Coordinated Path Following of Multiple UAVs for Time-Critical Missions in the Presence of Time-Varying Communication Topologies

NAVAL POSTGRADUATE SCHOOL

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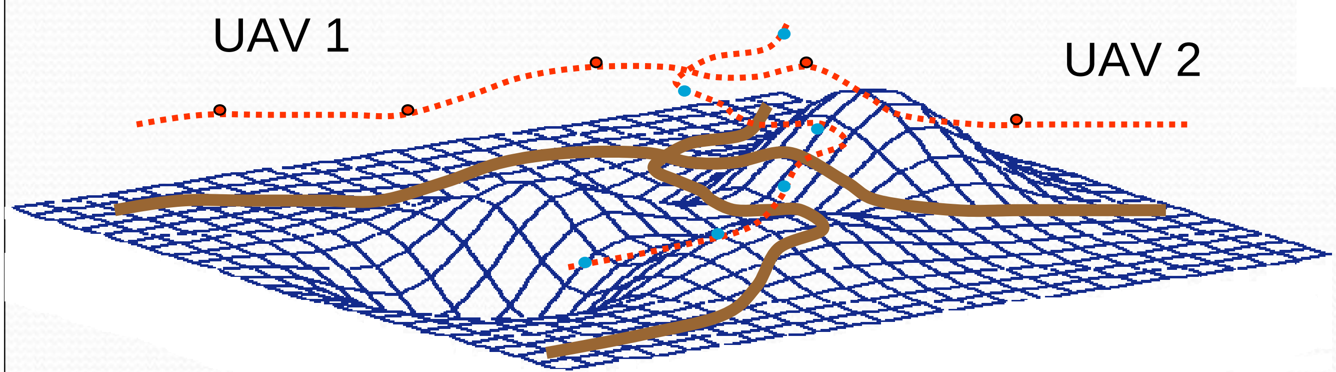
• Time Critical Applications for Multiple UAVs with spatial constraints

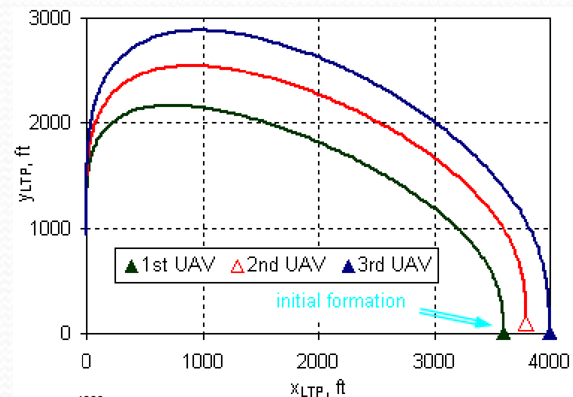
  • Sequential Autoland

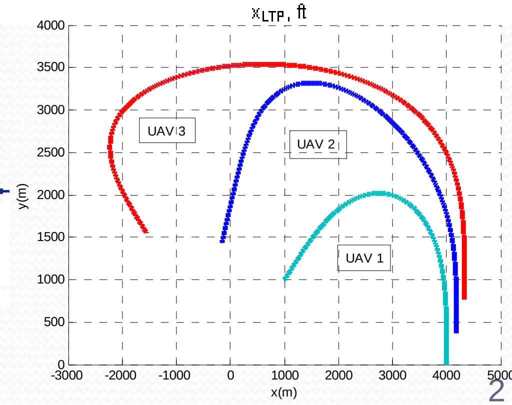
  • Coordinated Reconnaissance – synchronized high resolution pictures

  • Coordinated Road Search

• Coordinate on the arrival of the leader subject to deconfliction, network and spatial constraints







Introduction

•An integrated solution to **time-critical coordination** problems that includes

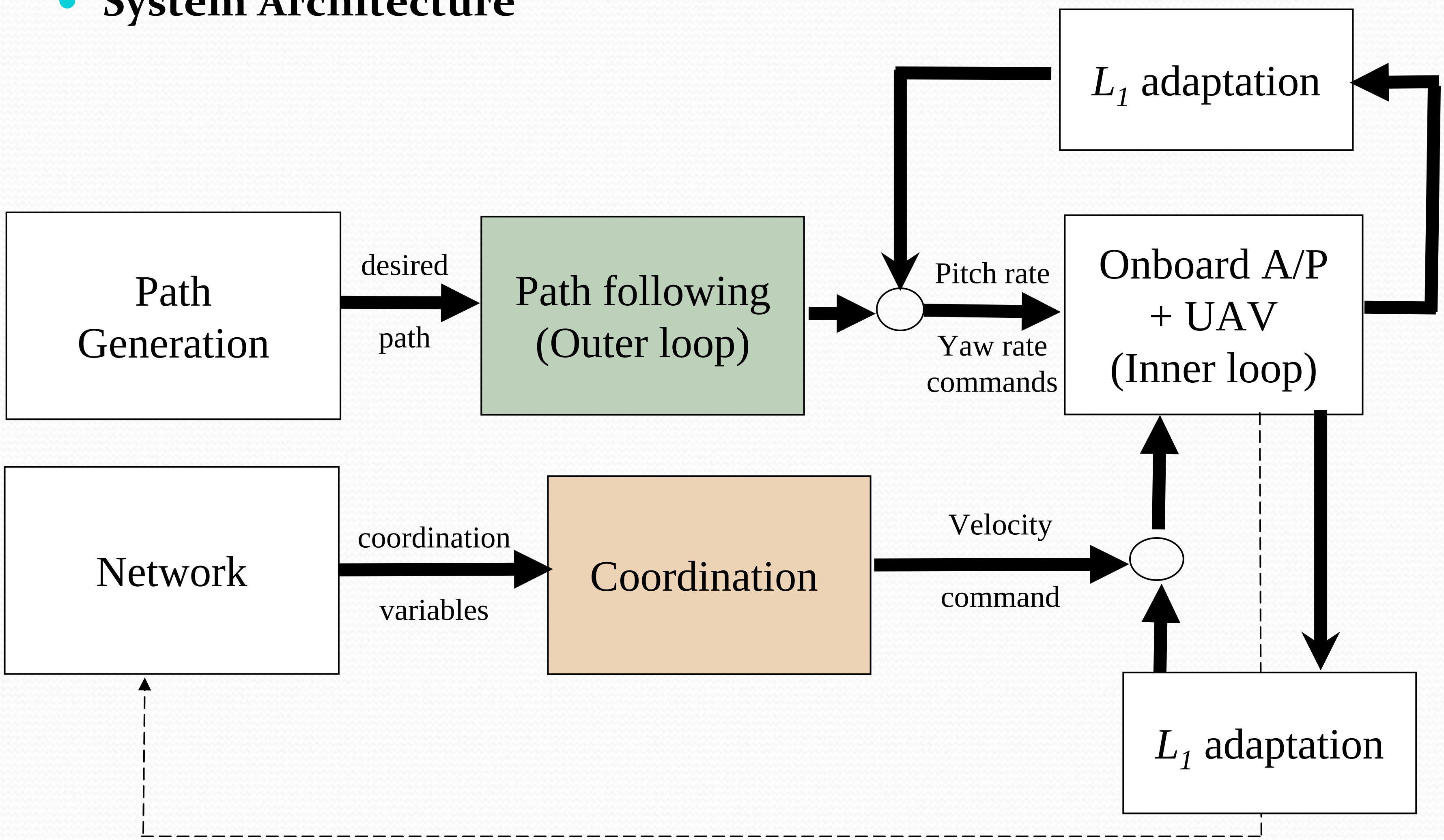
- **real-time (RT) path generation** accouting for
  - vehicle dynamics plus spatial and temporal coordination constraints
- **nonlinear path following** that relies on UAV attitude to follow the given path – leaving speed along the path as a degree of freedom

•***L_1* adaptation** to augment off-the-shelf autopilot to enable it to follow the paths it was not designed to follow

(a RT generated path is significantly more "aggressive" than a typical waypoint path these autopilots are designed to follow)

. **Time-critical coordination** controlling the speed of each vehicle over time varying faulty networks to provide robustness – account for the uncertainties that cannot be addressed in the path generation step

- **System Architecture**



- Assume polynomial paths
- *Decouple space and time in problem formulation* – drastic reduction in the number of optimization parameters (suitable for RT implementation), i.e let

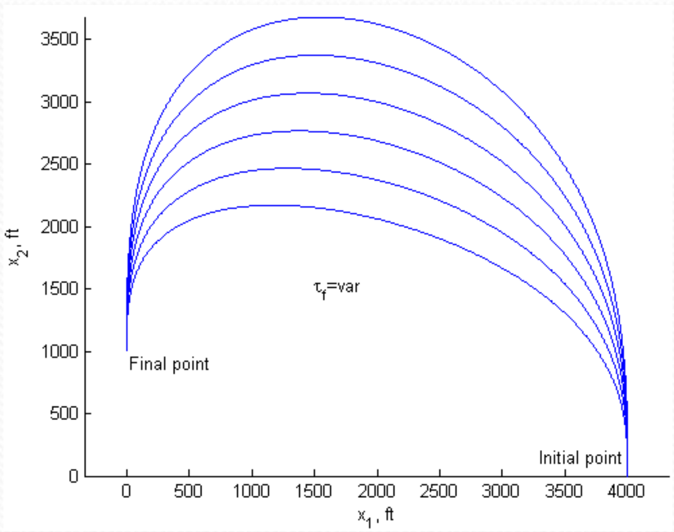
$p_c(\tau) := [x_1(\tau), x_2(\tau), x_3(\tau)]^T$ where $\tau \in [0; \tau_f]$ is e.g. virtual arc length

a_{ik}

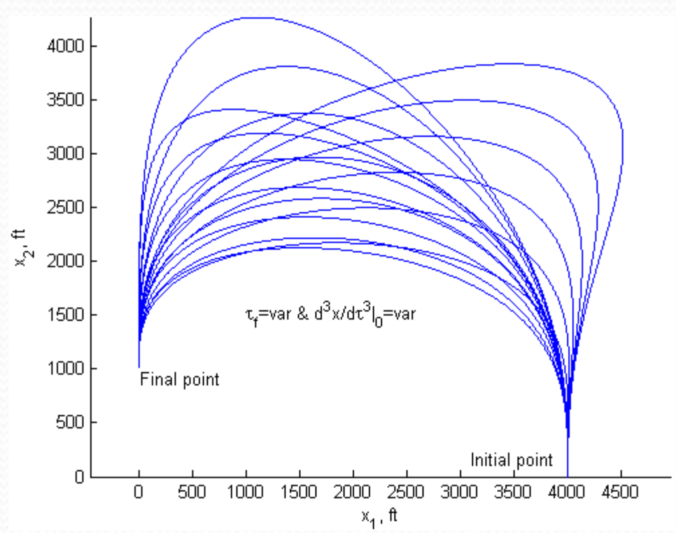
- $$x_i(\tau) = \sum_{k=0}^{N} a_{ik} \tau^k$$

Table 1. Examples of computation of the coefficients of polynomial trajectories.

Boundary conditions	x_{i0} , x'_{i0} , x''_{i0} , x_{if} , x'_{if} , x''_{if}	
d_0/d_f	2/2	3 (adding fictitious jerk x'''_{i0})/2
N^*/N	5/5	5/6
Linear algebraic matrix equation to solve for the coefficients a_{ik}	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & \tau_f & \tau_f^2 & \tau_f^3 & \tau_f^4 & \tau_f^5 \\ 0 & 1 & 2\tau_f & 3\tau_f^2 & 4\tau_f^3 & 5\tau_f^4 \\ 0 & 0 & 2 & 6\tau_f & 12\tau_f^2 & 20\tau_f^3 \end{pmatrix} \begin{pmatrix} a_{i0} \\ a_{i1} \\ a_{i2} \\ a_{i3} \\ a_{i4} \\ a_{i5} \end{pmatrix} = \begin{pmatrix} x_{i0} \\ x'_{i0} \\ x''_{i0} \\ x_{if} \\ x'_{if} \\ x''_{if} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 1 & \tau_f & \tau_f^2 & \tau_f^3 & \tau_f^4 & \tau_f^5 & \tau_f^6 \\ 0 & 1 & 2\tau_f & 3\tau_f^2 & 4\tau_f^3 & 5\tau_f^4 & 6\tau_f^5 \\ 0 & 0 & 2 & 6\tau_f & 12\tau_f^2 & 20\tau_f^3 & 30\tau_f^4 \end{pmatrix} \begin{pmatrix} a_{i0} \\ a_{i1} \\ a_{i2} \\ a_{i3} \\ a_{i4} \\ a_{i5} \\ a_{i6} \end{pmatrix} = \begin{pmatrix} x_{i0} \\ x'_{i0} \\ x''_{i0} \\ x'''_{i0} \\ x_{if} \\ x'_{if} \\ x''_{if} \end{pmatrix}$



