COORDINATED PATH FOLLOWING of MULTIPLE UNDERACTUATED VEHICLES WITH COMMUNICATION CONSTRAINTS

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Outline

- Motivating applications
- Coordinated path following (definition)
- Coordinated path following (wheeled robots): fixed communication topologies
- Coordinated path following
  - *general set-up* for path following
  - *underactuated* autonomous vehicles
  - *switching* communication topologies
Surface and underwater vehicles required to operate in a master / slave configuration
Coordinated Motion Control

The ASIMOV concept
(project ASIMOV, EC - 2000)
Coordinated Motion Control

Two AUVS carrying out a joint survey operation
Coordinated Motion Control

The quest for mid-water column hydrothermal vents, Azores, PT
Scientific Applications

- **Microsatellites**: imaging, remote sensing, interferometry

- **Autonomous Underwater Vehicles (AUVs)**: coordinated bathymetric mapping, seafloor imaging

- **Autonomous Surface Vessels (ASVs)**: coordinated ocean surveying

- **AUVs and ASVs**: coordinated control for fast communications and reliable navigation
Other applications

- **Unmanned Air Vehicles (UAVs):**
  - Formation control for increased aerodynamic efficiency
  - Coordinated search and rescue operations

- **Low cost small autonomous robots:**
  Exploration, mine detection, and neutralization

- **Cooperative manipulation**
How it all started at IST (1998) - ASIMOV

Dream

“Reality” (IST-NPS mission)

Theoretical problems: key issues

Coordinated Path Following while keeping inter-vehicle geometric constraints

Motion control in the presence of severe acoustic communication constraints (multipath, failures, latency, asynchronous comms, reduced bandwidth...)

Path Following

Inspired by the work of Claude Samson et al. for wheeled robots


✓ Use \textit{forward motion} to make the robot track a desired speed profile.

✓ Compute the \textit{closest point} on the path.

✓ Compute the \textit{Serret-Frenet (SF)} frame at that point.

✓ Use \textit{rotational motion} to align the body-axis with the SF frame and reduce the distance to closest point to zero.
Path Following

Important related work

R. Skjetne, T. I. Fossen, P. V. Kokotovic.
Robust output maneuvering for a class of nonlinear systems.

Avoiding Singularities!

Coordinated AUV / ASC behavior

Exploring an elegant concept introduced in


*Solution is too complex!*

Too much data exchanged between the vehicles

*“Combined Trajectory Tracking and Path Following: an Application to the Coordinated Control of Autonomous Marine Craft,” P. Encarnação and A. Pascoal, 40th IEEE Conference on Decision and Control, Orlando, Florida, USA, Dec. 2001*
Coordinated Path Following *(a fresh start)*
Coordinated Path Following *(a fresh start)*

**PATHS (HIGHWAYS TO BE FOLLOWED)**

*Initial configuration*

Reach (in-line) FORMATION at a desired speed $v_L$
IN-LINE FORMATION

Divide to Conquer Approach

Each vehicle runs its own PATH FOLLOWING controller to steer itself to the path

Vehicles TALK and adjust their SPEEDS in order to COORDINATE themselves (reach formation)

Coordination error
Coordination error
(in-line formation):

\[ s_{12} = s_1 - s_2 \]

Path lengths \( s_1 \) and \( s_2 \)
Coordinated Path Following (using the “inter-rabbit” distance)


They do not address communication constraints explicitly.
Communication Constraints

What is the communications topology? (GRAPH)

Bidirectional Links
→ undirected graphs

Non-bidirectional links
→ directed graphs

R. Murray [2002], B. Francis [2003], A. Jadbabaie [2003]
Communication Constraints

- Communication Delays
- Temporary Loss of Comms
- Switching Comms Topology
- Asynchronous Comms

Links with Networked Control and Estimation Theory
SINGLE VEHICLE, PATH FOLLOWING

1. Drive the distance from Q to the rabbit to zero;
2. Align the flow frame with the Serret-Frenet (align total velocity $V_t$ with the tangent to the path).

