Nonlinear Control Systems

Homework #3

(Due date: March 21, 2012)

March 14, 2012

1. The Euler equations of a rotating rigid spacecraft are given by

$$\begin{aligned} J_1\dot{\omega}_1 &= (J_2 - J_3)\omega_2\omega_3 + u_1 \\ J_2\dot{\omega}_2 &= (J_3 - J_1)\omega_3\omega_1 + u_2 \\ J_3\dot{\omega}_3 &= (J_1 - J_2)\omega_1\omega_2 + u_3 \end{aligned} \tag{1}$$

where ω_1 to ω_3 are the components of the angular velocity vector $\boldsymbol{\omega}$ along the principal axes, u_1 to u_3 are the torque inputs applied about the principal axes, and J_1 to J_3 are the principal moments of inertia.

a) Show that with $u_1 = u_2 = u_3 = 0$ the origin $\boldsymbol{\omega} = \mathbf{0}$ is stable. Is it asymptotically stable?

b) Suppose that the torque inputs are given by the feedback control $u_i = -k_i\omega_i$, $k_i > 0$. Show that the close loop system is GAS.

c) Consider $u_2 = u_3 = 0$, what can you say about the stability of the system?