} \frac{1}{\mathbf{e}_{i,j}(t)}(\mathbf{x}_j(t) - \mathbf{x}_i(t), \mathbf{y}_j(t) - \mathbf{y}_i(t)) & \text{if } \mathbf{e}_{i,j}(t) \neq 0 \\ (0,0) & Otherwise \end{cases}$$

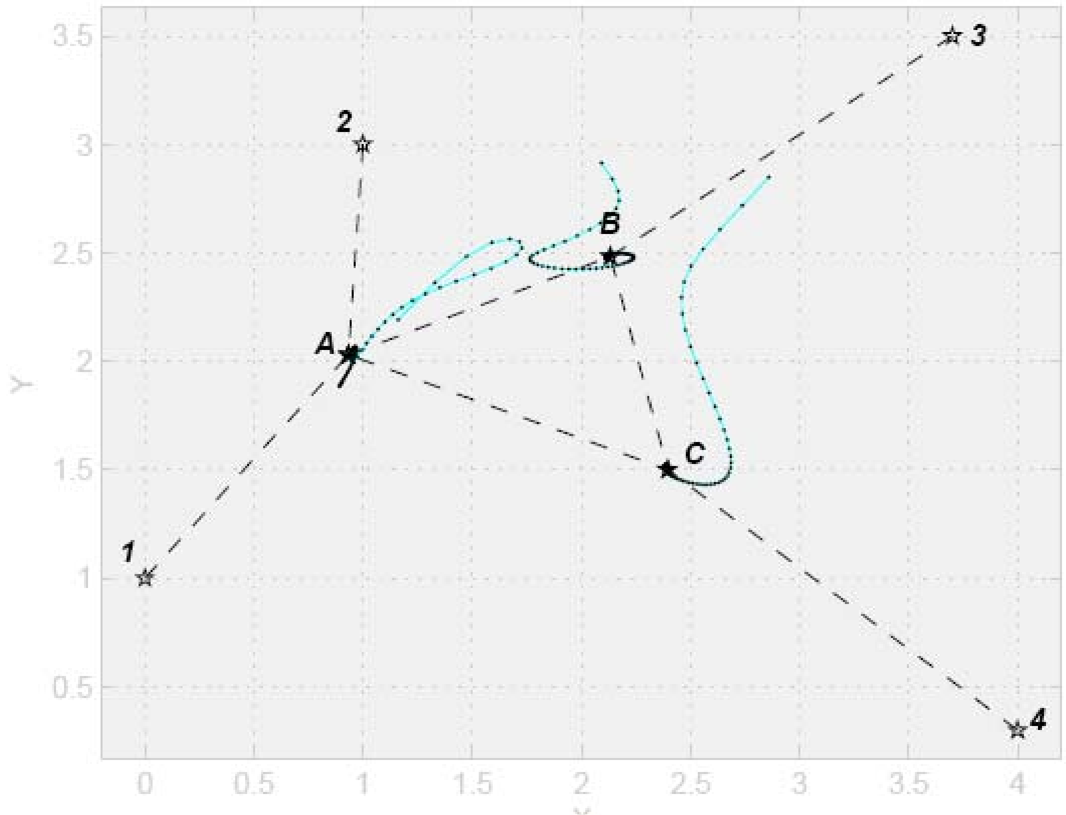
$$\mathbf{w}^*_{i,j}(k) \overset{def}{=} 10\log\varphi + \mathbf{P}^*_{i\to j}(k) + 10\alpha\mathbf{e}^*_{i,j}(k) - \mathbf{v}^*_{i\to j}(k)$$

Numerical Illustration

Consider an exampled with four fixed nodes, labeled from 1 to 4 and three Mobile nodes labeled from A to C.