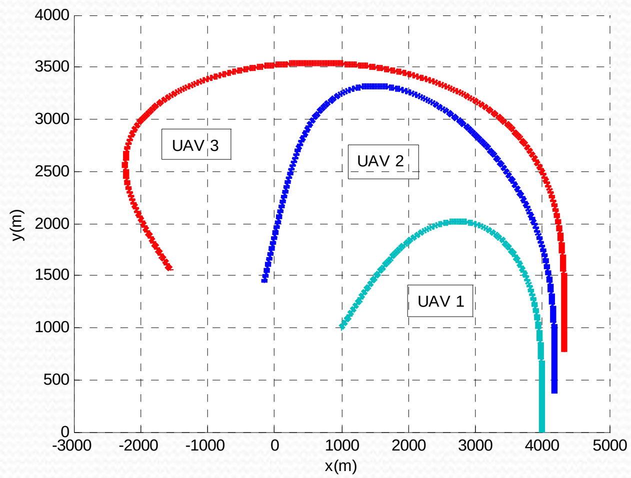
Impact of changing total path length

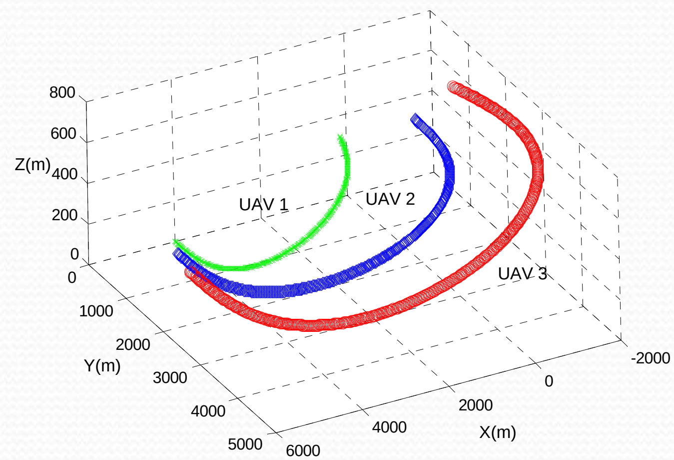


Impact of changing total path length and initial jerk

Time-Critical Coordination: RT Path Planning

- Deconfliction in space – sequentially assigned final conditions



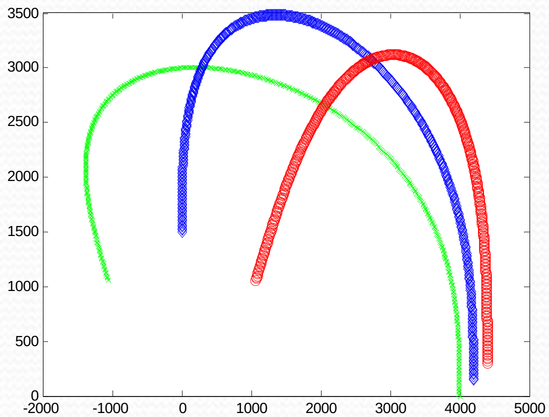


- essentially a 2D solution

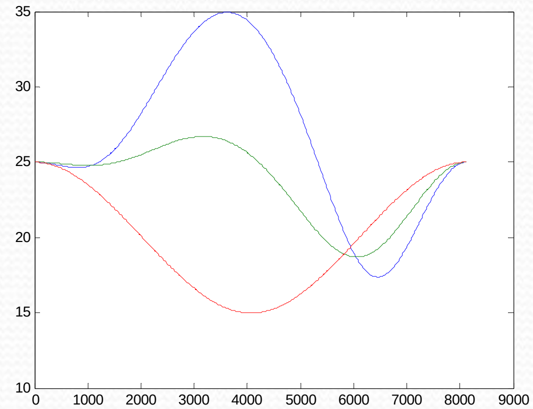
t_{f1}	dt	AM	v_{min}	v_1	v_2	v_3	v_{max}	d_{min}
235s	*.005s*	*48s*	*15 m/s*	*16m/s*	*21m/s*	*30.2m/s*	*35m/s*	*203m*

Time-Critical Coordination: RT Path Planning

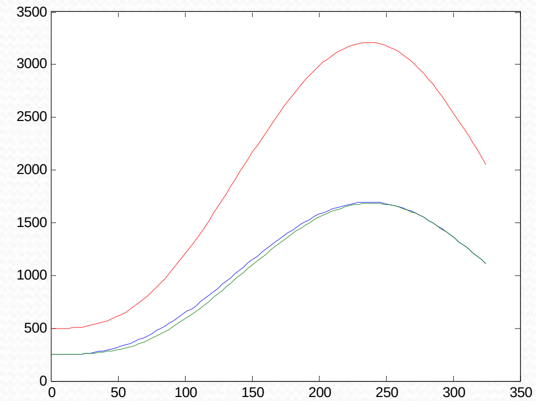
- Multiple UAVs – simultaneous arrival (deconfliction in time)



3D paths



Speed profiles



distances

$t_f = 324\ sec,\ AM = 271\ sec$

In this case the paths intersect, but if each vehicle follows the nominal speed profile they will maintain a minimum separation distance at all times – disturbance rejection is addressed at the **coordination level**

UAV Path Following Concept

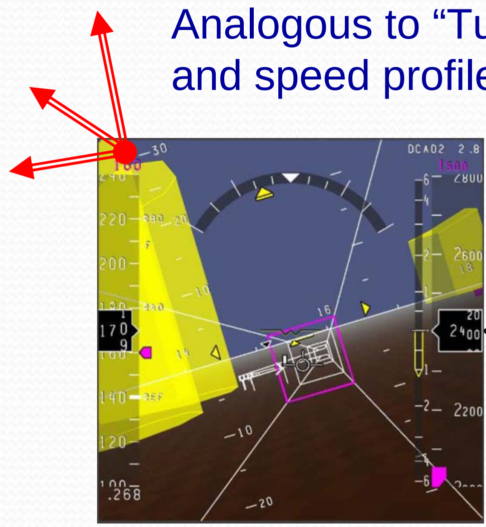
- **Objective**: follow predefined spatial 3D paths
  - ➔paths are time-independent:
    - decoupling between space and time = separation of 3D path and speed
  - ➔speed can be used as an additional DOF for time coordination

- **Limitation**: Traditional UAV AP is not designed to follow an aggressive 3D path.

- **Solution**: Adaptive augmentation without any modifications to commercial autopilot
  - ➔ Conventional solution – backstepping – requires modification of source code of A/P
  - ➔ Global results – but cancels inner loop

Intuitive Analogy

Analogous to “Tunnel in the Sky” concept familiar to pilots of 3D path and speed profile following

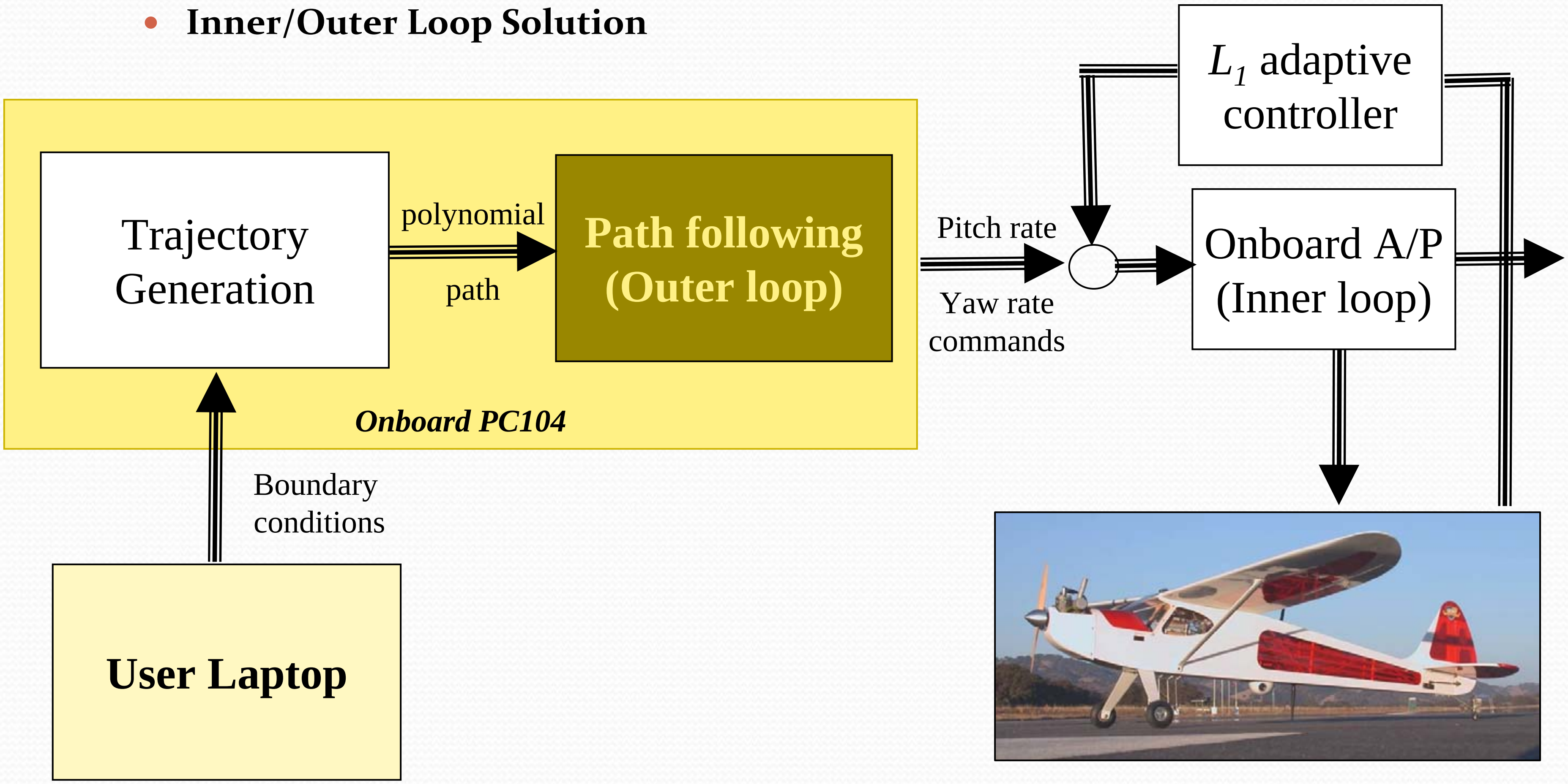


Eliminate path following (both distance and attitude) errors using angular rates

Follow speed profile to keep the a/c within its dynamic limitations.

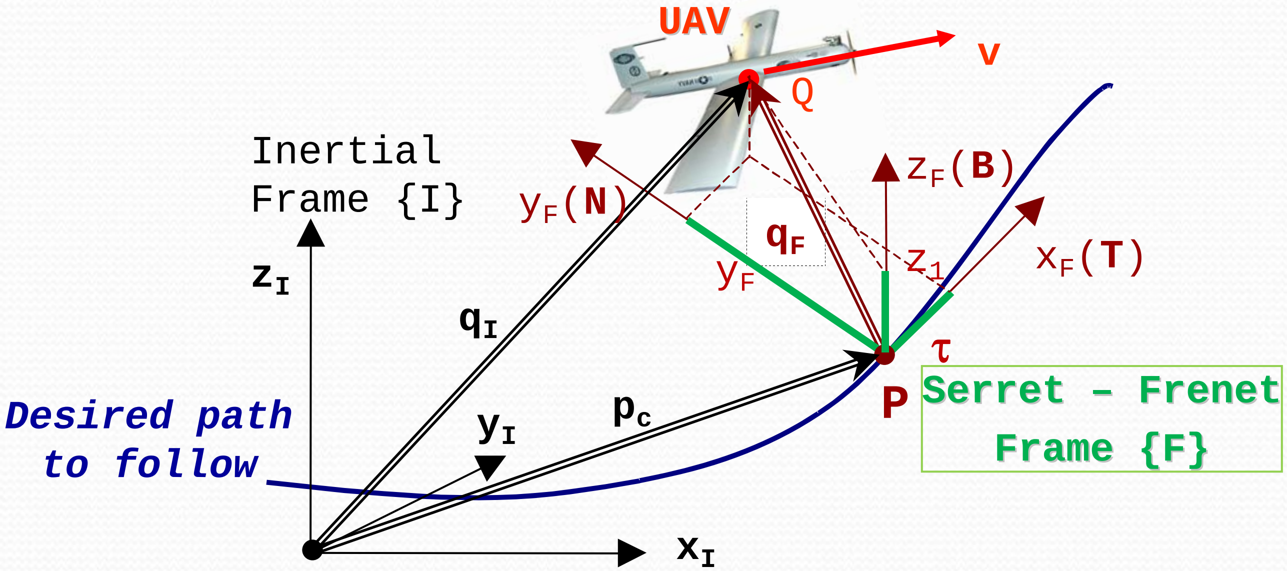
UAV Path Following

- System Architecture
  - **Inner/Outer Loop Solution**



Problem Geometry

- **F: Serret-Frenet frame**
- **W: wind frame**
- **I: inertial** frame
- $p_c(l)$ desired trajectory
- $v(t)$ UAV speed
- γ flight path angle
- ψ heading angle
- τ path length
- $q_I = [x_I \quad y_I \quad z_I]$ position of UAV in inertial frame



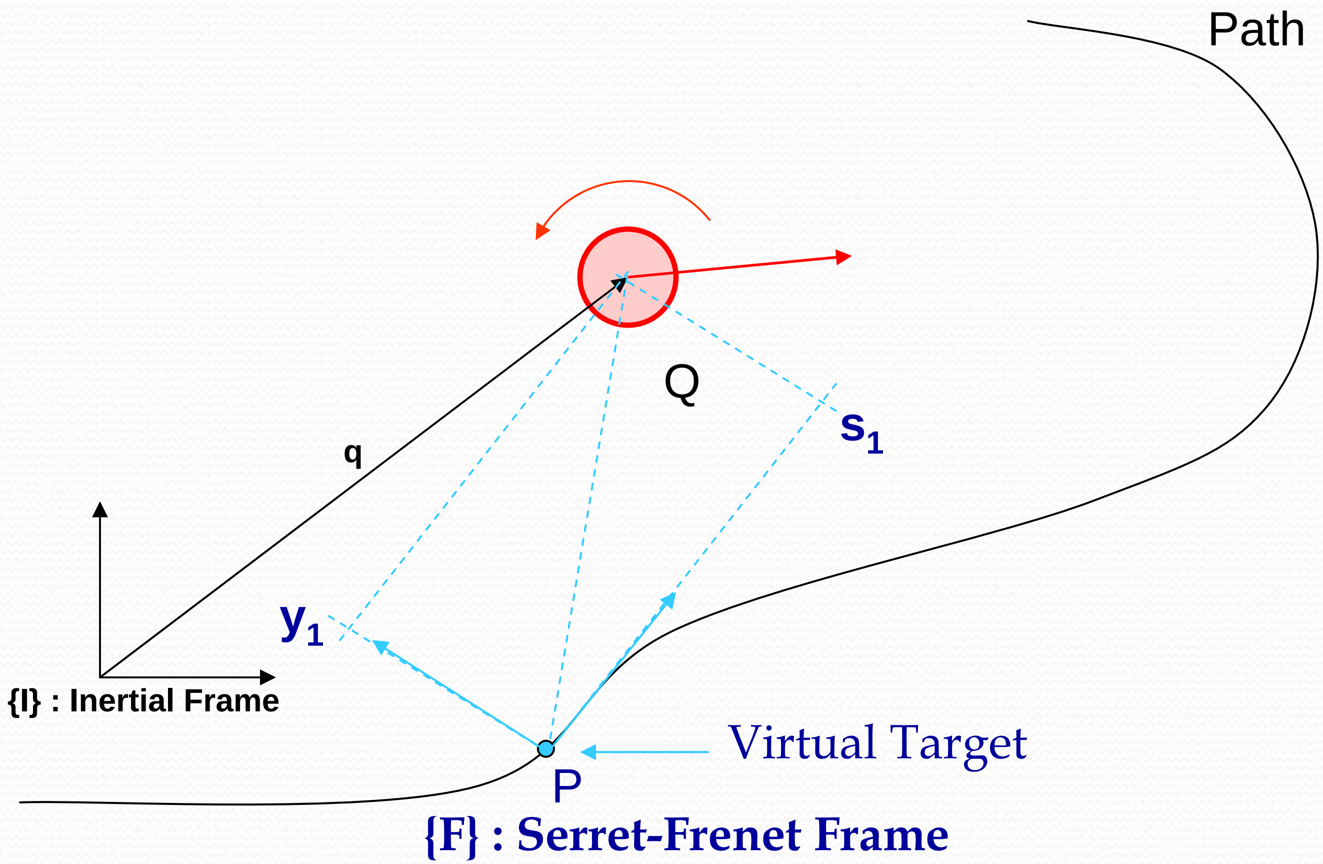
3D Kinematics Equations

$$dx_I / dt = v\cos\gamma\cos\psi$$
$$dy_I / dt = -v\cos\gamma\sin\psi$$
$$dz_I / dt = v\sin\psi$$
$$\begin{bmatrix} d\gamma / dt \\ d\psi / dt \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \cos^{-1}\gamma \end{bmatrix} \begin{bmatrix} q \\ r \end{bmatrix}$$
$$d\tau / dt = \tau_v$$