This will make the vehicle follow the path

“guide” (rabbit) moving along the path – “a mind of its own” (control variable)
COORDINATED PATH FOLLOWING

Triangle formation

In-line formation

More general formations and paths
COORDINATED PATH FOLLOWING

Coordination Error = error between the “rabbits”

Generalizable to multiple vehicles and other formation patterns, and paths
COORDINATED PATH FOLLOWING

KEY INGREDIENTS:

PATH FOLLOWING for each vehicle

+ Inter-vehicle COORDENATION

(driving the coordination errors to zero:
speed adjustments based on
VERY LITTLE INFO EXCHANGED)

(space-time decoupling … maths work out!)
Divide to Conquer Approach

*PATH FOLLOWING (each vehicle on its own)*, PF

*ALONG-PATH COORDINATION*, CC

But, they co-exist.

Analyze in detail!
Key results

Coordination achieved with

- fixed communication networks (ICAR’05, CDC’05, IJACSP)
- brief connectivity losses, general comm. losses, and time delays (SIAM-to be submitted, CDC’06, MCMC’06)
Fixed comm. networks

(ICAR’05, CDC’05, IJACSP)

Outline

- Path following: single vehicle
- Coordination error & path reparameterization
- Coordination dynamics
- Communication constraints & graphs
- Coordination control
Path following (single vehicle)

**Vehicle**: wheeled robot  
(underactuated vehicle w/ no side-slip)

**Control signal**: angular speed

**Stability**: Semi-global asymptotic convergence to the path.

**Condition**: \( \int_{0}^{\infty} |v(t)| \, dt = \infty \)

*Exponential convergence if* \( v \geq v_m > 0 \)
Path following (kinematics)

- Path following error vector and kinematics

\[
\begin{align*}
\dot{x}_e &= (y_e c_c(s) - 1) \dot{s} + \nu \cos \psi_e \\
\dot{y}_e &= -x_e c_c(s) \dot{s} + \nu \sin \psi_e \\
\dot{\psi}_e &= r - c_c(s) \dot{s}
\end{align*}
\]

- control signals
- exogenous signal

\(c_c(s)\) path curvature at
**Path following (problem)**

\[ \dot{r} = \frac{1}{J} N; \quad \dot{v} = \frac{1}{m} F \]

- **Problem:** Given a spatial path and a desired temporal profile \( v_d \) for the speed, derive feedback laws for \( N \) and \( \dot{s} \) to drive \( x_e, y_e, \psi_e, v - v_d \) to zero.

\[ N \text{ drives } x_e, y_e, \psi_e \text{ to zero (heading control)} \]
Path following, results

**MAIN result:** existence of control laws that solve the PF problem: error convergence is guaranteed

- if \( v(t) \) is uniformly continuous and does not vanish asymptotically.

**OR**

- \( \int_{0}^{\infty} |v(t)| \, dt = \infty \)
Path following (control strategy)

- Lyap. func. 
  \[ V_p = \frac{1}{2} x_e^2 + \frac{1}{2} y_e^2 + \frac{1}{2} (\psi_e - \sigma(y_e))^2 \]

- approach angle 
  \[ \sigma(y_e) = -\text{sign}(v)\sin^{-1}\left(\frac{k_2y_e}{|y_e| + \varepsilon}\right) \]

- time derivative 
  \[ \dot{V}_p = -k_1x_e^2 - k_2|v(t)|\frac{y_e^2}{|y_e| + \varepsilon} + -k_3(\psi_e - \sigma)^2 \]
  for some \( r \) and \( s \)

- do back stepping to find \( N \)
Coordination state / coord. error

- Use the RABBITS to define the coordination error!
- Coordination is achieved if the coordination states (CSs) are equal,
- The CS is a geometrical variable: arc length (shifted paths), angle (circumferences), or even more general
Coordination State / Path Reparametrization

- Paths parameterized by $\xi_i$.
  
  Vehicles $i$ and $j$ are coordinated if $\xi_i = \xi_j$.