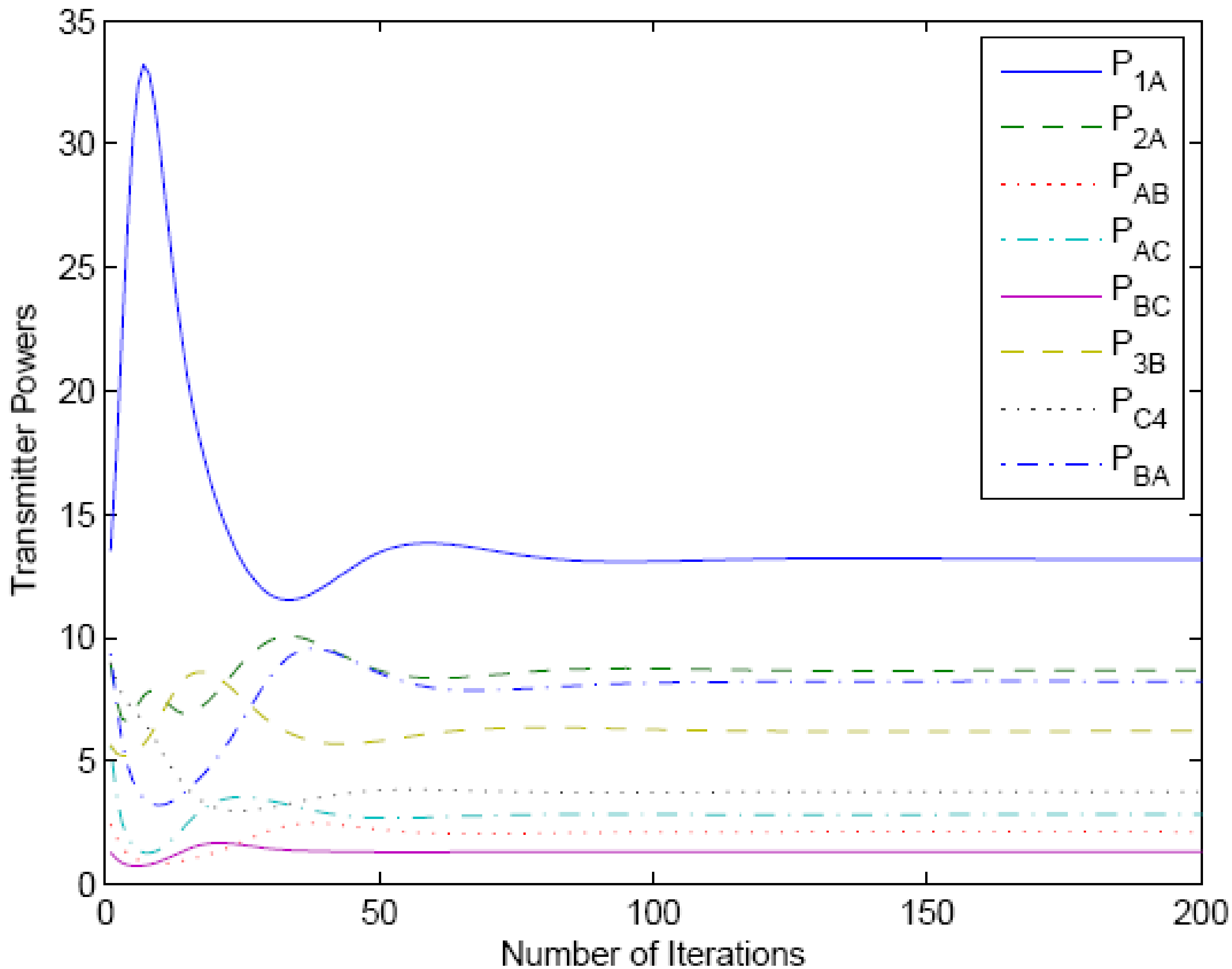
We want to find the constellation with minimum power that satisfies the following link table with a 15dB constraint on the SINR