Input: q r τ_v

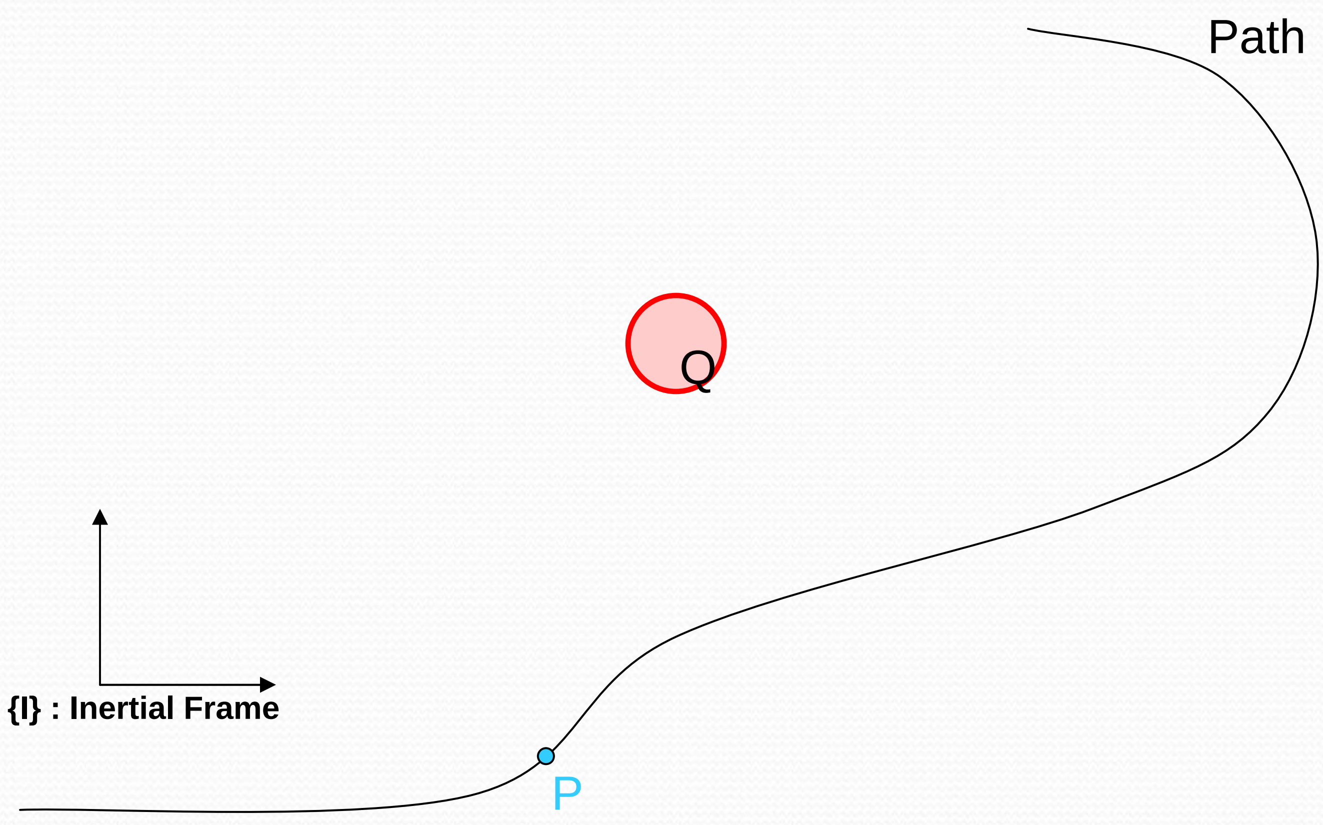
UAV Path Following

- Key idea: use virtual target to determine desired location on the path
  - Minimize the distance from the UAV to the virtual target on the path
  - Reduce the angle between the vehicle velocity vector and local tangent to the path
- Virtual target's motion – ***extra degree of freedom***



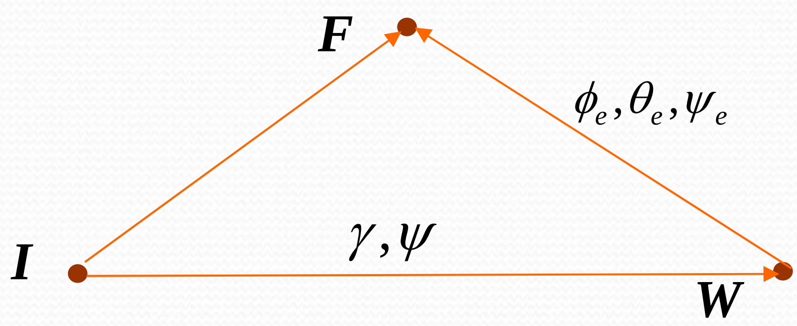
UAV Path Following (cont.)

Control the evolution of the virtual target : added degree of freedom



Kinematics

Coordinate systems



$q_F = [x_F \quad y_F \quad z_F]$

- difference between q_I and $p_c(\tau)$ in F

ϕ_e, θ_e, ψ_e

- Euler angles from F to W

Kinematics equations in *I*

$$\begin{aligned} \dot{x}_I &= v\cos\gamma\cos\psi \\ \dot{y}_I &= -v\cos\gamma\sin\psi \\ \dot{z}_I &= v\sin\gamma \\ \begin{bmatrix} \dot{\gamma} \\ \dot{\psi} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & \cos^{-1}\gamma \end{bmatrix} \begin{bmatrix} q \\ r \end{bmatrix} \\ \dot{\tau} &= \tau_v \end{aligned}$$

where

ζ - torsion

κ - curvature

Error Equations in *F*

$$G_e := \begin{cases} dx_F/dt = -\tau_v(1-\kappa(\tau)y_F) + v\cos\theta_e\cos\psi_e \\ dy_F/dt = -\tau_v(\kappa(\tau)x_F - \zeta(\tau)z_F) + v\cos\theta_e\sin\psi_e \\ dz_F/dt = -\zeta(\tau)\tau_v y_F - v\sin\theta_e \\ \begin{bmatrix} d\theta_e/dt \\ d\psi_e/dt \end{bmatrix} = D(t,\theta_e,\psi_e) + T(t,\theta_e)\begin{bmatrix} q \\ r \end{bmatrix} \\ d\tau/dt = \tau_v \end{cases}$$

Kinematic Control Law

$$V_c = \frac{1}{2c_1}(x_F^2 + y_F^2 + z_F^2) + \frac{1}{2c_2}(\theta_e - \delta_\theta)^2 + \frac{1}{2c_3}(\psi_e - \delta_\psi)^2$$

Desired shaping functions $\delta_\theta(t) = \sin^{-1}\left(\frac{z_F(t)}{|z_F(t)|+d_1}\right), \quad \delta_\psi(t) = \sin^{-1}\left(\frac{y_F(t)}{|y_F(t)|+d_2}\right)$

$$\begin{bmatrix} q_c \\ r_c \end{bmatrix} = T^{-1}(t,\theta_e)\left(\begin{bmatrix} u_{\theta_c} \\ u_{\psi_c} \end{bmatrix} - D(t,\theta_e,\psi_e)\right)$$

Path following control laws

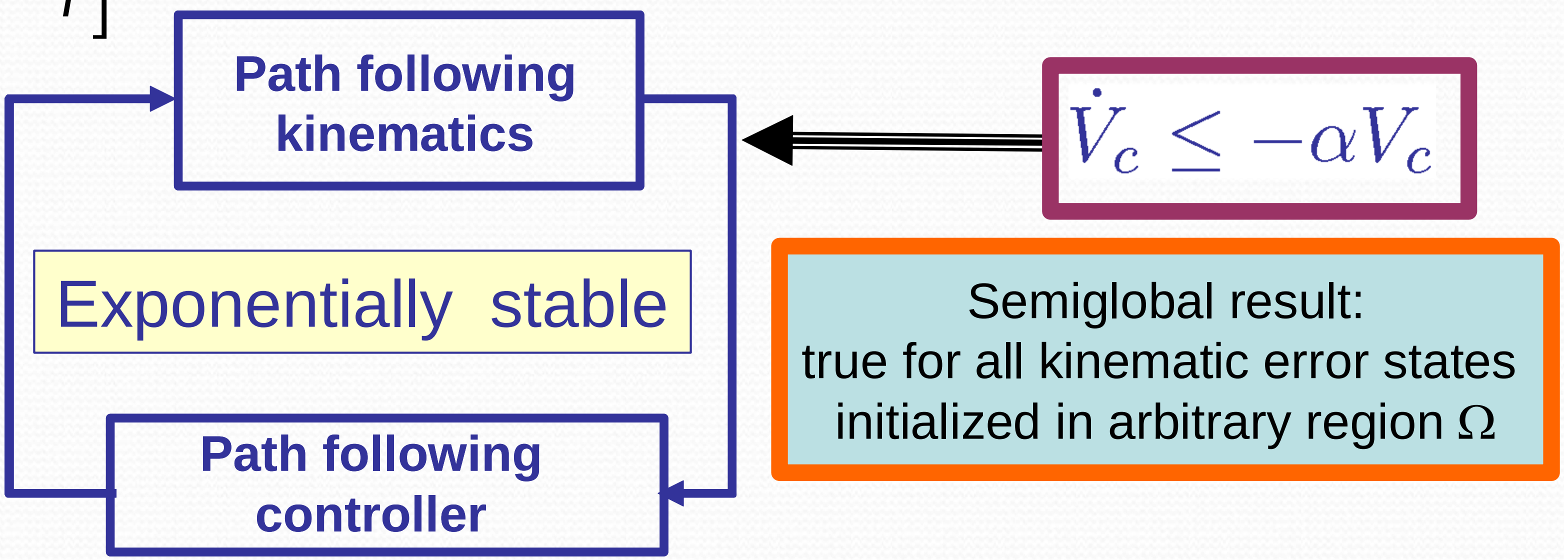
$$\dot{\tau} = K_1 x_F + v\cos\theta_e \cos\psi_e$$

$$u_{\theta_c} = -K_2(\theta_e - \delta_\theta) + \frac{c_2}{c_1} z_F v \frac{\sin\theta_e - \sin\delta_\theta}{\theta_e - \delta_\theta} + \dot{\delta}_\theta$$

$$u_{\psi_c} = -K_3(\psi_e - \delta_\psi) - \frac{c_2}{c_1} y_F v\cos\theta_e \frac{\sin\psi_e - \sin\delta_\psi}{\psi_e - \delta_\psi} + \dot{\delta}_\psi$$

