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Reinforcement learning in general state spaces

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Reading group on Sequential Decision Making

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Outline of the presentation

- **Background**
- Some history on RL with FA
- TD(0) with linear function approximation
- Interpolated Q-learning
- References

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Background

A *Markov chain* is a pair $(\mathcal{X}, \mathsf{P})$ where

- $\mathcal{X}$ is the (general) state-space;
- P is a *transition probability kernel*:

$$\mathsf{P}(x, U) = \mathbb{P}\left[X_{t+1} \in U \mid X_t = x\right];$$

- *Positive chains* admit an invariant probability measure μ;
- *Geometrically ergodic chains* converge exponentially fast to μ:

$$\sum_{t=0}^{\infty} \rho^t \left\|\mathsf{P}^t(x, \cdot) - \mu(\cdot)\right\| \leq \infty,$$

for all $x \in \mathcal{X}$, with $\rho > 1$.

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Background (2)

An *MDP* is a tuple $(\mathcal{X}, \mathcal{A}, \mathsf{P}, r, \gamma)$ where

- $\mathcal{X}$ is the (general) state-space;
- $\mathcal{A}$ is the finite action-space;
- P is a controlled *transition probability kernel*:

$$\mathsf{P}_a(x, U) = \mathbb{P}\left[X_{t+1} \in U \mid X_t = x, A_t = a\right];$$

- $r : \mathcal{X} \times \mathcal{A} \times \mathcal{X} \to \mathbb{R}$ is a reward function;
- γ is a discount factor.

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Background (3)

- The optimal value function is

$$V^*(x) = \max_{\{A_t\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R(X_t, A_t) \mid X_0 = x\right];$$

- V^* verifies the Bellman optimality equation

$$V^*(x) = \max_{a \in \mathcal{A}} \int_{\mathcal{X}} \big[r(x,a,y) + \gamma V^*(y)\big] \mathsf{P}_a(x, dy);$$

- The optimal Q-function is simply

$$Q^*(x,a) = \int_{\mathcal{X}} \big[r(x,a,y) + \gamma V^*(y)\big] \mathsf{P}_a(x, dy).$$

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Background (4)

- TD(0) evaluates a policy δ using the update

$$V_{t+1}(X_t) = V_t(X_t) + \alpha_t\big[R_t + \gamma V_t(X_{t+1}) - V_t(X_t)\big];$$

- Q-learning determines the optimal Q^* using the update

$$Q_{t+1}(X_t, A_t) = Q_t(X_t, A_t) + \alpha_t\big[R_t + \gamma \max_{b \in \mathcal{A}} Q_t(X_{t+1}, b) - Q_t(X_t, A_t)\big].$$

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Some history

- Samuel's pioneer works in machine learning (late 50's/early 60's) describe an artificial checker's player and a "feature"-based approximation [19, 20];
- In 1992, Tesauro combined $TD(\lambda)$ and non-linear function approximation (using a neural network). Its results boosted the interest int the problem of generalization [27, 28, 29];
- In 1993, Thrun and Schwartz discuss the use of reinforcement learning with function approximation [30];
- Singh et al. propose the use of *soft-state aggregation* with reinforcement learning, *proving convergence w.p.1* of the obtained method;

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Some history (2)

- Soft-state aggregation was further addressed by Gordon [12] and Tsitsiklis and Van Roy [31];
- Baird [3] and Gordon [11] provide divergent counter-examples for Q-learning and SARSA. Baird proposes the use of gradient ascent algorithms, to overcome the convergence limitations of standard RL methods with function approximation [2, 4];
- Boyan and Moore [7] and Sutton [23] experimentally evaluate several approximation architectures;
- Tsitsiklis and Van Roy provide a fundamental analysis of TD(λ) with function approximation, establishing convergence w.p.1 for linear function approximation [32];
- The fundamental insight was further explored in other works [5, 8, 9]

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Some history (3)

Recent days have witnessed several new and exciting works, namely:

- Functional Q-learning [6];
- Off-policy TD(λ) with established convergence [17];
- Kernel-based reinforcement learning [16];
- Interpolation based Q-learning [26];
- Other [1, 13, 15, 25].