- Define the function (arc length) $s_i = s_i(\xi_i)$.

- Define $R_i = \frac{\partial s_i}{\partial \xi_i}$.

The choice of $\xi_i$ must yield positive and bounded $R_i$.

Shifted paths:

$$s_i = \xi_i; \quad R_i(\xi_i) = 1$$

Circumferences:

$$s_i = R_i \xi_i, \quad R_i(\xi_i) = R_i \text{ (radii)}$$
Coordination State / Path Reparametrization

**45-degree example**

\[ \xi_1 = s_1; \quad \xi_2 = \sqrt{2}s_2 \]
\[ R_1 = 1; \quad R_2 = \sqrt{2} \]

**Sinusoidal example**

\[ \xi_1 = x_1 = s_1; \quad \xi_2 = x_2 \]
\[ R_1 = 1; \quad R_2 = \frac{ds_2}{d\xi_2} = \sqrt{1 + \cos^2 \xi_2} \]
Coordination state dynamics-1

- The rabbit’s dynamic for vehicle $i$
  \[ \dot{s}_i = v_i + (\cos \psi_{e,i} - 1)v_i + k_1 x_{e,i} \]

- Dynamics of coordination state $i$
  \[ \dot{\xi}_i = \frac{1}{R_i(\xi_i)} \dot{s}_i \implies \dot{\xi}_i = \frac{1}{R_i(\xi_i)} v_i + d_i \]

IMPORTANT: $d_i$ is guaranteed to vanish at the path following level \**IF**
$v_i$ does not blow up and $v_i$ does not tend to 0 (CAVEAT!)

The effect of the PF subdynamics appears as a “vanishing” disturbance in the Coo subdynamics.
Coordination state dynamics

- Objectives: \( \xi_i - \xi_j = 0 \)
  \[
  \dot{\xi}_i = v_L
  \]

- Making \( d_i = 0 \) from
  \[
  \dot{\xi}_i = \frac{1}{R_i(\xi_i)} v_i + d_i
  \]

  desired speed of vehicle \( i \) equals \( R_i(\xi_i) v_L \)

- define the speed tracking error
  \[
  \eta_i := v_i - R_i v_L
  \]
  \[
  \dot{\eta}_i = f_i := \frac{1}{m} F_i + \frac{d}{dt} R_i v_L
  \]
Coordination subsystem

Complete Fleet of Vehicles

\[ \dot{\eta} = f \quad \text{CONTROL VARIABLE} \]
\[ \dot{\xi} = C\eta + \nu_L 1 + d \]

- Make \( d \) equal to 0.
- Bring it into the picture at a later stage.

\[ C \] is a state-driven varying matrix:
\[ c_{i} = \frac{1}{R_i(\xi_i)} \]
\[ 0 < c_1 \leq C(\xi) \leq c_2 \]

- Problem: Derive control law for \( f \) so that
\[ \eta_i, \xi_i - \xi_j \] converge asymptotically to zero.

Communication topology comes into play!
use Graph theory
Communication Constraints

Adjacency Matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Degree Matrix $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

V1 receives info from *neighbours* V2 and V3
V2 receives info from *neighbour* V1
V3 receives info from *neighbour* V1
Communication Constraints

\[ L = D - A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \]

\[
\begin{align*}
L \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} &= \begin{pmatrix} (\xi_1 - \xi_2) + (\xi_1 - \xi_3) \\ \xi_2 - \xi_1 \\ \xi_3 - \xi_1 \end{pmatrix} \\
L \xi &= 0 \Rightarrow \xi_1 = \xi_2 = \xi_3
\end{align*}
\]

Properties:
- \( L1 = 0 \)
- Graph is connected \( \Rightarrow \) rank\( L = n - 1 = 2 \)
Communication constraints

• info available from a subset of the fleet: the neighboring vehicles, sets \( N_i \)

• bi-directional or directional comm.