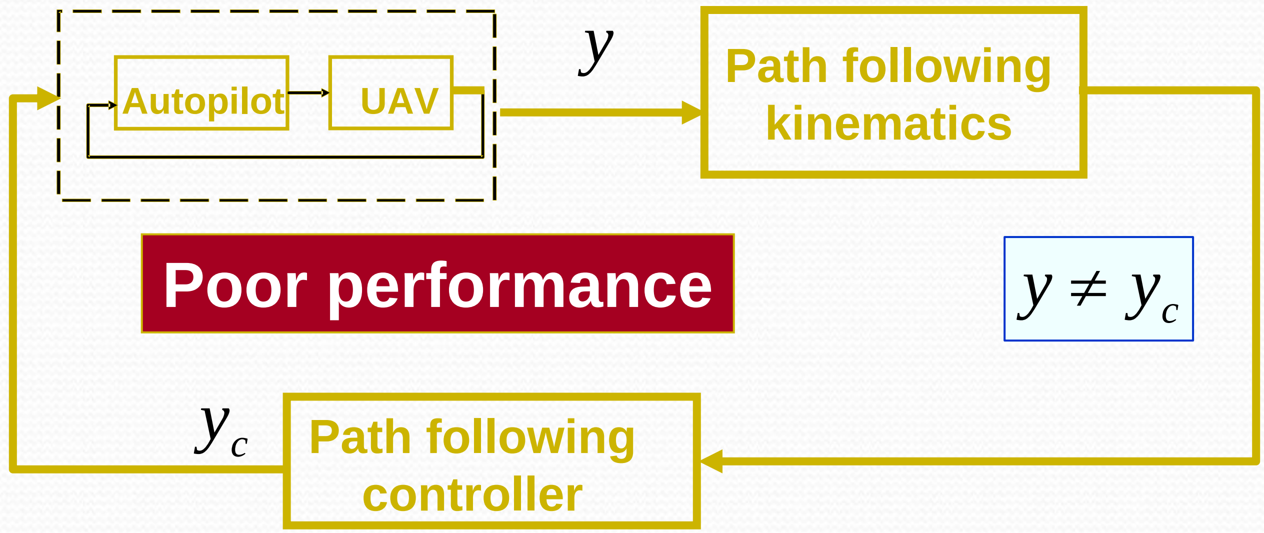
Kinematic Control Law

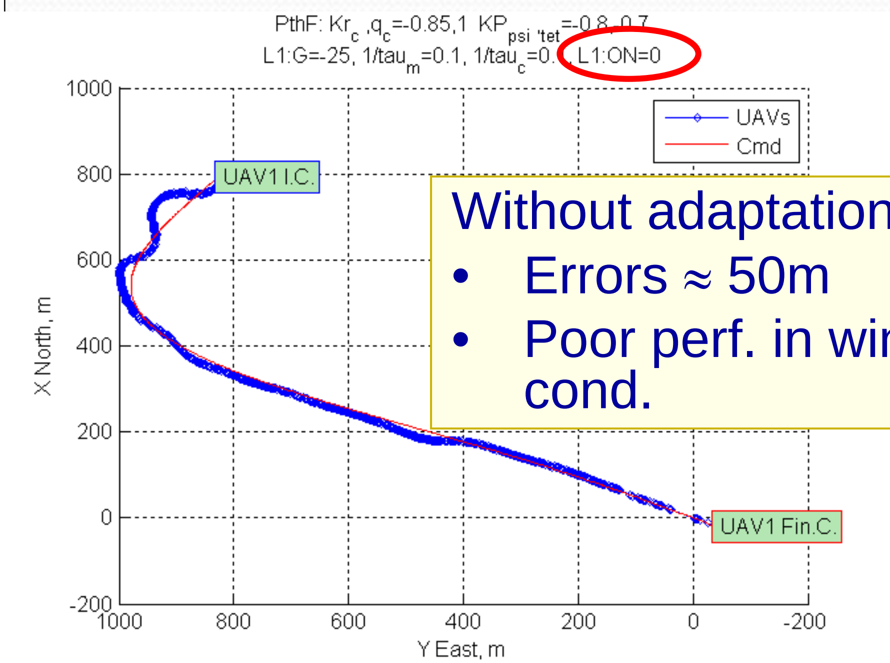
$$y = [q \quad r]^T$$



$$y_c = [q_c \quad r_c]^T$$

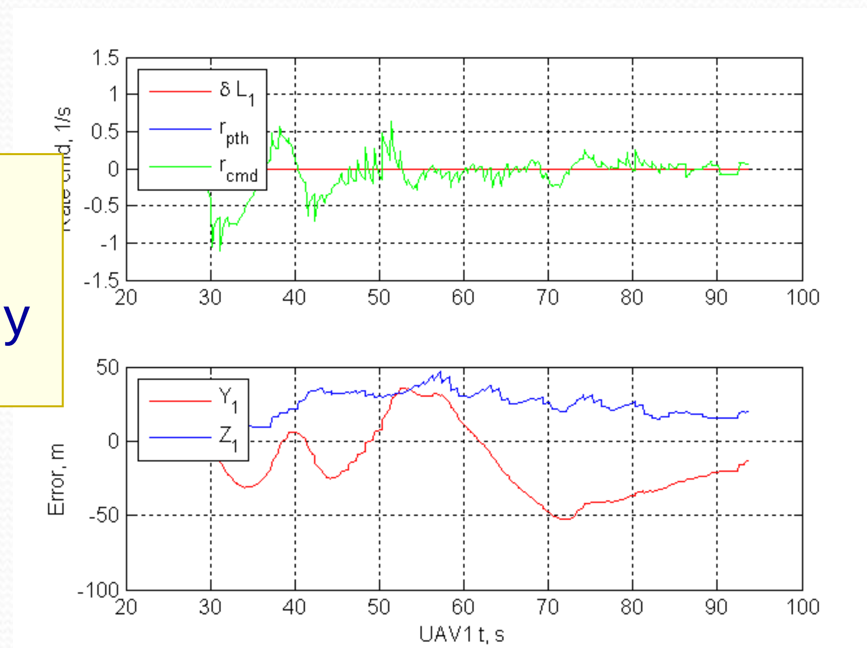
Degradation of Performance



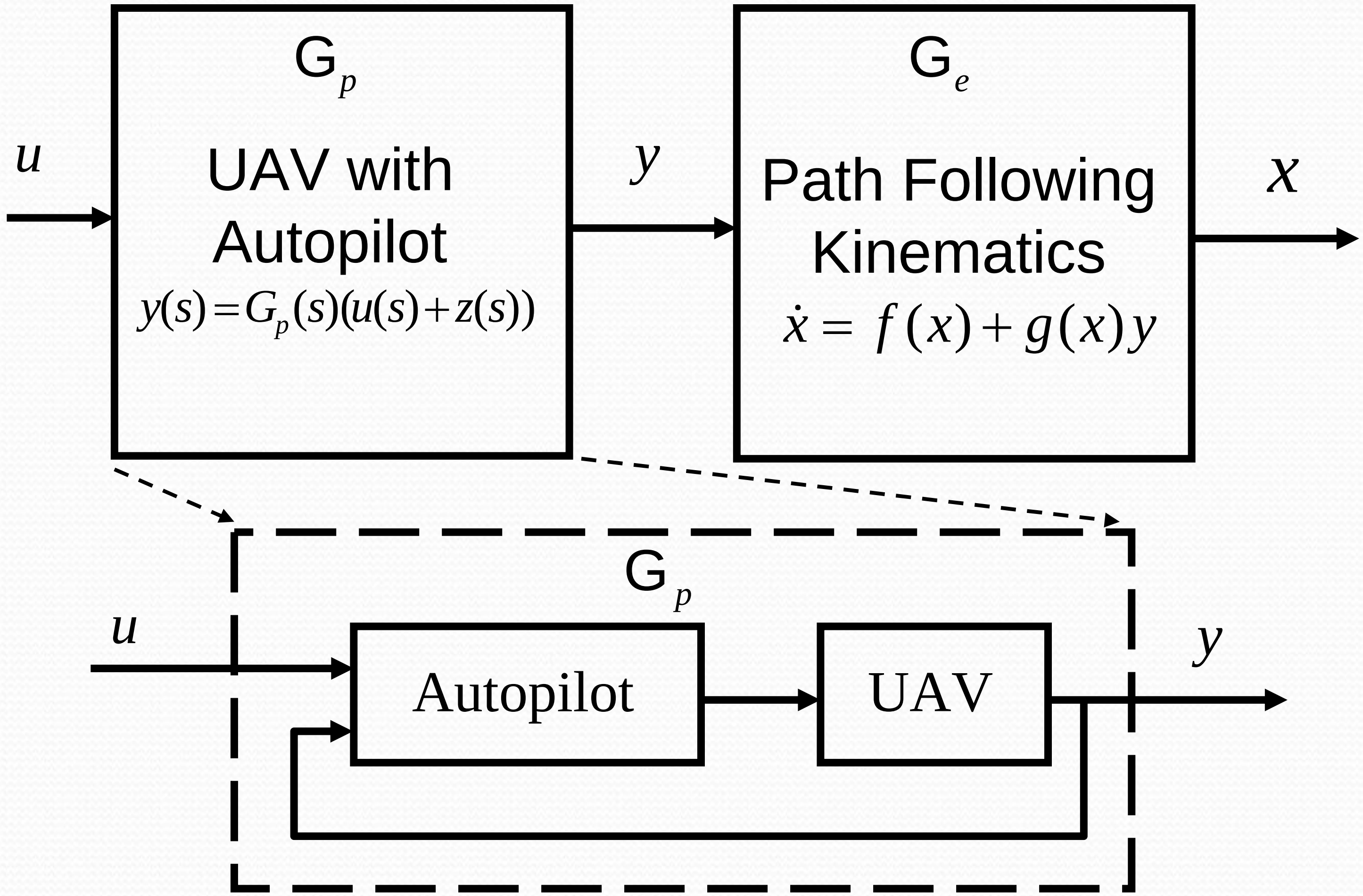


Without adaptation:

- Errors ≈ 50m
- Poor perf. in windy cond.



Cascaded System



- Conventional solution – backstepping – requires modification of source code of A/P
- Global results – but cancels inner loop

Problem Reformulation

UAV with autopilot

$$q(s) = G_q(s)\left(q_c(s) + z_q(s)\right)$$

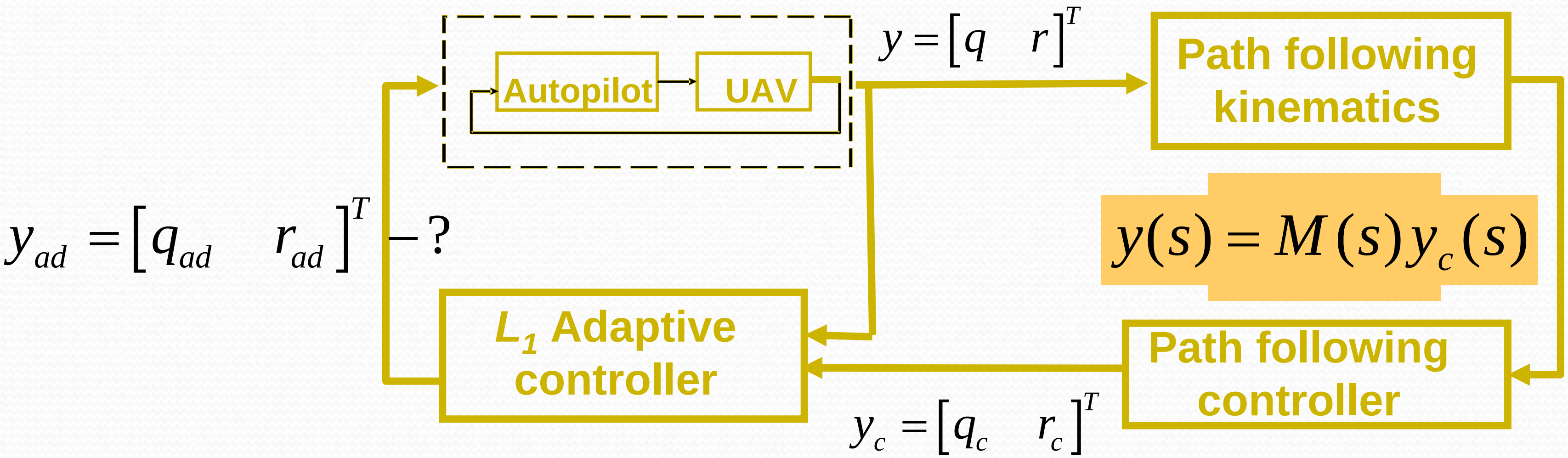
$$r(s) = G_r(s)\left(r_c(s) + z_r(s)\right)$$

