Non-coherent Communication in Multiple-antenna Systems: Receiver Design, Codebook Construction and Capacity Analysis

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Outline

- ▷ High SNR regime
  - – deterministic fading channel (PEP analysis and codebook construction)
    - → conference paper published in IEEE ICASSP'2006
    - → journal paper submitted to IEEE Transactions on Signal Processing
- ▷ Low SNR regime
  - – random fading channel (mutual information analysis)
    - → conference paper submitted to IEEE ICASSP'2007
  - – deterministic fading channel (PEP analysis and codebook construction)
    - → conference paper published in IEEE SPAWC'2006
    - → conference paper submitted to IEEE ICASSP'2007
    - → journal paper in preparation for IEEE Trans. on Signal Processing
- ▷ Future work

Part 1: High SNR regime

Problem Formulation

▷ Data model: $\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{H}^H + \boldsymbol{E}$

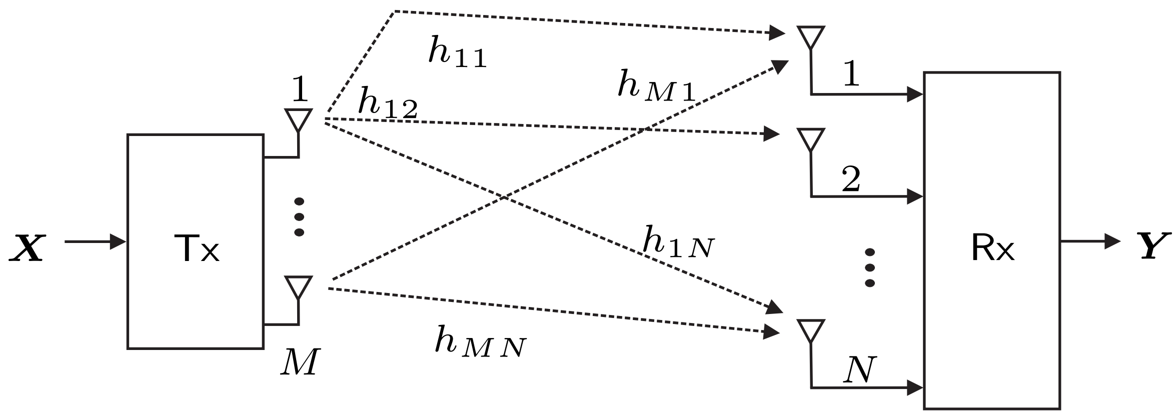


Figure 1: MIMO system

▷ $\boldsymbol{Y}, \boldsymbol{E}$: $T \times N$, $\boldsymbol{X}$: $T \times M$, $\boldsymbol{H}$: $N \times M$

▷ Codebook : $\mathcal{X} = \{\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_K\}$ is a point in the manifold

$$\mathcal{M} = \{(\boldsymbol{X}_1, \ldots, \boldsymbol{X}_K) : \text{tr}(\boldsymbol{X}_k^H \boldsymbol{X}_k) = 1\}$$

▷ Contribution: design codebook when $\boldsymbol{H}$ deterministic, unknown and vec $(\boldsymbol{E}) \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Upsilon})$ (colored noise)

Problem Formulation

▷ GLRT receiver:

$$\widehat{k} = \underset{k=1,2,\ldots,K}{\text{argmax}} \; p(\boldsymbol{y}|\boldsymbol{X}_k, \widehat{\boldsymbol{g}}_k) = \underset{k=1,2,\ldots,K}{\text{argmin}} \; ||\boldsymbol{y} - \widetilde{\boldsymbol{X}_k}\widehat{\boldsymbol{g}}_k||^2_{\boldsymbol{\Upsilon}^{-1}}$$

$\widetilde{\boldsymbol{X}_k} = \boldsymbol{I}_N \otimes \boldsymbol{X}_k$, $\widehat{\boldsymbol{g}}_k = (\widehat{\boldsymbol{X}_k}^H \widehat{\boldsymbol{X}_k})^{-1} \widehat{\boldsymbol{X}_k}^H \boldsymbol{\Upsilon}^{-\frac{1}{2}} \boldsymbol{y}$ (ML channel estimate),
$\widehat{\boldsymbol{X}_k} = \boldsymbol{\Upsilon}^{-\frac{1}{2}} \widetilde{\boldsymbol{X}_k}$, $||\boldsymbol{z}||^2_{\boldsymbol{A}} = \boldsymbol{z}^H \boldsymbol{A} \boldsymbol{z}$, $\boldsymbol{y} = \text{vec}\,(\boldsymbol{Y})$

▷ PEP analysis: it can be shown that (see [6]) for high SNR

$$P_{\boldsymbol{X}_i \to \boldsymbol{X}_j} = \mathcal{Q}\left(\frac{1}{\sqrt{2}} \sqrt{\boldsymbol{g}^H \boldsymbol{L}_{ij} \boldsymbol{g}}\right) \leq \mathcal{Q}\left(\frac{1}{\sqrt{2}} ||\boldsymbol{g}|| \sqrt{\lambda_{\min}(\boldsymbol{L}_{ij})}\right) \qquad (1)$$

where $\boldsymbol{g} = \text{vec}(\boldsymbol{H}^H)$, $\boldsymbol{L}_{ij}(\mathcal{X}) = \widehat{\boldsymbol{X}}_i^H \underbrace{\left(\boldsymbol{I}_T - \widehat{\boldsymbol{X}}_j \left(\widehat{\boldsymbol{X}}_j^H \widehat{\boldsymbol{X}}_j\right)^{-1} \widehat{\boldsymbol{X}}_j^H\right)}_{\Pi_j^\perp} \widehat{\boldsymbol{X}}_i$