T ＼ R	4	A	B	C
1		X		
2		X		
3			X	
4				
A			X	X
B		X		X
C	X			

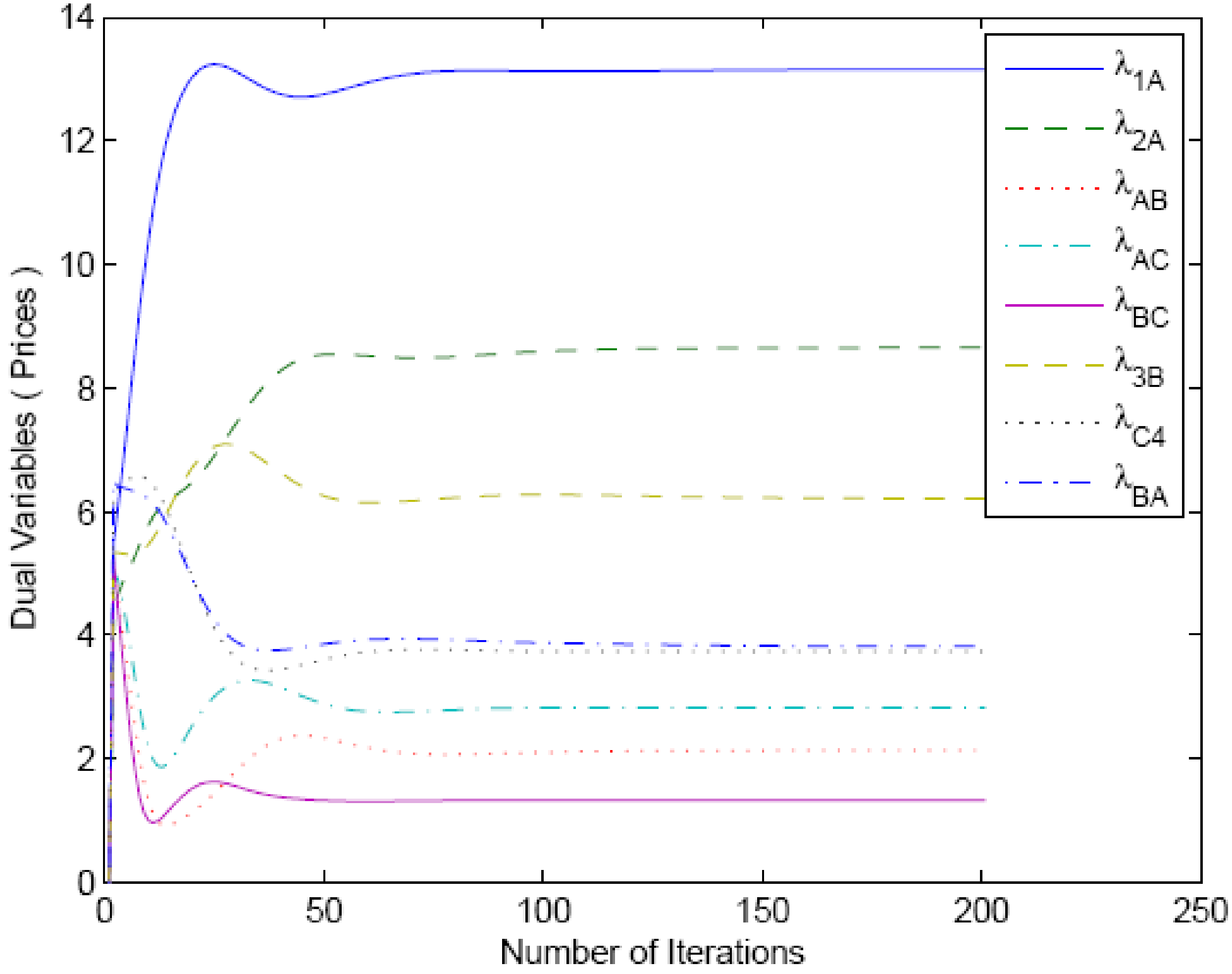


Node trajectories with respect to primal-dual iterations.

Numerical Illustration



Numerical Illustration



Conclusions

- We have proposed a new framework which extends existing results on optimal power allocation so as to include placement.

- A geometric programming approach was proposed for providing a computationally efficient way for computing the optimum with an arbitrary degree of accuracy.

- In the presence of a neighborhood structure, we provided a decentralized algorithm that converges to an optimum. The underlying mechanism operates via price exchanges in the neighborhoods only.

- Examples were provided for the case of exponential path loss. A discussion of the validity of such modeling approximation were also provided.

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Details can be found in:

"Jointly Optimal power allocation and constrained node placement in wireless networks of agents," S. Firouzabadi and N. Martins, U. Maryland, ISR, Tech. Report, 08