Design objective

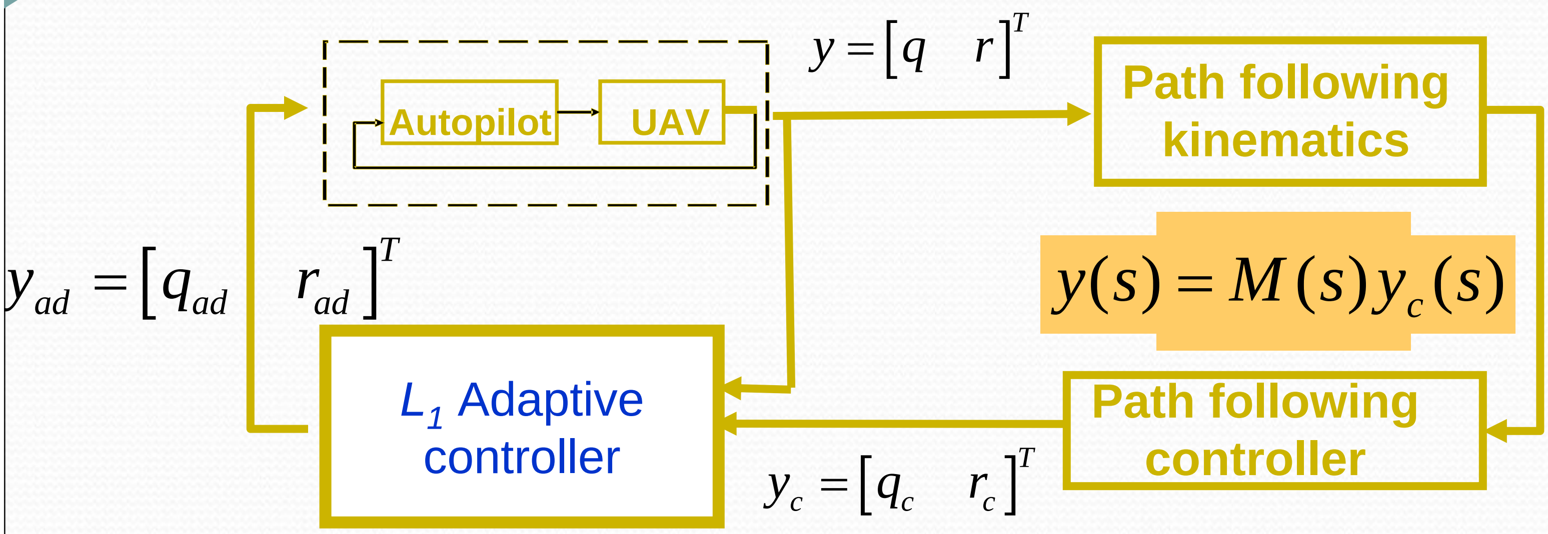
$$q(s) \approx M(s)\,q_c(s)$$

$$r(s) \approx M(s)\,r_c(s)$$

$$M(s) = \frac{m}{s+m}$$

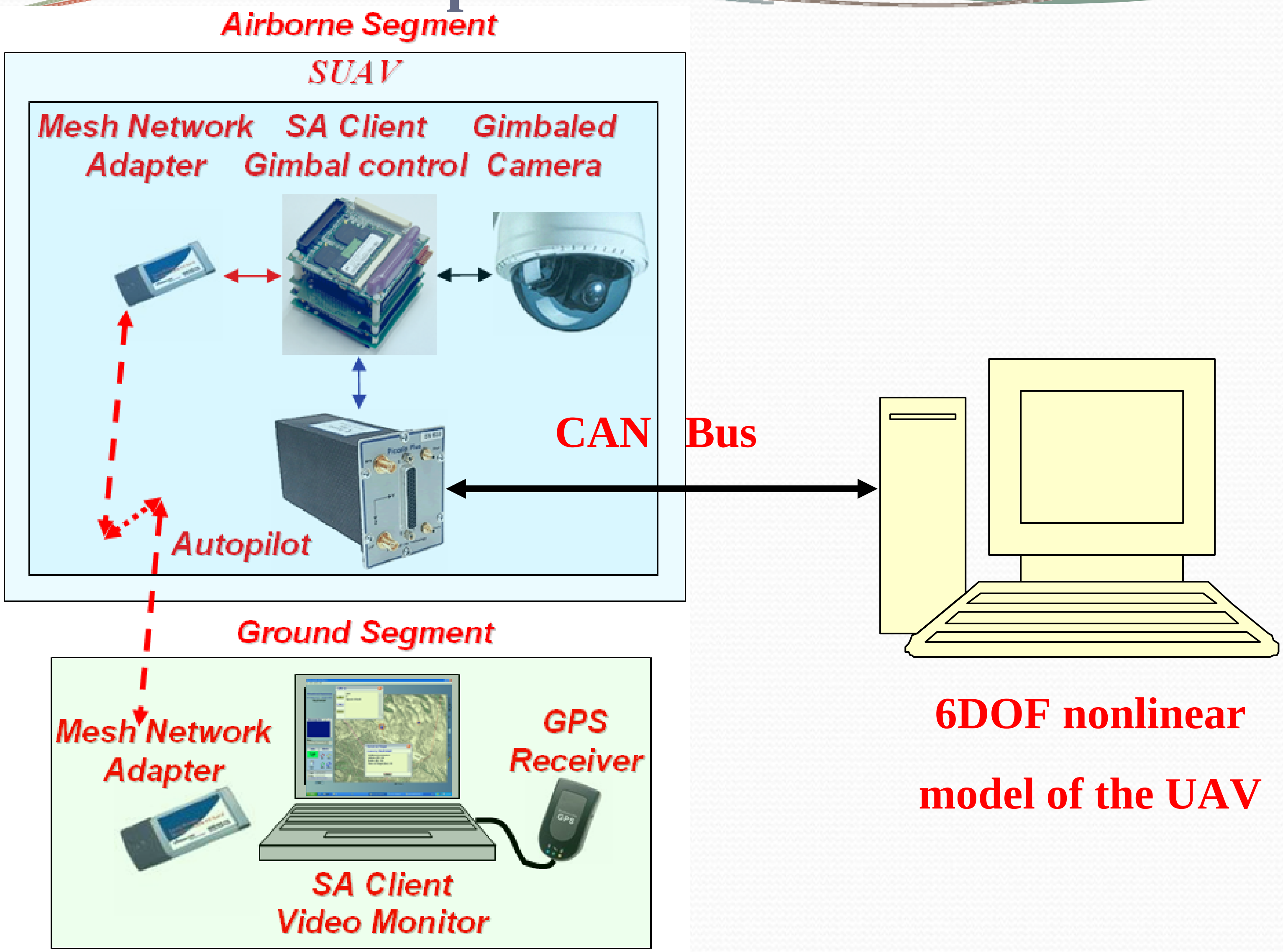


Path Following: Summary



- The cascaded system is UUB
- The UUB can be reduced via selection of the filter bandwidth and the reference system bandwidth
  - ➔ Reducing the UUB leads to reduced robustness
- **L1 guarantees that the region of attraction for kinematic errors does not change**

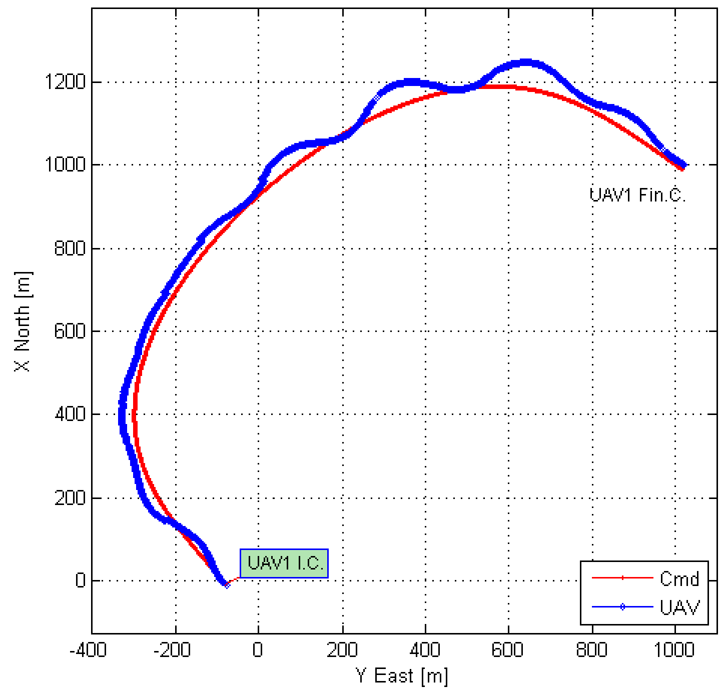
Hardware-in-the-Loop Simulation



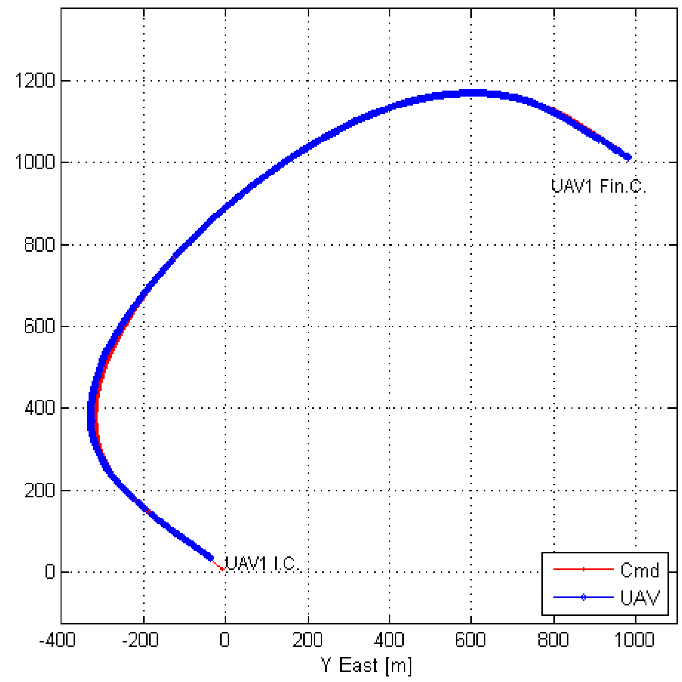
Hardware-in-the-Loop Simulation

- comparison: with and without adaptation

Path following
w/o L_1adaptation

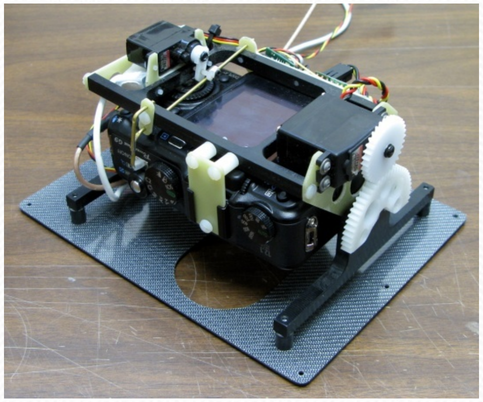


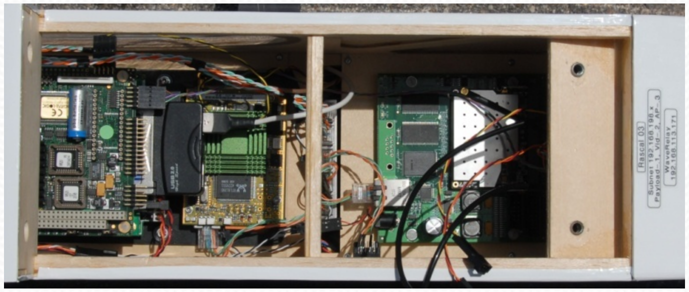
Path following
with L_1adaptation



Hardware (Second Generation)

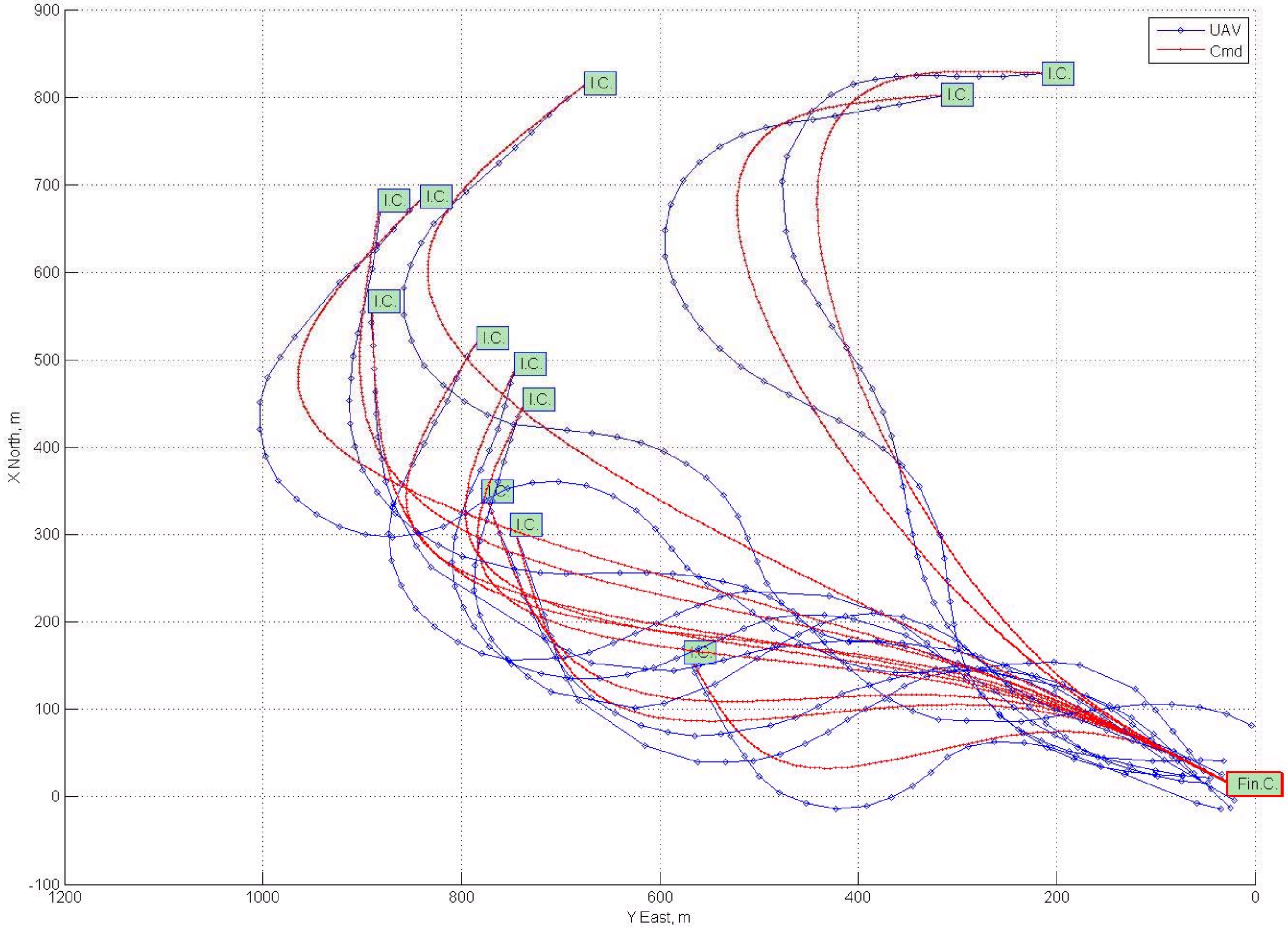
- **Networked Aircraft**
  - Airframe; Sig Rascal 110
    - 2.8 meter span, 8 kg
    - 26 cc gas engine
    - 2-3 hour endurance
    - 15-35 m/s velocity
  - Payload:
    - Cannon G9 12Mp gimbaled Camera
    - PC104 with Wave Relay Mesh card
    - PC104 for gimbal control and AP interface
      - ADL MSMT3SEG
    - PELCO-NET 350 video server



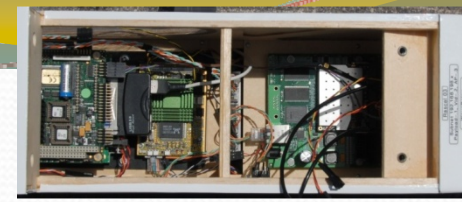


Flight Test Results: Path Following

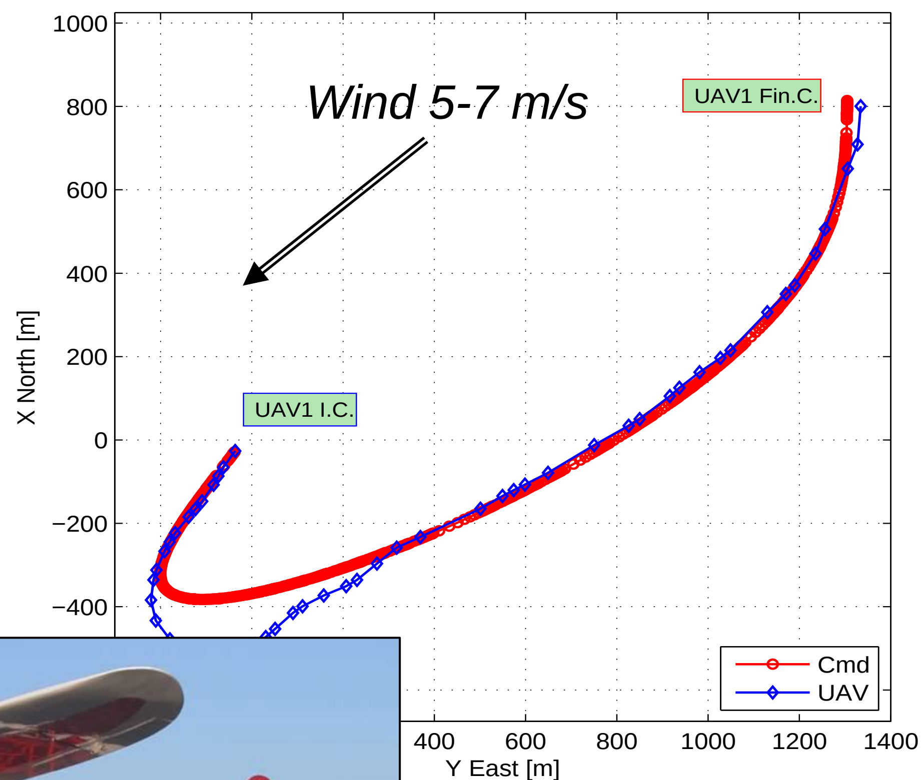
- no adaptation – effect of gain changes