• use graph Laplacian \( L \) to model comm. constraints declared by sets \( N_i \)
Coordination strategies

Complete Fleet of Vehicles (for $d = 0$)

\[ \dot{\eta} = f \]
\[ \dot{\xi} = C\eta + \nu_L 1 \]

- Comm. graph is **undirected** and **connected**
- **(MAIN results)** either of the following control laws solve the CC problem

\[ f = -A\eta - BCL\xi \]
\[ f = -(LC + C + A)\eta - AL\xi \]
\[ f = -(A^{-1}L + A)C\eta - B\text{sat}(\eta + A^{-1}L\xi) \]

- $L$ : underlying comm. graph Laplacian
- $A, B$ : positive diagonal matrices
- sat(.) : saturation function
Coordination strategies

\[ f = -A\eta - BCL\xi \]

vehicle \( i \), decentralized form \[ f_i = a_i \eta_i - \frac{b_i}{R_i} \sum_{j \in N_i} \xi_i - \xi_j \]

Challenges:

DONE!

1) when \( C(\xi) \) is varying
2) prove \( v_i(t) \) satisfies required conditions when putting together PF and CC

Switching comm. / failures / time delays are very important issues → next part of the talk
Part I- summary (contributions)

- Several solutions for coordination control
  - Bi-directional network (\( C(\xi) \) varying)
  - Non-bidirectional networks (\( C(\xi) = C \) constant)
- PF-CC interconnected system: error trajectories are bounded and converge to zero asymptotically. Namely, \( \nu(t) \) satisfies the required conditions.
General Path Following algorithms
Switching communications

Outline

• General under-actuated vehicle, PF
• Coordination under
  – Brief connectivity losses
  – General communications losses
    (Time delays)


**Path following**

- **Path-following problem**
  - Given a geometric path \( \{ y_d(\gamma) \in \mathbb{R}^3 : \gamma \in \mathbb{R} \} \) and a speed assignment \( v_r(t) \), we want

  - the position of the vehicle to converge to and remain inside an arbitrarily thin tube centered around the desired path
  - satisfy (asymptotically) the desired speed assignment, i.e.,
    \[
    \gamma \rightarrow v_r \text{ as } t \rightarrow \infty
    \]
Path following
(a very general set-up)

Vehicle dynamics

\[
\begin{aligned}
\dot{x}_i &= f_i(x_i, u_i) \\
y_i &= h_i(x_i)
\end{aligned}
\]

Following error

\[
e_i(t) = y_i(t) - y_{d,i}(\gamma_i(t))
\]

Speed tracking error

\[
\eta_i(t) = \dot{\gamma}_i(t) - v_{r,i}(t)
\]
Coordination, problem

**Coo. Dyn.** \( \dot{\gamma}_i = v_{r,i} + \eta_i \quad i = 1, \ldots, n \)

\( \eta_i \) a signal from PF closed-loop dyn.

**Coordination problem:**
- Derive a control law for \( v_{r,i} \)
- such that asympt. \( |\gamma_i - \gamma_j| \to 0, |\dot{\gamma}_i - v_L| \to 0 \)
- \( v_L(t) \): a given formation speed profile

Comm. needed to *exchange information*
Comm. subjected to *change* and time *delays*
Coordination, control law

**Coo. Dyn.** \[ \dot{\gamma}_i = v_{r,i} + \eta_i \quad i = 1, \ldots, n \]

**Proposed control**
\[ v_{r,i}(t) = v_L(t) - k_i \sum_{j \in N_{i,p(t)}} \gamma_i(t) - \gamma_j(t) \]

\( p(t) \) : a vector indicating which edge is active at time \( t \)

\( N_{i,p(t)} \) : Neighbors of vehicle \( i \) at time \( t \)

info. arrives with time delay
Coordination, closed-loop

Closed-loop dyn. in vector form no delays

\[ \dot{y} = -KL_{p(t)}y + \nu_L 1 + \eta \]

Closed-loop dyn. in vector form with delays

\[ \dot{y} = -KD_{p(t)}y(t) + KA_{p(t)}y(t-\tau) + \nu_L 1 + \eta \]

