NONLINEAR KALMAN BASED FILTERING FOR POSE ESTIMATION OF A ROBOTIC VEHICLE FROM DISCRETE ASYNCHRONOUS RANGE MEASUREMENTS

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Abstract: We consider the problem of estimating the position and orientation of a robotic vehicle moving in a 2D plane using range measurements from the vehicle to fixed beacons that arrive synchronously/asynchronously at discrete instants of time. In this setup we assume that the vehicle is equipped with only one range measuring sensor and that the set composed by the measurements at each fixed time may not be complete. We analyze the observability of the system and in particular we show that there exist conditions under which the system is not observable. Two nonlinear Kalman based filtering algorithms are proposed: the extended Kalman filter (EKF) and the unscented Kalman filter (UKF). Simulation results are used to compare the performance of the filters. The results show that when the measurements arrive synchronously, both the filters exhibit good performance if the initial estimation errors are small. In this case we did not notice any major difference in the estimation accuracy between the two filters. When the range measurements are not received at the same time, the performance of the filters degrade as it is expected.

Keywords: Nonlinear estimation, Extended Kalman filter, Unscented Kalman filter, Asynchronous Measurements

1. INTRODUCTION

The problem of state estimation of a given process using range measurements arises in many practical situations and has been addressed by several authors (Athans *et al.*, 1968; Wishner *et al.*, 1969; Julier *et al.*, 2000; Gustafsson and Gunnarsson, 2005; Li *et al.*, 2006; Mao *et al.*, 2007).

In (Athans *et al.*, 1968; Wishner *et al.*, 1969), the localization problem of a falling body using discrete noisy radar range measurements was addressed. The authors propose a suboptimal state estimation that address the continuous time dynamics of the process with the discrete noisy measurements. In (Wishner *et al.*, 1969) three nonlinear filters have been compared. Julier *et al.* (Julier *et al.*, 2000) focused on the same problem comparing the extended Kalman filter (EKF) results with the proposed nonlinear filter named unscented Kalman filter (UKF).

In (Gustafsson and Gunnarsson, 2005) the authors have addressed the problem of localization of a cellular mobile phone using time of arrival of signals in a wireless network, which turns out to be a problem of localization using range measurements. The same problem has been tackled by Li *et al.* (Li *et al.*, 2006) using asynchronous sensors.

In field of aerial vehicles, Mao *et al.* (Mao *et al.*, 2007) propose an EKF approach to estimate the location of an unmanned aerial vehicle (UAV) when its GPS

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connection is lost, using inter-UAV distance measurements. Preliminary experimental results are also presented.

In this paper, we address the problem of estimating the pose (position and orientation) of a robotic vehicle moving in a 2D plane using range measurements that arrive synchronously/asynchronously at discrete instants of time. In this setup we assume that the vehicle is equipped with only one range measuring sensor. This implies that there exist conditions under which the system is not observable. In fact, due to the nature of the sensor used and the dynamics of the vehicle, the observability (of the states of the system) depends on the control input signals. This is not the case for linear systems. For a survey on observability of nonlinear systems, the reader is referred to e.g., (Hermann and Krener, 1977; Diop and Fliess, 1991; Sedoglavic, 2002; Diop and Wang, 1993) and the references therein.

To solve the estimation problem we propose and compare two nonlinear Kalman based filtering algorithms: the extended Kalman filter and the unscented Kalman filter.

The EKF is a widely used method for estimating the state of a nonlinear system. It is obtained by linearizing the nonlinear dynamics and the observation along the trajectory of the estimate in order to compute the filter gain. In many situations it is quite efficient, robust, and practical. Recently, Julier *et al.* (Julier *et al.*, 1995) proposed a similar (in structure) algorithm called Unscented Kalman filter (UKF) where roughly speaking the linearization used to compute the predicted mean and covariance in the EKF is replaced by Monte-Carlo formulas but with a small number of carefully chosen deterministic sample points called sigma points.