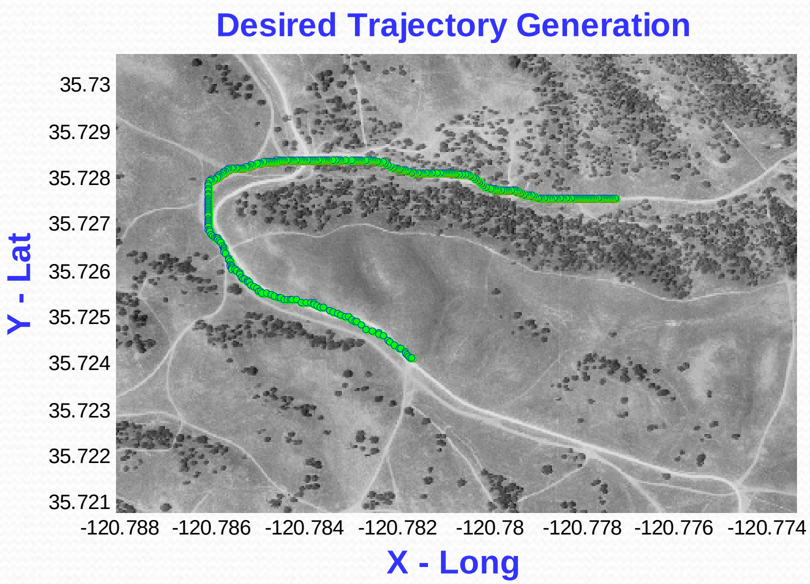
Flight Test Results: Path Following



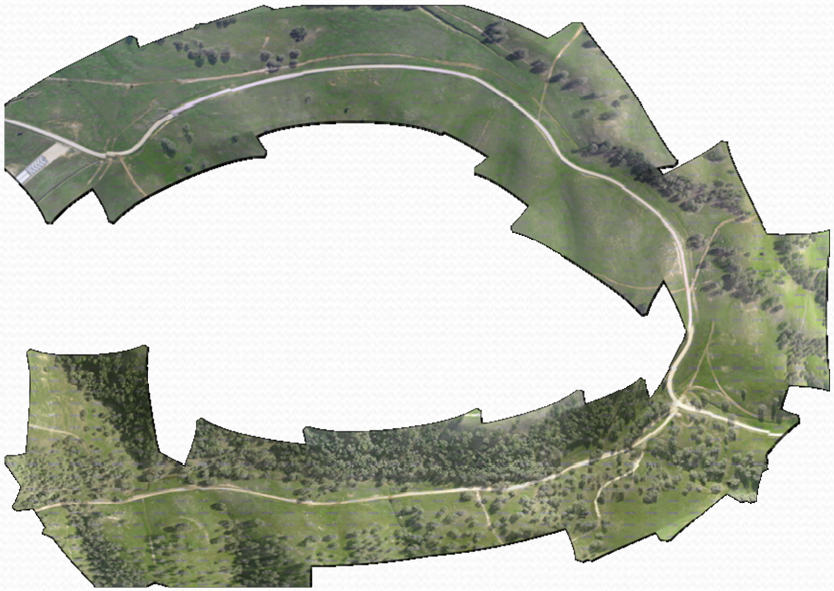
L1 ON



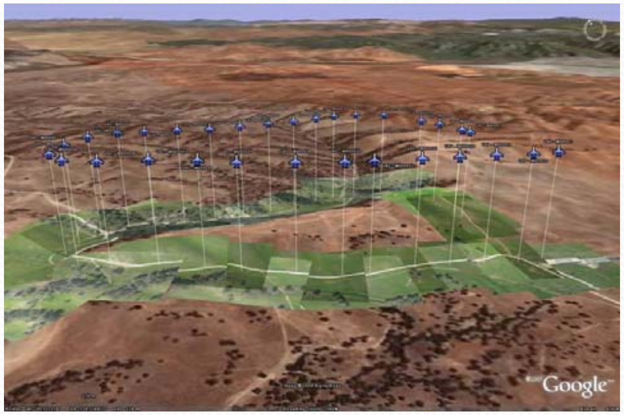
Feature Following using Path Following



Feature Selection



Resulting Mosaic using Overlaid G9 images



Google Earth Overlay

Time-Critical Coordinated Motion Control

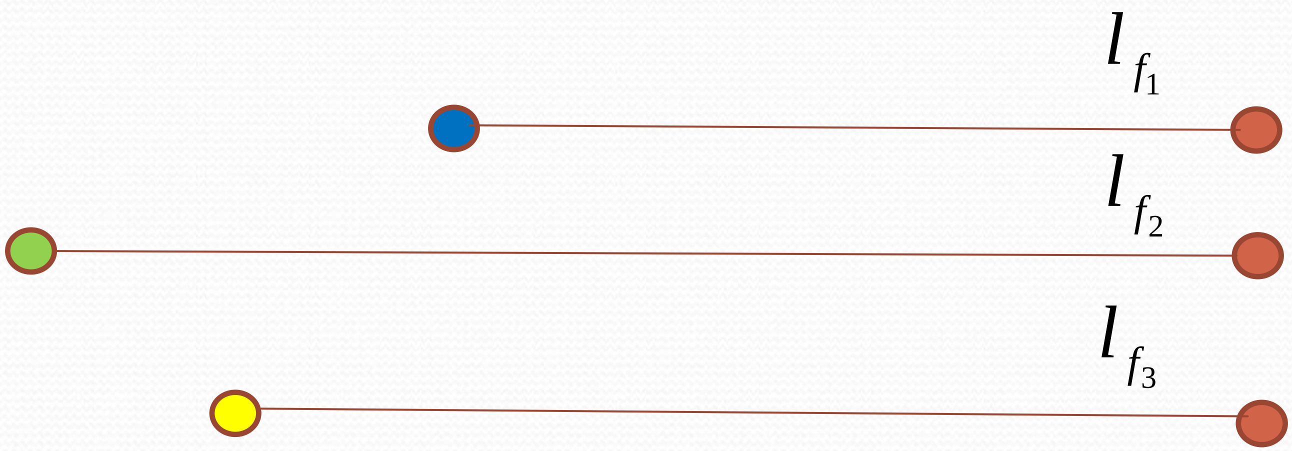
Target positions

,

UAVs : initial positions

UAVs must arrive at the same time;
absolute time is not a priority!

Time-Critical Coordinated Motion Control



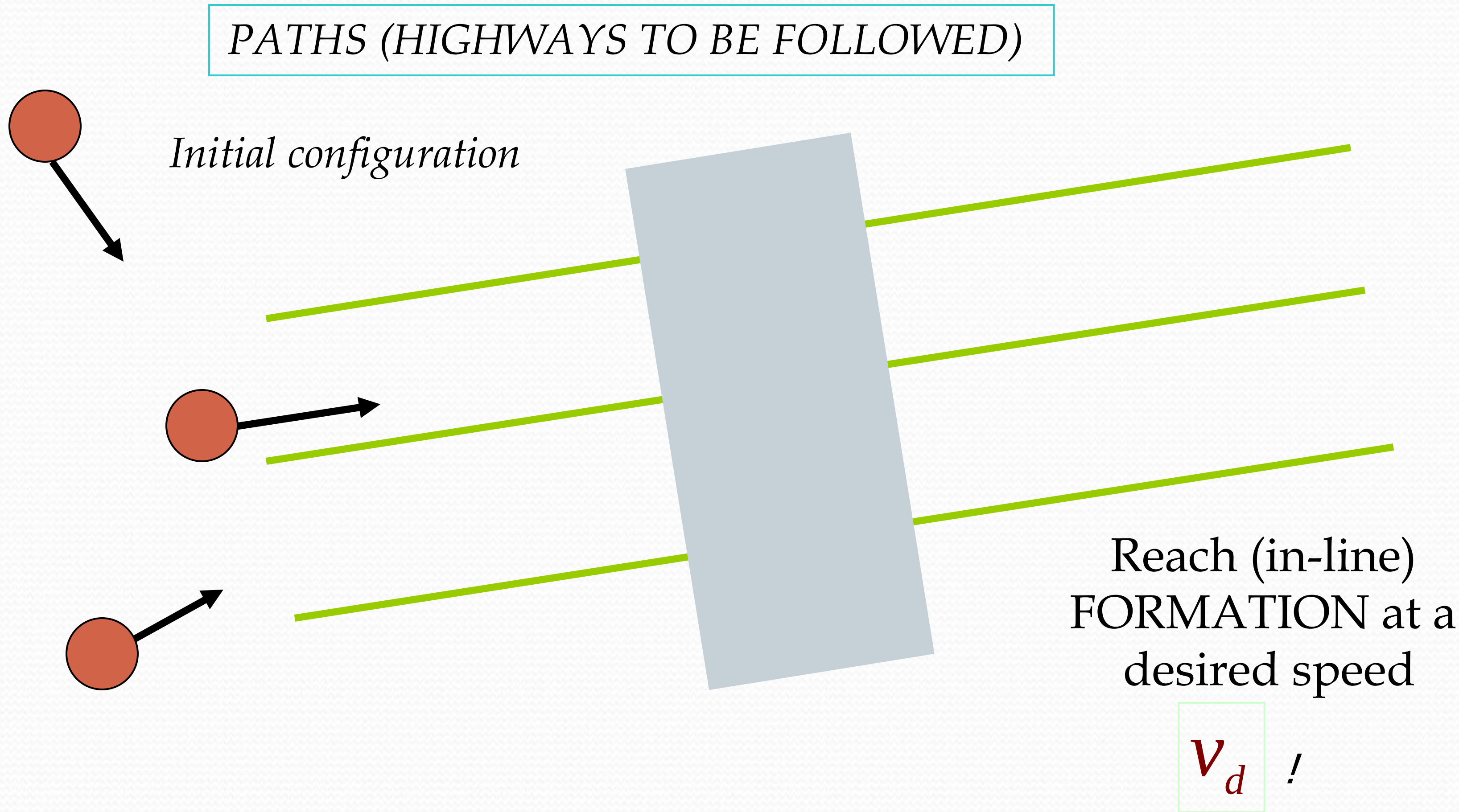
Paths parameterized by length (τ=l)

l_{f_i} - length of path for vehicle i

t_f - time of arrival (unknown)

$$l'_{f_i}(t) = l_i(t) / l_{f_i};\ l'_{f_i}(t_f) = 1,\ \forall \mathrm{i}$$

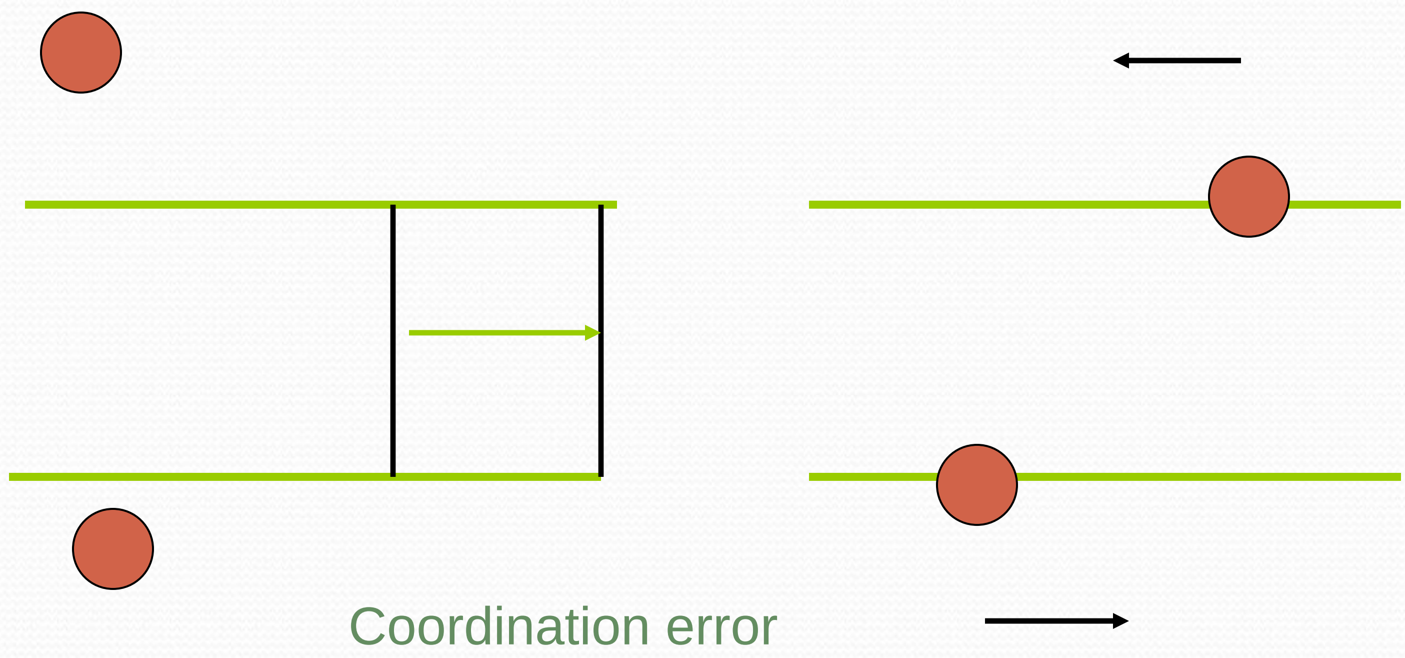
Coordinated Path Following



Divide to Conquer Approach

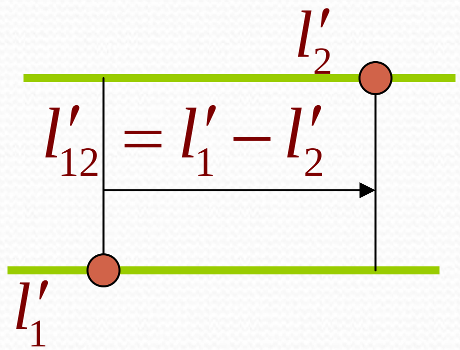
Each vehicle runs its own PATH FOLLOWING controller to steer itself to the path