Two types of switching comm. considered
Switching Communication
brief connectivity losses

\[ p_1 = 1, p_2 = 1, p_3 = 0 \]

\[ p_1 = 1, p_2 = 0, p_3 = 1 \]

\[ p_1 = 0, p_2 = 1, p_3 = 0 \]

\[ L \] is a function of \( p \), denoted \( L_p \)
Brief Connectivity Losses

- Inspired by the concept of “brief instabilities” - Hespanha et. Al. IEEE Transactions AC 04

- **BCL:** the communication graph is connected and disconnected alternatively
Brief connectivity losses

Charac. function of switching topology:
\[ \chi(p) = \begin{cases} 
0 & \text{graph is connected} \\
1 & \text{graph is disconnected} 
\end{cases} \]

Connectivity loss time over \([t_1, t_2]\):
\[ T_p(t_1, t_2) = \int_{t_1}^{t_2} \chi(p(t)) \, dt \]

The comm. Network has BCL if \( T_p \leq \alpha(t_2 - t_1) + (1 - \alpha)T_0 \)

Asympt. connectivity loss rate: \( 0 \leq \alpha \leq 1 \)

Connectivity loss upper bound: \( T_0 > 0 \)

Example: periodically
- 10 sec connected
- 40 sec disconnected
\[ \alpha = 20\% \]
\[ T_0 = 40 \]
Brief connectivity losses

Connectivity loss time over $[t_1, t_2]$:

$$T_p(t_1, t_2) = \int_{t_1}^{t_2} \chi(p(t))dt$$

$$T_p \leq \alpha(t_2 - t_1) + (1 - \alpha)T_0$$

$$\frac{T_p}{t_2 - t_1} \leq \alpha + \frac{(1-\alpha)T_0}{t_2 - t_1} \implies \lim_{t_2 - t_1 \to \infty} \frac{T_p}{t_2 - t_1} \leq \alpha$$

If the graph is disconnected over $[t_1, t_2]$:

$$T_p = t_2 - t_1$$

$$T_p \leq \alpha T_p + (1 - \alpha)T_0 \implies T_p \leq T_0$$
Switching Communication
“uniform” connected in mean

\[ V_1 \rightarrow V_2 \quad V_1 \rightarrow V_3 \quad V_2 \rightarrow V_3 \]

\[ p_1 = 1, p_2 = 0, p_3 = 0 \quad p_1 = 0, p_2 = 0, p_3 = 0 \quad p_1 = 1, p_2 = 0, p_3 = 1 \]

\[ L_{p_1} + L_{p_2} + L_{p_3} \]

the union graph over time interval \( T \) is connected

\[ L_{[t,t+T]} = L_{p_1} + L_{p_2} + L_{p_3} \Rightarrow \]

\[ \begin{cases} \text{rank } L_{[t,t+T]} = n - 1 = 2 \\ L_{[t,t+T]}1 = 0 \end{cases} \]
Uniform Connected in Mean

• Inspired by work of
  – Moreau (CDC’04)
  – Lin, Francis, Maggiore (SIAM recent)

• **UCM:** there is a $T > 0$, such that the union communication graph is connected over any time interval of length $T$

  We assume a switching *dwell time* $\tau_D > 0$
  (time clearance between two consecutive switches)
Error space

Coo. dyn:\[\dot{y} = -KL_{p(t)}y + v_L1 + \eta\]

Error vector:\[\tilde{y} = L_\beta y\]

\[L_\beta = I - \frac{1}{\beta^T1}1\beta^T, \quad \beta = K^{-1}1\]

Important properties:\[\tilde{y} = 0 \iff y \in \text{span}\{1\}\]

\[\beta^T\tilde{y} = 0\]

\[\dot{\tilde{y}} = -KL_p\tilde{y} + L_\beta\eta\]

