Nonlinear Control Systems

Homework #7

(Due date: May 14th, 2012)

May 7, 2012

1. Consider the system

$$\dot{x} = f(x) + G(x)[u + \delta(x, u)],$$

and suppose that there are known smooth functions $\phi(x)$, $V(x)$, and $\rho(x)$, all vanishing at $x = 0$, and a known constant k such that

$$c_1 \leq \|x\|^2 \leq V(x) \leq c_2\|x\|^2, \qquad \frac{\partial V}{\partial x}[f(x) + G(x)\phi(x)] \leq -c_3\|x\|^2$$

and

$$\|\delta(x, \phi(x) + v)\| \leq \rho(x) + \kappa_0\|v\|, \ 0 \leq \kappa_0 < 1, \ \forall x \in \mathbb{R}^n, \forall v \in \mathbb{R}^p$$

where c_1 to c_3 are positive constants.

a) Show that it is possible to design a continuous state feedback controller $u = \gamma(x)$ such that the origin of

$$\dot{x} = f(x) + G(x)[\gamma(x) + \delta(x, \gamma(x))],$$

is globally exponentially stable.

b) Apply the result of part (a) to the system

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= (1 + a_1)(x_1^3 + x_2^3) + (1 + a_2)u \end{aligned}$$

where a_1 and a_2 are unknown constants that satisfy $|a_1| \leq 1$ and $|a_2| \leq 1/2$

2. Using backstepping, design a state feedback controller to globally stabilize the system

$$\begin{aligned} \dot{x}_1 &= x_2 + a + (x_1 - a^{1/3})^3 \\ \dot{x}_2 &= x_1 + u \end{aligned}$$

where a is a known constant.