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Main idea

- Require the sampling policy to yield a geometrically ergodic chain;
- Aproximate the algorithm

$$\theta_{t+1} = \theta_t + \alpha_t H(\theta_t, X_{t+1})$$

by considering the "average" algorithm

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t + \alpha_t \mathbb{E}_\mu \left[H(\theta_t, X_{t+1}) \right].$$

- Writing $h(\theta) = \mathbb{E}_\mu \left[H(\theta, X_{t+1}) \right]$, analyze the ODE

$$\dot{\theta}_t = h(\theta_t).$$

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TD(0) with linear function approximation

- Consider a fixed policy δ;
- Aproximate the function V^δ as a linear combination of basis functions ξ_i, $i = 1, \ldots, M$:

$$V^\delta(x) \approx \tilde{V}(x, \theta) = \sum_{i=1}^{M} \xi_i(x)\theta_i = \xi^\top(x)\theta.$$

- Iterate in θ using the update

$$\theta_{t+1} = \theta_t + \alpha_t \xi(X_t)\big[R_t + \gamma V(X_{t+1}, \theta_t) - V(X_t, \theta_t)\big].$$

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TD(0) with linear function approximation

- Using the previous approach, the ODE becomes

$$\dot{\theta}_{t+1} = \mathbb{E}_\mu\left[\mathbf{A}(X_t)\right]\theta_t + \mathbb{E}_\mu\left[\mathbf{b}(X_t)\right],$$

with a negative definite matrix A. Then,

$$\theta^* = -\mathbb{E}_\mu\left[\mathbf{A}(X_t)\right]^{-1}\mathbb{E}_\mu\left[\mathbf{b}(X_t)\right]$$

is a *globally asymptotically stable equilibrium point* of the ODE;

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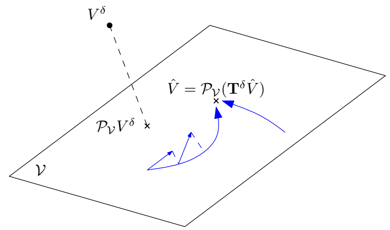
TD(0) with linear function approximation

The limit point is the fixed point $\hat{V} = \mathcal{P}_\mathcal{V}\mathbf{T}^\delta\hat{V}$.



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Problems with Q-learning:

The fixed point $\hat{V} = \mathcal{P}_{\mathcal{V}} \mathbf{T}^{\delta} \hat{V}$ exists because:

- $\mathbf{T}^{\delta}$ is a *contraction* in the 2-norm;
- $\mathcal{P}_{\mathcal{V}}$ is a *non-expansion* in the 2-norm;
- The combined operator $\mathcal{P}_{\mathcal{V}} \mathbf{T}^{\delta}$ is a contraction in the 2-norm.

But the operator $\mathbf{H}$ associated with Q^* is a contraction *in the sup-norm*...

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Interpolation-based Q-learning

- An idea is to use a "projection-like" operator $\mathcal{P}$ that is a non-expansion in the sup-norm;
- Interpolation-based Q-learning defines a set of points $I = \{(x_1, a_1), \ldots, (x_M, a_M)\}$ and uses the projection-like operator

$$(\mathcal{P}q)(x, a) = \mathcal{F}_{\theta}(x, a),$$

where $\mathcal{F}_{\theta}$ is a *convex interpolator* and θ is a vector such that $\theta_i = q(x_i, a_i)$;

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Interpolation-based Q-learning

- The parameters θ are updated using the rule

$$\theta_{t+1} = (1 - \alpha)\theta_t + \alpha_t g_{\varepsilon}(X_t, A_t)\big[R_t + \gamma \max_{b \in \mathcal{A}} F_{\theta_t}(X_{t+1}, b), \theta_t)\big];$$

- The limit point is now the fixed point $\hat{Q} = \mathcal{P}\hat{\mathbf{H}}\hat{Q}$.

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