We compare through extensive computer simulations the performance of the filters. The results show that in synchronous mode and when the set of range measurements are complete, that is, the observer receives at least three range measurements, both the filters exhibit good performance if the initial estimation errors are small. In this case we did not notice any major difference in the estimation accuracy between the two filters. When the range measurements are not received at the same time, the performance of the filters degrade as it is expected.

The paper is organized as follows: In section 2 we formulate the estimation problem. Section 3 presents an observability study. In particular, we show under what conditions the system is observable. In Section 4 and 5 we describe the EKF and the UKF algorithm. Section 6 presents the simulation results and concluding remarks are given in Section 7.

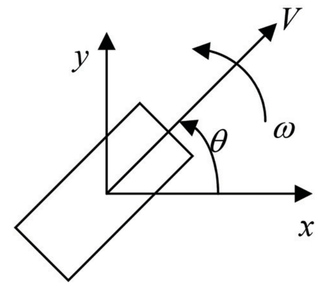

Fig. 1. Definition of the state variables of the robotic vehicle

2. PROBLEM STATEMENT

In this section we formulate the pose (position and orientation) estimation problem of a robotic vehicle that moves in a 2D plane. Let $(x, y) \in \mathbb{R}^2$ denote the position of the center of mass of the vehicle and $\theta \in \mathbb{S}^1$ its orientation with respect to the x axis (see Fig. 1). The kinematics and dynamics of the vehicle are described by the following equations:

$$\begin{cases} \dot{x} = V\cos\theta \\ \dot{y} = V\sin\theta \\ \dot{\theta} = \omega \\ \dot{V} = u_{\mathrm{V}} + w_{\mathrm{V}} \\ \dot{\omega} = u_{\omega} + w_{\omega} \end{cases} \tag{1}$$

where V and ω are the linear and angular velocities, respectively. The control inputs are the pushing force u_{V} and the steering torque u_{ω}. For simplicity, we have assumed that the mass and the moment of inertia are unitary.

The signals w_{V} and w_{ω} denote unknown input disturbances and are assumed to be stationary, Gaussian, zero mean white noise processes, mutually independent, with covariances $E[w_{\mathrm{V}}(t)w_{\mathrm{V}}(s)] = Q_{\mathrm{V}}\delta(t-s)$ and $E[w_{\omega}(t)w_{\omega}(s)] = Q_{\omega}\delta(t-s)$ where delta is the Kronecker delta function. Using Euler approximation to the continuous-time nonlinear system (1) we obtain

$$\begin{cases} x_{\mathrm{k+1}} = x_{\mathrm{k}} + TV_{\mathrm{k}}\cos\theta_{\mathrm{k}} \\ y_{\mathrm{k+1}} = y_{\mathrm{k}} + TV_{\mathrm{k}}\sin\theta_{\mathrm{k}} \\ \theta_{\mathrm{k+1}} = \theta_{\mathrm{k}} + T\omega_{\mathrm{k}} \\ V_{\mathrm{k+1}} = V_{\mathrm{k}} + Tu_{\mathrm{V,k}} + w_{\mathrm{V,k}} \\ \omega_{\mathrm{k+1}} = \omega_{\mathrm{k}} + Tu_{\omega,\mathrm{k}} + w_{\omega,\mathrm{k}} \end{cases} \tag{2}$$

where the constant $T > 0$ is the sampling period assumed to be small. The sequences $w_{\mathrm{V,k}}$ and $w_{\omega,\mathrm{k}}$ are stationary zero mean white Gaussian sequences of random variables, mutually independent. The covariance of these discrete-time sequences are

$$\begin{aligned} E[w_{\mathrm{V,k}}w_{\mathrm{V,j}}] &= Q_{\mathrm{V}}T\delta_{\mathrm{kj}} \\ E[w_{\omega,\mathrm{k}}w_{\omega,\mathrm{j}}] &= Q_{\omega}T\delta_{\mathrm{kj}} \end{aligned} \tag{3}$$

