Nonlinear Control Systems

Homework #5

(Due date: April 11, 2012)

April 2, 2012

1. Consider the nonautonomous system

$$\dot{x} = f(t, x), \tag{1}$$

where $f \in C^1$, $\frac{\partial f}{\partial x}$ is bounded, uniformly in t, and $x = 0$ is GES.

The system is perturbed such that

$$\dot{x} = f(t, x) + g(t, x). \tag{2}$$

For γ small enough, $||g(t, x)|| \leq \gamma ||x|| + \delta$, $g(t, 0) = 0$, and $\delta > 0$. **Without** using ISS:

a) Show that $x = 0$ is globally uniformly ultimately bounded.

b) Find a expression for b, the ultimate bound.

c) Assume $\delta = 0$. Use the converse theorem to show that $x = 0$ is GES.

2. Consider the following systems

$$\dot{x}_1 = -x_1^3 + x_2 \tag{3}$$

$$\dot{x}_2 = -x_2^3. \tag{4}$$

Show that the origin of the system (3) and (4) is GAS using ISS of cascaded systems.

3. For the following systems investigate $\mathcal{L}_\infty$ and finite-gain $\mathcal{L}_\infty$ stability:

1. $\dot{x}_1 = -x_1 + x_1^2 x_2 \qquad \dot{x}_2 = -x_1^3 - x_2 + u \qquad y = x_1$