Vehicles TALK and adjust their SPEEDS in order to COORDINATE themselves (reach formation)

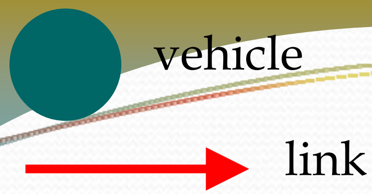


Coordination state / error

Coordination error (in-line formation): l'_{12}

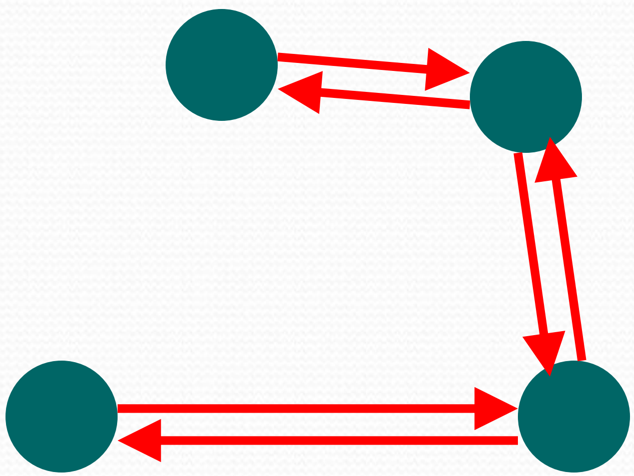


Normalized Path lengths l'_1 and l'_2

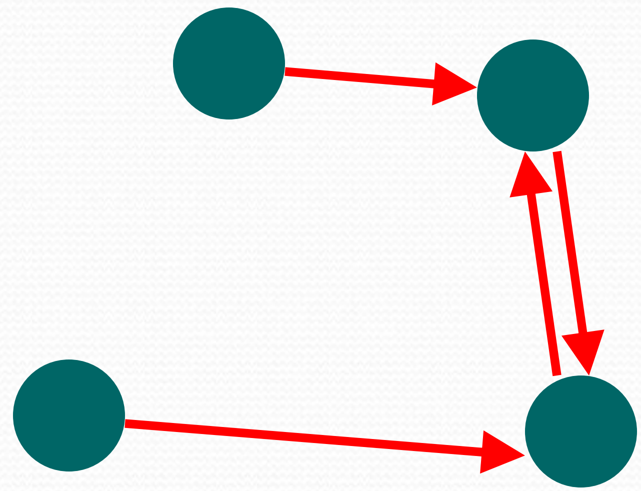


Communication Constraints

What is the communications topology? (**GRAPH**)



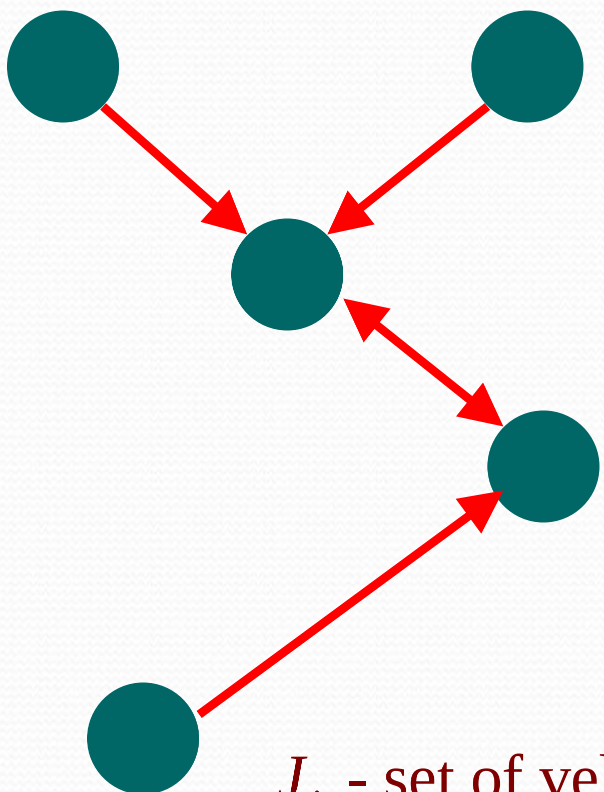
Bidirectional Links
→ *undirected graphs*



Non-bidirectional links
→ *directed graphs*

R. Murray [2002], B. Francis [2003], A. Jadbabaie [2003]

Communication Constraints



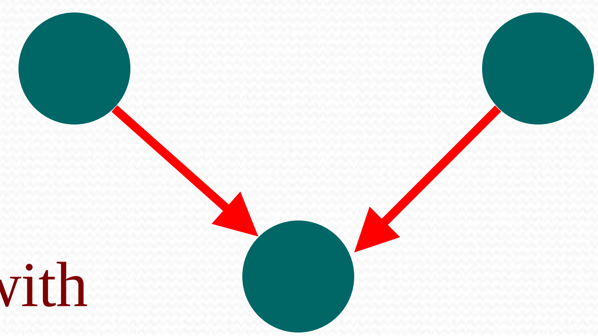
Communication Delays

Temporary Loss of Comms

Switching Comms Topology

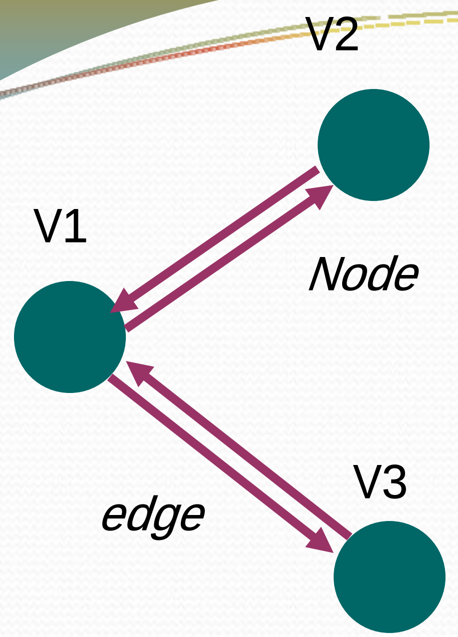
Asynchronous Comms

J_i - set of vehicles that vehicle i communicates with



Links with Networked Control and Estimation Theory

Communication Constraints



Adjacency Matrix A $= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

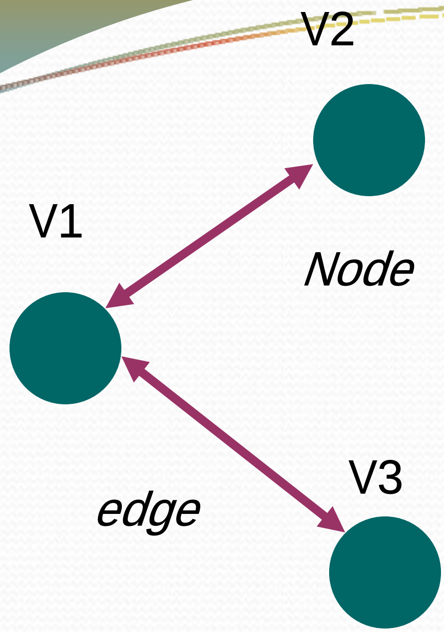
Degree Matrix D $= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

V1 receives info from *neighbours* V2 and V3

V2 receives info from *neighbour* V1

V3 receives info from *neighbour* V1

Communication Constraints



Laplacian

$$L = D - A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$L\begin{pmatrix} l_1' \\ l_2' \\ l_3' \end{pmatrix} = \begin{pmatrix} (l_1' - l_2') + (l_1' - l_3') \\ l_2' - l_1' \\ l_3' - l_1' \end{pmatrix}$$

Neighbour set 1={ V2 , V3}

Neighbour set 2={ V1}

Neighbour set 3={ V1}

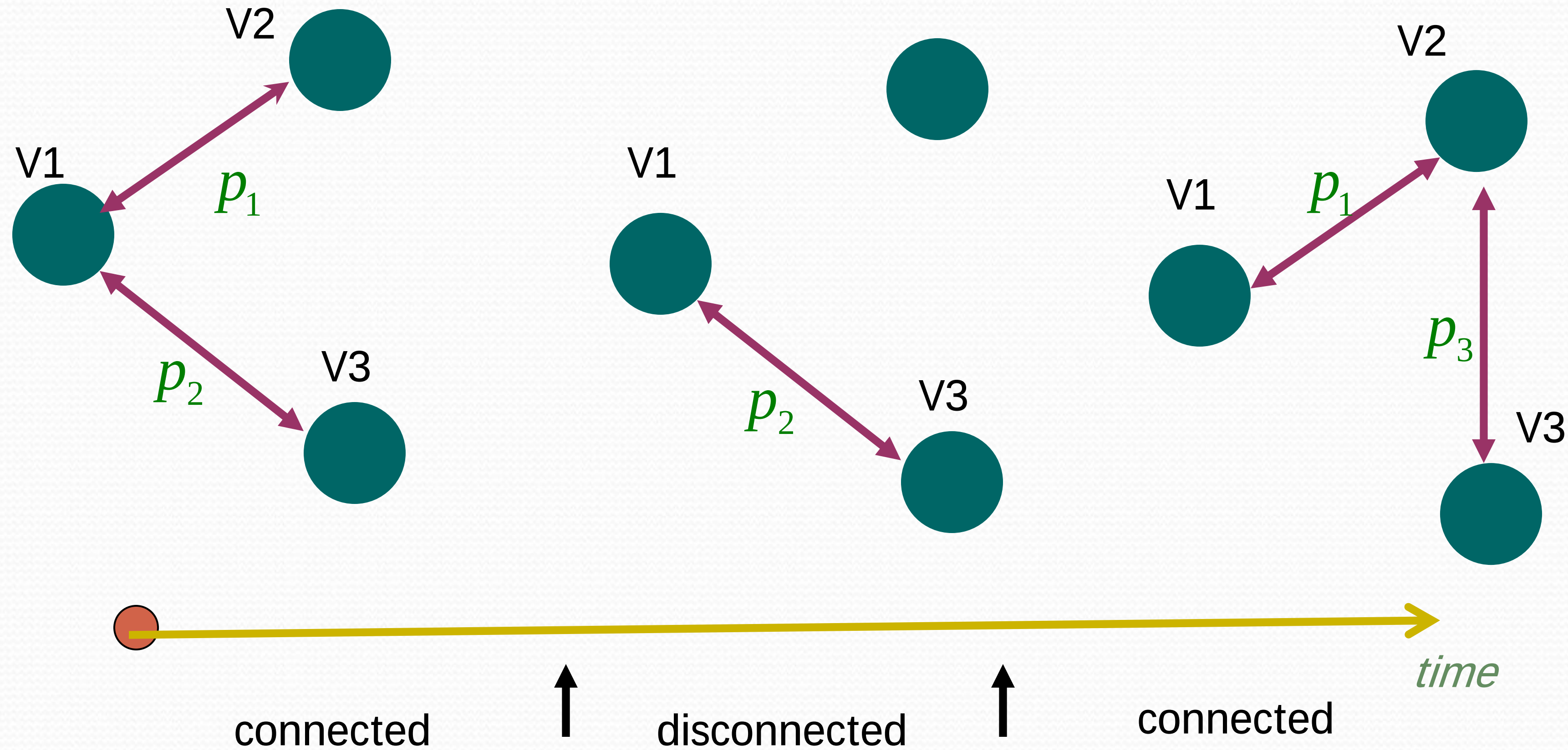
Properties:

- $L1 = 0$

-Graph is connected $\Rightarrow \text{rank} L = n - 1 = 2$

$$Ll' = 0 \Rightarrow l_1' = l_2' = l_3'$$

Switching Communications: brief connectivity losses



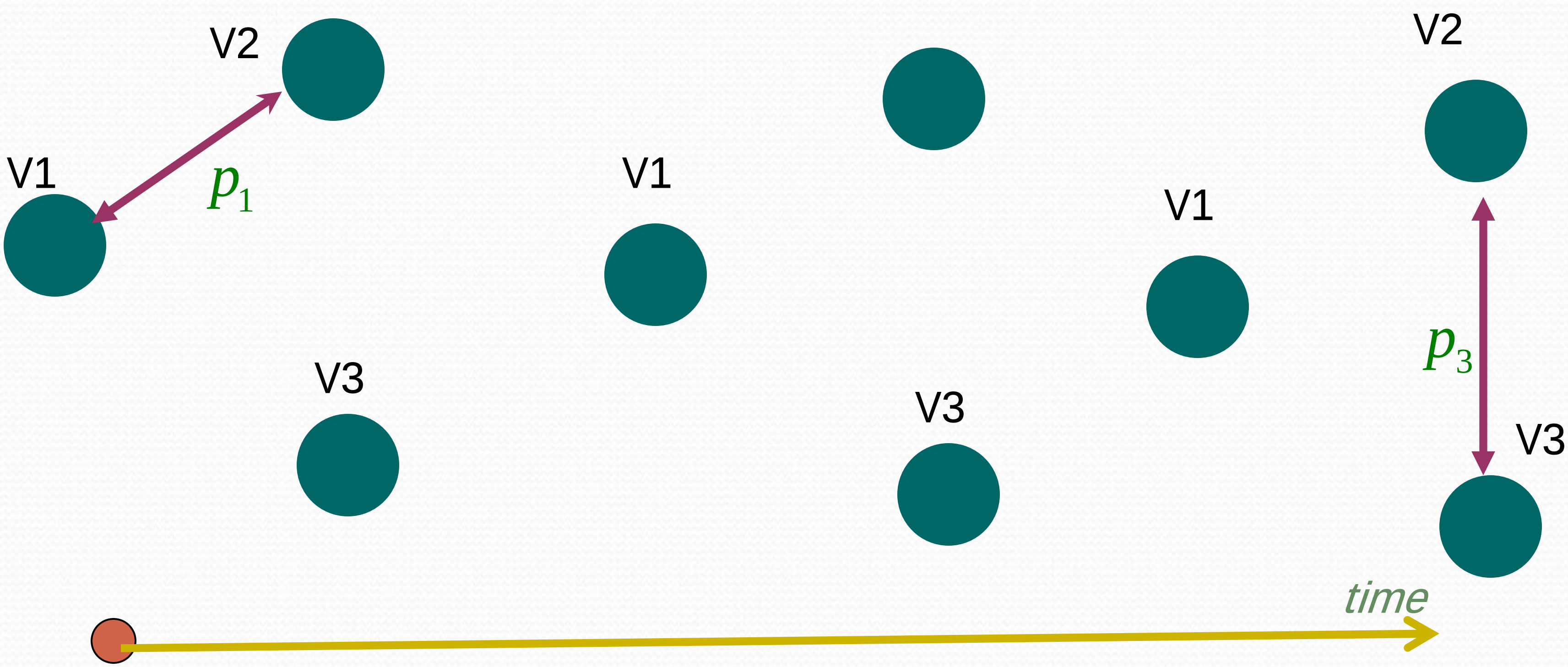
$p_1 = 1, p_2 = 1, p_3 = 0$

$p_1 = 0, p_2 = 1, p_3 = 0$

$p_1 = 1, p_2 = 0, p_3 = 1$

L is a function of p, *denoted* L_p

Switching Communications: "uniformly" connected in mean



$p_1 = 1, p_2 = 0, p_3 = 0$ ↑ $p_1 = 0, p_2 = 0, p_3 = 0$ ↑ $p_1 = 1, p_2 = 0, p_3 = 1$

L_{p_1} $\quad L_{p_2}$ $\quad L_{p_3}$

the union graph over time interval T is connected

$$L_{[t,t+T]} = L_{p_1} + L_{p_2} + L_{p_3} \Rightarrow \begin{cases} \operatorname{rank} L_{[t,t+T]} = n - 1 = 2 \\ L_{[t,t+T]} 1 = 0 \end{cases}$$

Time-Critical Coordinated Motion Control

Coordination Control Law

$$dl_i / dt = \kappa_1 x_{Fi} + v_i \cos\theta_{e.i} \cos\psi_{e.i}$$

Remember from Path Following

Adjust at the Coordination Level!

$$v_{c_i} = \frac{u_{\mathrm{coord}_i} l_{fi} - k_1 x_{F_i}}{\cos\theta_{e,i} \cos\psi_{e,i}}, \quad i = 1, \ldots, n,$$

Time-Critical Coordinated Motion Control

Coordination Control Law

Speed Asssignment

$$u_{\text{coord},1} = -a \sum_{j \in J_1} (l'_1 - l'_j) + \frac{v_{d,1}}{l_{f_1}}$$

$$u_{\text{coord},i} = -a \sum_{j \in J_i} (l'_i - l'_j) + \chi_{I,i}\,, \qquad i = 2, \ldots, n$$

$$\dot{\chi}_{I,i} = -b \sum_{j \in J_i} (l'_i - l'_j)\,, \qquad i = 2, \ldots, n$$

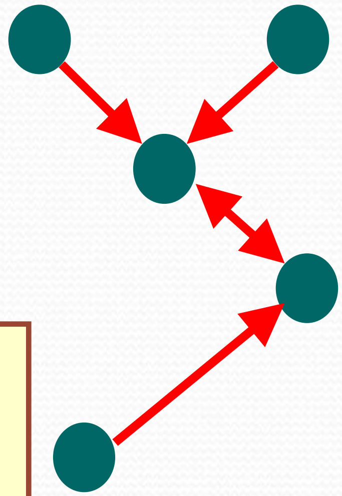
Integral term

Communications Topology

$$u_{\text{coord}}(t) = -aL(t)l'(t) + \begin{bmatrix} v_{d,1}(t)/l_{f_1} \\ \chi_I(t) \end{bmatrix}$$

$$\dot{\chi}_I(t) = -bCL(t)l'(t),$$

Time-Critical Coordinated Motion Control



Key results (example)

Suppose the Graph satisfies

$$\frac{1}{T}\int_t^{t+T} x^T Q L(\tau) Q^T x \, d\tau \geq \mu, \ \forall t \geq 0, \ \forall x \in R^{n-1}$$

$$\textit{for} \ \text{some T}, \ \mu > 0$$

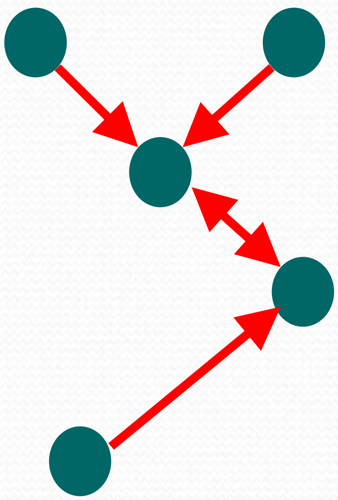
(graph does not even have to be connected at any time!)

⇓

"sensible" performance of the complete
Path Following + Coordination Controllers

Time-Critical Coordinated Motion Control

$$\|\zeta(t)\| \leq k_1 \|\zeta(0)\| e^{-\lambda t} + k_2 \sup_{\tau \in [0,t)} |\dot{v}_{d,1}(\tau)|$$

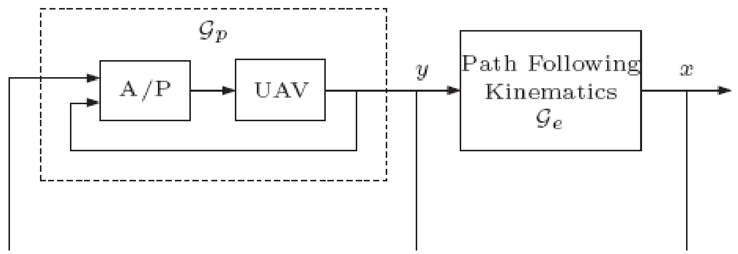


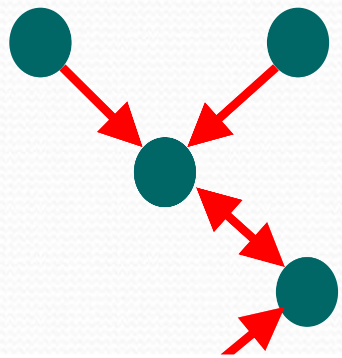
A function of normalized path lengths

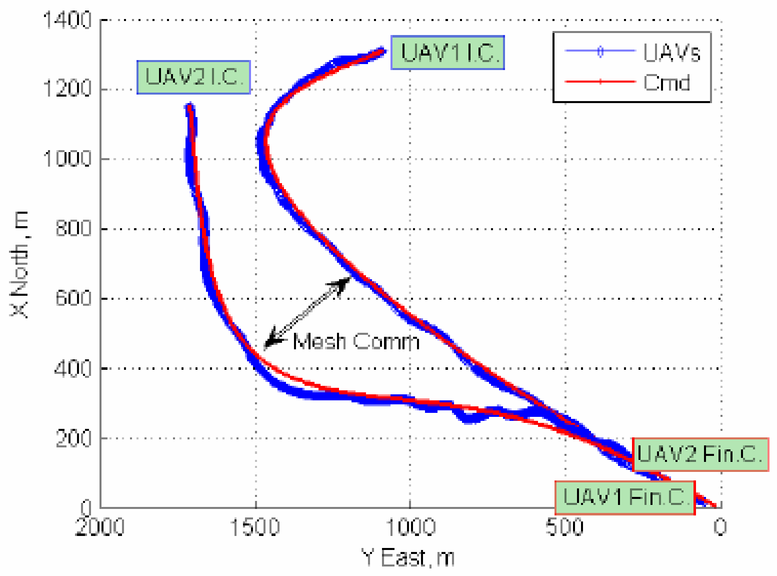
$$\lim_{t\to\infty} \sup \left| l_i'(t) - l_j'(t) \right| \leq k_4 \lim_{t\to\infty} \sup |\dot{v}_{d,1}(t)|$$

An ISS-like result

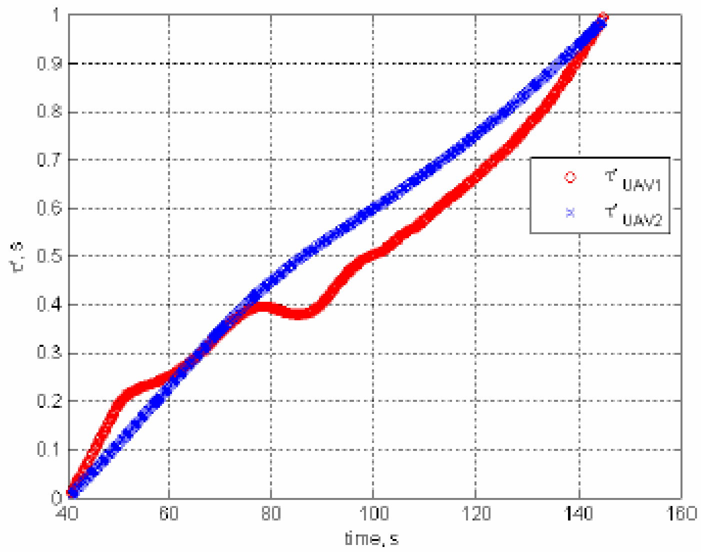
Time-Critical Coordinated Motion Control (HIL)





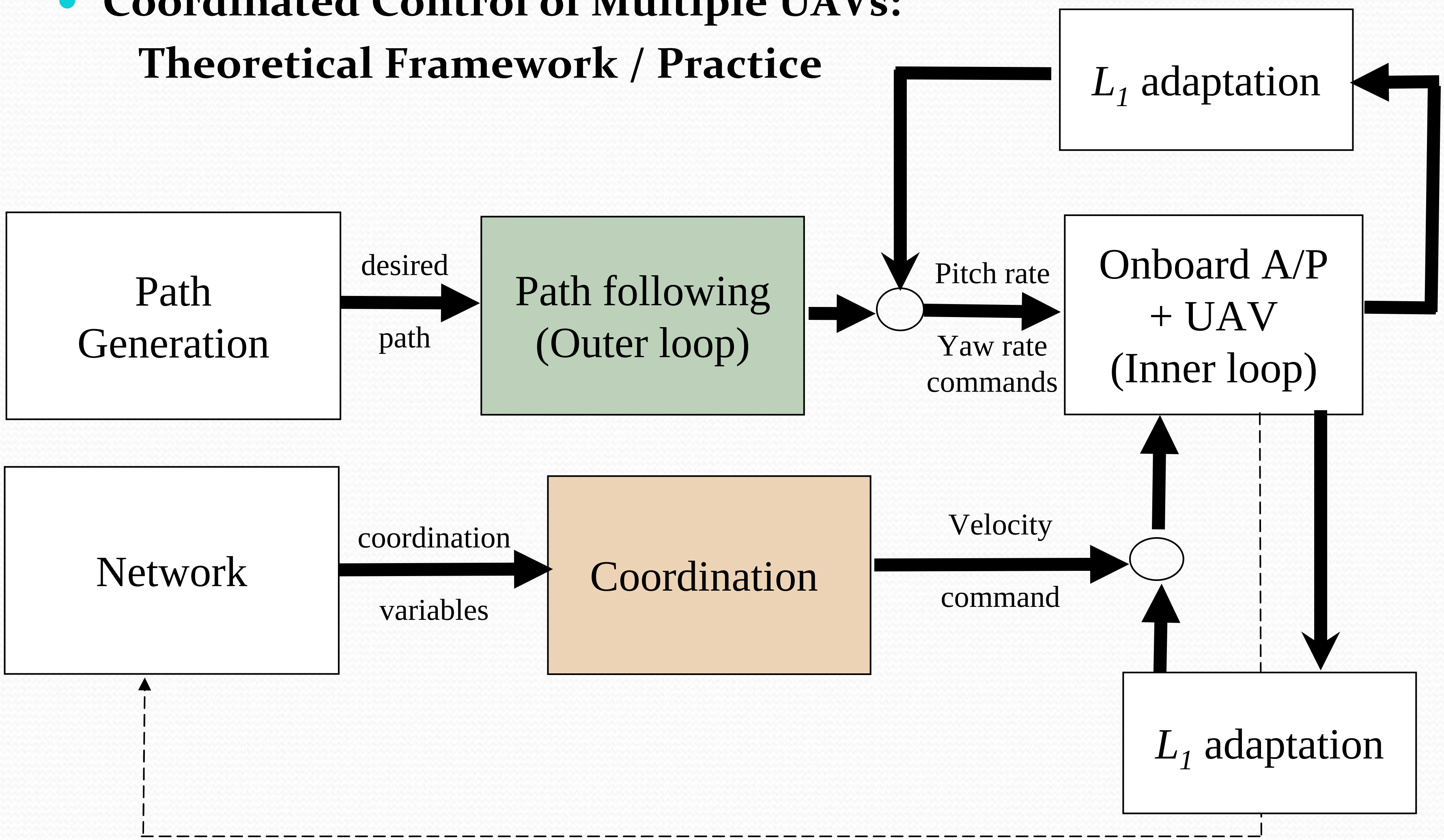


(a) UAV trajectories: 2D projection.



(b) Coordination states $l_i'(t)$.

- **Coordinated Control of Multiple UAVs: Theoretical Framework / Practice**



Coordinated Path Following of Multiple UAVs for Time-Critical Missions in the Presence of Time-Varying Communication Topologies

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