This means that the discrete-time model of the vehicle can be described as

$$\begin{cases} x_{\mathrm{k+1}} = x_{\mathrm{k}} + TV_{\mathrm{k}}\cos\theta_{\mathrm{k}} \\ y_{\mathrm{k+1}} = y_{\mathrm{k}} + TV_{\mathrm{k}}\sin\theta_{\mathrm{k}} \\ \theta_{\mathrm{k+1}} = \theta_{\mathrm{k}} + T\omega_{\mathrm{k}} \\ V_{\mathrm{k+1}} = V_{\mathrm{k}} + Tu_{\mathrm{V,k}} + W_{\mathrm{V,k}}\sqrt{T} \\ \omega_{\mathrm{k+1}} = \omega_{\mathrm{k}} + Tu_{\omega,\mathrm{k}} + W_{\omega,\mathrm{k}}\sqrt{T} \end{cases} \tag{4}$$

The sequences $W_{\mathrm{V,k}}$ and $W_{\omega,\mathrm{k}}$ are mutually independent, stationary zero mean white Gaussian sequences of random variables, with covariances Q_{V} and Q_{ω} respectively.

The vehicle is equipped with only one sensor that provides at discrete instants of time t_{j} , $j = 0, 1, ...$ the distances between the position of the vehicle and a set of N beacons located at $R_{\mathrm{i}} = \left[\, x_{\mathrm{i}} \;\, y_{\mathrm{i}} \;\, z_{\mathrm{i}} \,\right]^T$, $i = 1, ..., N$ known positions.

Stated mathematically the output equation is given by

$$r_{\mathrm{ij}} = \sqrt{(x_{\mathrm{j}} - x_{\mathrm{i}})^2 + (y_{\mathrm{j}} - y_{\mathrm{i}})^2 + (z_0 - z_{\mathrm{i}})^2} + v_{\mathrm{ij}}$$
$$\text{with}\,,\; i \in \mathcal{I}_{\mathrm{j}} \qquad (5)$$

where r_{ij} denotes $r_{\mathrm{i}}(t_{\mathrm{j}})$, z_0 is a constant value (the vehicle moves only in a horizontal plane), and v_{ij} is a stationary zero mean white Gaussian sequence of random variables that models the distance measurement errors.

Notice that in asynchronous mode the measurements arrive at asynchronous instants of time t_{j} and may not be complete, that is, at a given time t_{j} only the outputs r_{i} with $i \in \mathcal{I}_{\mathrm{j}}$ are available, where $\mathcal{I}_{\mathrm{j}} \subseteq \{1, ..., N\}$ and the inclusion may be strict when some measurements are missing.

The problem under consideration is to design an observer which estimates the states $\mathbf{x} := [x, y, \theta, V, \omega]$ governed by (1), given the discrete measurements (5).

To solve the problem we propose and compare two nonlinear Kalman based filtering algorithms: the EKF and the UKF.

3. OBSERVABILITY ANALYSIS

In this section we investigate if the nonlinear system composed by (1) and (5) admits a convergent observer. To this effect, we introduce the following definition:

Two states x_0^1 and x_0^2 are said to be *distinguishable* by an input $u(t)$ for a given nonlinear system

$$\dot{x} = f(x, u) \qquad (6)$$
$$y = h(x, u) \qquad (7)$$

if the outputs $y^1(t)$, $y^2(t)$ satisfying the initial conditions $x_0 = x_0^1$, $x_0 = x_0^2$ differ at some $t \geq 0$. The system is said to be *observable* if every pair x_0^1, x_0^2 can be distinguished by some input $u(t)$.

For autonomous linear systems a necessary and sufficient condition for observability is that the observability matrix is of full column rank. In that case (A,C) is said to be an observable pair. Notice that in this case the input $u(t)$ is not relevant to study the observability. However, this may not be the case for nonlinear systems. The following result shows under what conditions we can determine the states of (1) from the knowledge of inputs and outputs.