When \(p\) declares a connected graph

or

when \(L_p\tilde{y} \neq 0\)

\[\Rightarrow \tilde{y}^T L_p\tilde{y} \geq \lambda\tilde{y}^T\tilde{y}\]
Convergence, switching topology

\[ V = \frac{1}{2} \tilde{\gamma}^T K^{-1} \tilde{\gamma} \Rightarrow \dot{V} \leq \begin{cases} -\lambda_1 V + k \| \eta \|^2 & \text{when } L_p \tilde{\gamma} \neq 0 \\ \lambda_2 V + k \| \eta \|^2 & \text{otherwise} \end{cases} \]

We show that for both
- Brief Connectivity Losses with param. \( \alpha, T_0 \)
- Uniform Connected in Mean with param. \( T \)

\[ \begin{cases} \lambda_1 = \lambda_1 (1 - \alpha) & \text{for BCL} \\ \lambda_1 = \lambda_1 \left( \frac{\tau_D}{T + 2\tau_D} \right) & \text{for UCM} \end{cases} \]

We assumed that the PF control law guarantees \( \| \eta \| \rightarrow 0 \quad t \rightarrow \infty \)

but...
Path following

Vehicles dyn.
\[
\begin{cases}
    \dot{x}_i = f_i(x_i, u_i) \\
    y_i = h_i(x_i)
\end{cases}
\]

PF error
\[ e_i(t) = y_i(t) - y_{d,i}(\gamma_i(t)) \]

Speed tracking error
\[ \eta_i(t) = \dot{y}_i(t) - v_{r,i}(t) \]

a simple situation:
- exact following
  \[ \dot{y}_i(t) = v_{r,i}(t) = v_L(t) - k_i \sum_{j \in N_{i,p(t)}} \gamma_i(t) - \gamma_j(t) \]

The PF control law requires \( \frac{d}{dt}(y_{d,i}(\gamma_i)) \) and higher derivatives then, it requires \( \gamma_i, \dot{\gamma}_i \)

Only \( \dot{v}_L \) is available
\[ v_{ri} = v_L + \tilde{v}_{ri} \]
Path following results

• **New results**: For a general underactuated vehicle, there is a control law $u_i$ that uses $v_L$, $\dot{v}_L$ instead of $\gamma = v_L + \tilde{v}_{ri}$ such that the closed-loop PF is ISS with input $\tilde{v}_{ri}$.
PF and CC interconection

Proof of convergence.

Key Ingredient: a new small gain theorem for systems with brief instabilities
PF and CC interconection

Main Result A (Brief Connectivity Losses)

For any choice of connectivity parameters $T$ and $\alpha$, there exist PT and CC gains that yield convergence of the complete system error trajectories to an arbitrarily small neighborhood of the origin.
Main Result B (Connected in Mean)

For any choice of average connectedness time $T$, there exist PT and CC gains that yield convergence of the complete system error trajectories to an arbitrarily small neighborhood of the origin.
Summary

• Path following control for a general underactuated vehicle: ISS from input $\tilde{V}_r$

• Coordination strategies under switching communications
  – Brief connectivity losses
  – Uniform connected in mean

• PF-CC interconnection
Simulations (ASCraft, fully actuated)

[João Almeida, MSc, IST-2006]

In-line formation

Triangle formation
Simulations (Underactuated AUVs)
Simulations, no failures

coordination errors

path following errors
Simulations, 75% failure

coordination errors

path following errors

With 75% of communications failures occurring periodically with $T=20s$
Contributions, Coordination control

- Fixed networks
  - Proper set-up for coordinated path following, general coordination patterns and paths.
  - Several solutions for bidirectional and non-bidirectional networks.
  - Convergence guaranteed when putting together PF and CC systems (dynamics of the vehicle directly taken into account!)
Contributions, Coordination control

- Switching networks
  - Coordination guaranteed under switching communications
    - Brief connectivity losses
    - Uniform connectedness in mean
  - Switching communications with delays
  - Small gain theorem for systems with brief failures
    - Convergence guaranteed when putting together PF and CC systems
Future work

- Information exchange in discrete set-up, asynchronous
- Non-equal and varying time delays
- Coordinated navigation