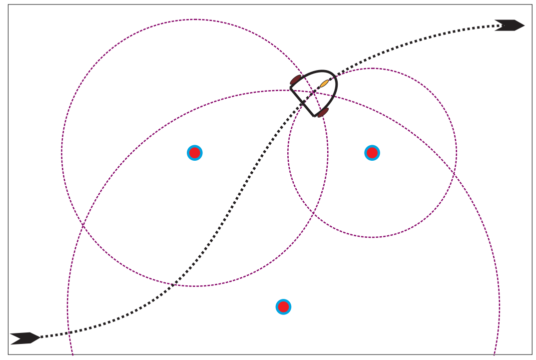

Fig. 2. Range sensors and the position of vehicle.

Theorem 1. Consider the nonlinear system described by (1) without the process noise together with the following output function.

$$r_{\mathrm{i}}(t) = \sqrt{(x(t) - x_{\mathrm{i}})^2 + (y(t) - y_{\mathrm{i}})^2 + (z_0 - z_{\mathrm{i}})^2},$$
$$i = 1, ..., N$$

Let $N = 3$ with $R_1 \neq R_2 \neq R_3$ and assume that z_0 is known and the vehicle is always moving, that is, $V(t) \neq 0\,, \forall t \geq 0$.

Then, from the knowledge of $u(t) = [u_{\mathrm{V}}, u_{\omega}]$, and $r_{\mathrm{i}}(t)$ we can determine uniquely the state $x(t)$.

PROOF. Each r_{i} (distance from the vehicle to beacon R_{i}) defines a circumference of possible locations of the vehicle. From this fact, it can be concluded that three intersections are enough to obtain uniquely the position (x,y).

Fig. 2 illustrates this fact graphically. Bucher and Misra (Bucher and Misra, 2002) have demonstrated mathematically for 3D that with four different range measurements, the position of vehicle can be computed from a linear set of equations. Using the same approach it is straightforward to show that for 2D, 3 different ranges are sufficient because z_0 is assumed to be known. To obtain the orientation θ we use the fact that

$$\theta = \text{atan2}(\dot{y}, \dot{x}) \qquad (8)$$

Thus, while the vehicle is moving, it is possible to compute its orientation.

From the knowledge of $\dot{x}$, $\dot{y}$, and θ and using the kinematics of the vehicle, the linear velocity V can be derived from

$$\begin{aligned} V &= (V\cos\theta)\cos\theta + (V\sin\theta)\sin\theta \\ &= \dot{x}\cos\theta + \dot{y}\sin\theta \end{aligned} \qquad (9)$$

To obtain the angular velocity ω we differentiate θ and use (8) to obtain

$$\omega = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\dot{x}^2 + \dot{y}^2} \qquad (10)$$

Thus, with 3 range measurements and the assumption of $V(t) \neq 0$, it is possible to infer all the states of the system. $\square$

We should mention that if the vehicle stops we will lose observability. However, if the vehicle is moving, it is possible to get (local) convergence of the state estimate to a neighborhood of the true ones even when the range measurements arrive at asynchronous instants of time t_{j} with $\mathcal{I}_{\mathrm{j}}$ being a strictly subset of $\{1, ..., N\}$ (i.e., some measurements missing). In that case, we conjecture that a persistence of excitation like condition that implies "observability" in an integral sense has to be satisfied.

4. THE EXTENDED KALMAN FILTER

The Kalman filter (Anderson and Moore, 1979; Simon, 2006) provides an efficient recursive procedure and is the optimal state estimate algorithm in a well defined sense when the process is linear and the noises are assumed to be linear stochastic.

For nonlinear systems of the type:

$$\begin{aligned} x_{\mathrm{k}} &= f(x_{\mathrm{k}-1}, u_{\mathrm{k}-1}) + w_{\mathrm{k}-1} \\ y_{\mathrm{k}} &= h(x_{\mathrm{k}}) + v_{\mathrm{k}} \end{aligned} \tag{11}$$

where x_{k} is the state of the system at time t_{k}, u_{k} is the control input, y_{k} the measurement signal and w_{k} and v_{k} the noisy signals, the EKF is a widely used algorithm in practice to obtain an estimate of the state.

The basic idea is to linearize the nonlinear system (Eqn. 11) at each time instant around the most recent state estimate and propagate an approximation of the conditional expectation and covariance.

The linearized state-space model equations consist of stochastic errors in the process and the measurement models. The process noise w_{k} represents random errors in the state transition model whereas the measurement noise v_{k} denotes random errors in observation model. The EKF assumes the random errors due to w_{k} and v_{k} to be independent zero-mean white Gaussian noises with the respectively covariance matrices as Q and R. Q accounts for model inaccuracy, process disturbances and noise introduced by the actuation instrumentations. R reflects the measurement noise introduced by the available sensors and measuring devices.

For the complete description of the EKF reader can refer to (Anderson and Moore, 1979; Simon, 2006).

5. THE UNSCENTED KALMAN FILTER

The extended Kalman filter requires the linearization of the nonlinear system at each sampling time. From this fact, two important potential drawbacks can arise (Julier and Uhlmann, 1997; Simon, 2006).

- The derivation of the Jacobian matrices, that is, the linear approximations to the nonlinear functions, can be complex causing implementation difficulties.
- This linearization can lead to filter instability if the estimation time-step intervals are not sufficiently small.

Julier and Uhlmann (Julier *et al.*, 1995; Julier and Uhlmann, 1997) developed the UKF algorithm which overcomes the linearization issue. Basically, the UKF makes use of a small set of deterministic points called sigma points that are carefully chosen. By computing the statistics of the transformed set we obtain an estimative of the state mean and covariance that does not require to linearize the general nonlinear system dynamics in (11).

Due to space limitations, we omit the description of the UKF algorithm. It can be found in (Julier *et al.*, 1995; Julier and Uhlmann, 1997; Julier *et al.*, 2000; Merwe and Wan, 2001; Simon, 2006).

6. SIMULATION RESULTS

In this section we illustrate and compare the performance of the proposed EKF and UKF algorithms for the following two scenarios:

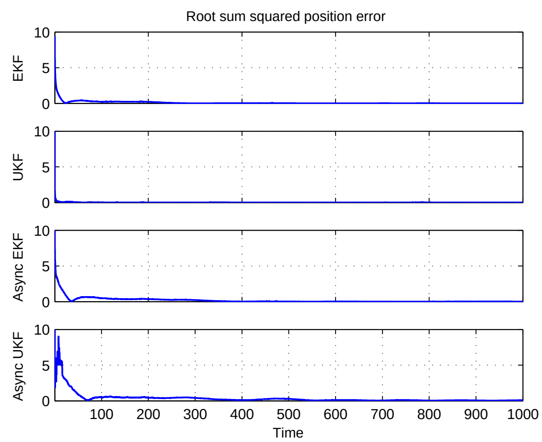

Fig. 3. Time evolution of the root sum squared position error $\|(\hat{x}, \hat{y}) - (x, y)\|_2$.

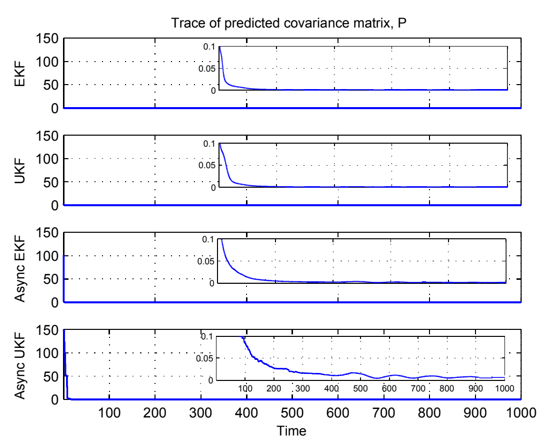

Fig. 4. Time evolution of the trace of the predicted covariance matrix P.

- Synchronous measurements: At every instant of time t_{k}, the observer receives all the range measurements from four beacons.
- Asynchronous measurements: At every t_{k}, the possibility of receiving a range measurement from one of the four beacons is modeled by a Bernoulli distribution with $p_{\mathrm{i}} = \frac{1}{4}$, $i \in \{1, ..., 4\}$.

The simulation parameters including the initial state errors are shown in Table 1. These parameters are the same for all the scenarios. For the asynchronous case, the filters are implemented with a constant sampling time T (equal to the ones in the asynchronous mode) and assumed that the instants of time t_{j} (which corresponds to the instants of time that the filters receive measurements) are multiples of T. The periods of time that there are no measurements available the filters only execute the "predict step". Fig. 3 and Fig. 4 show the time-evolution of numerical averages of the estimation errors and the trace of the predicted covariance matrix P for 10 Monte Carlo simulations. From these and other simulations not reported here, we can conclude that both filters, even for the asynchronous case, exhibit good performance for moderate errors in the initial states (see Table 2).

Table 1. Simulation parameters

Initial state errors	$[-4, 15, -15°, 0, 0]$
Initial variance	$[16, 225, 0.0685, 10^{-4}, 10^{-6}]$
Q_{V}, Q_ω (known input)	$[1, 1] \times 10^{-10}$
Q_{V}, Q_ω (unknown input)	$[100, 4] \times 10^{-6}$
Maximum V, ω	± 0.1
u_{V}, u_ω	$\pm 10^{-3}$
Measurement variance	$[0.5576, 0.5, 0.1917, 0.1520]$
UKF parameters (α, β, κ)	1, 2, 0
Sampling time (T)	$0.1s$

To illustrate the influence of the input signals data, Figs. 5 and 6 display the estimated trajectories when the input signals (u_{V} and u_ω in Eqn. 1) are known and unknown, respectively. In the case for the unknown input signals, we had to increase the process noise covariances to enforce the filters to not trust too much on the model. The figures show that the performance of the filters are still acceptable.

7. CONCLUDING REMARKS

In this paper two different nonlinear filters were developed to solve the problem of localization of a moving robotic vehicle using only range measurements. The output equation (5) shows that the relation between the position of the vehicle and the range measurements is nonlinear. To check the "degree" of this nonlinearity and infer how well the Kalman based algorithms proposed would perform, we decided to compute the mean and covariance of the range using several methods. Table 3 displays the results for the case that the position of the vehicle is assumed to be a random variable with Gaussian distribution of the form $(x, y) \sim N(5, 0.35^2)$. From Table 3 we can see that the sigma points method has a slight superior accuracy on estimating the mean and covariance when compared with the other methods.

Table 3. Estimation of range mean and variance

Method	$E(r)$	$\sigma^2(r)$
Exact values	7.079	0.122
Linearization	7.071	0.122
Mont Carlo (N=10)	7.039	0.111
Mont Carlo (N=100)	7.063	0.127
Sigma points ($(\bar{x}, \bar{y}) \pm \sqrt{2}\sigma$)	7.079	0.122

From the simulation results and consistent with the results in Table 3, we could conclude that for the synchronous measurements scenario both the EKF and the UKF exhibit similar performance. In asynchronous scenario, the results show that both the EKF and UKF are sensitive to the presence or not of measurements, which leads to performance degradation when compared with the synchronous case. However, it is worth to point out that we have set a probability of receiving data from each beacon at each instant of time to be $\frac{1}{4}$. This implies that the probability of not receiving any data at each instants of time is $(\frac{3}{4})^4 = 0.32$, which corresponds to an extreme situation. Preliminary experimental results (not reported here) also suggest that the UKF seems to be harder to tune and is much dependent to the initial measurement noise, process noise and states covariances, when compared with the EKF.

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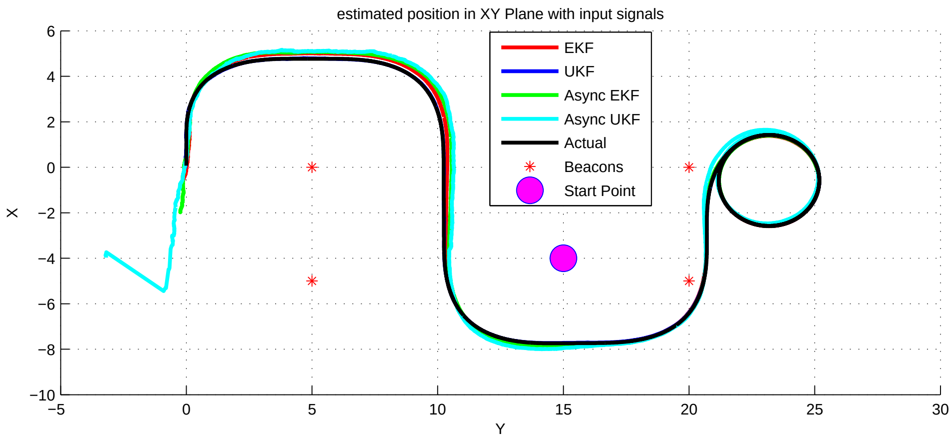

Fig. 5. Estimated trajectory of the vehicle for the two scenarios after 15 seconds (the input signals are known).

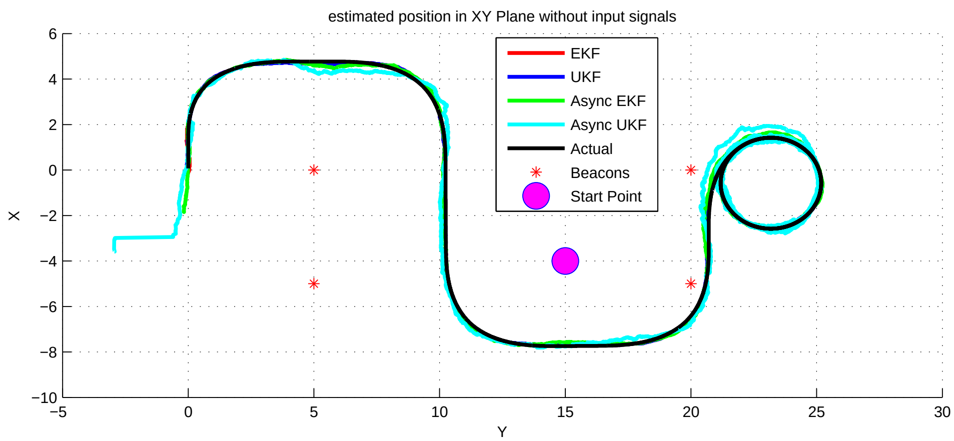

Fig. 6. Estimated trajectory of the vehicle for the two scenarios after 15 seconds (the input signals are unknown).

Table 2. Summary of averaged 10 Monte Carlo simulations

Method	$E(\hat{x}-x)$	$\sigma(\hat{x}-x)$	$E(\hat{y}-y)$	$\sigma(\hat{y}-y)$	$E(\hat{\theta}-\theta)$	$\sigma(\hat{\theta}-\theta)$
EKF	0.0371	0.1061	0.0230	0.0502	-0.2108	2.5097
UKF	-0.0029	0.0169	-0.0023	0.0097	-0.2162	0.8144
Async EKF	0.0292	0.2166	0.0637	0.1118	-0.7069	2.7328
Async UKF	-0.0246	0.4972	0.0529	0.1846	-1.6216	3.8842

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