Problem Formulation

- Optimization problem: result (1) suggests the codebook merit function

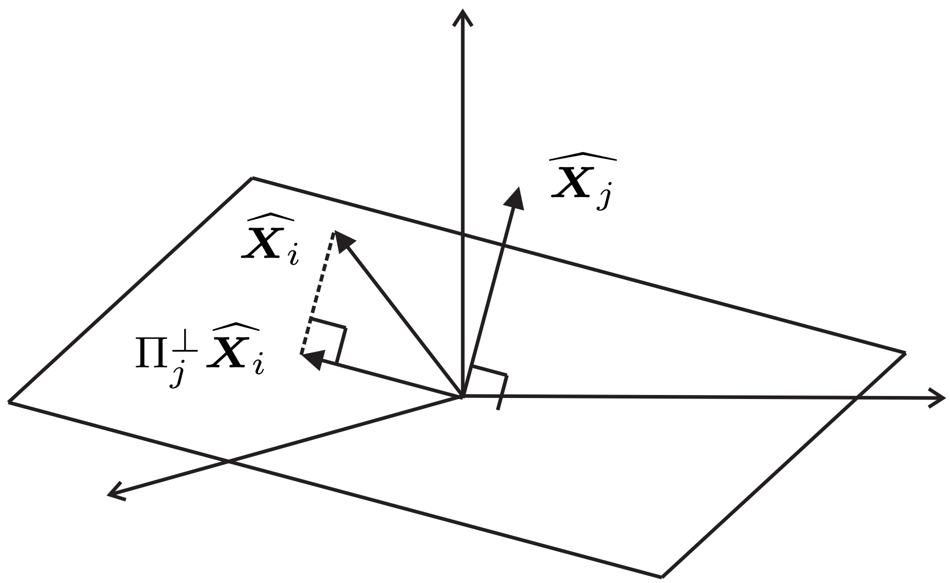
$$\mathcal{X}^* = \underset{\mathcal{X} \in \mathcal{M}}{\arg\max} \quad \underbrace{\mathsf{min}\{\lambda_{\min}(\boldsymbol{L}_{ij}(\mathcal{X})) : 1 \leq i \neq j \leq K\}}_{f(\boldsymbol{X}_1, \ldots, \boldsymbol{X}_K)} \tag{2}$$

- The problem in (2) is a high-dimensional, non-linear and non-smooth optimization problem!

  e.g. for $K = 256$, $T = 8$, $M = 2$: $K(K-1) = 65280$ $\boldsymbol{L}_{ij}(\mathcal{X})$ functions and $2KTM = 8192$ real variables to optimize

Codebook design : geometrical interpretation

▷ $\widehat{\boldsymbol{X}_i}$ should lie in the orthogonal complement of span$\{\widehat{\boldsymbol{X}_j}\}$



▷ $f(\boldsymbol{X}_1, \ldots, \boldsymbol{X}_K) = f(\boldsymbol{X}_1 e^{i\theta_1}, \ldots, \boldsymbol{X}_K e^{i\theta_K})$: packing in complex projective space

Codebook Construction

- ▷ Two-phase methodology to tackle the optimization problem in (2)
- ▷ Phase I: solves a convex semi-definite programming (SDP) relaxation
- ▷ Incremental approach: Let $\mathcal{X}_{k-1}^* = \{\boldsymbol{X}_1^*, ..., \boldsymbol{X}_{k-1}^*\}$ be the codebook at the $k-1^{\underline{th}}$ stage. The new codeword is found by solving

$$\boldsymbol{X}_k^* \quad = \quad \underset{\mathrm{tr}(\boldsymbol{X}_k^H \boldsymbol{X}_k) = 1}{\arg\max} \; \min_{1 \leq i \leq k-1} \{\lambda_{\mathsf{min}}(\boldsymbol{L}_{ik}),\, \lambda_{\mathsf{min}}(\boldsymbol{L}_{ki})\} \tag{3}$$

for $k = 2, ..., K$

Codebook Construction - Phase I

- The optimization problem (3) is equivalent to (see [6])

$$(\widehat{\boldsymbol{Y}}^*, \widetilde{\boldsymbol{X}}^*, t^*) = \arg\max \quad t \tag{4}$$

with the following constraints

$$\begin{bmatrix} \mathsf{tr}(\boldsymbol{N}_i \boldsymbol{A}_1 \widehat{\boldsymbol{Y}} \boldsymbol{B}_1) - t & \cdots & \mathsf{tr}(\boldsymbol{N}_i \boldsymbol{A}_{MN} \widehat{\boldsymbol{Y}} \boldsymbol{B}_1) \\ \vdots & & \vdots \\ \mathsf{tr}(\boldsymbol{N}_i \boldsymbol{A}_1 \widehat{\boldsymbol{Y}} \boldsymbol{B}_{MN}) & \cdots & \mathsf{tr}(\boldsymbol{N}_i \boldsymbol{A}_{MN} \widehat{\boldsymbol{Y}} \boldsymbol{B}_{MN}) - t \end{bmatrix} \succeq \mathbf{0},$$

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{Z}_i \\ \boldsymbol{Z}_i^H & \boldsymbol{P}_i \end{bmatrix} \succeq \mathbf{0} \, \forall_{1 \leq i \leq k-1}, \boldsymbol{K} \widehat{\boldsymbol{Y}} \boldsymbol{K}^H = \widetilde{\boldsymbol{X}}, \mathsf{tr}(\widetilde{\boldsymbol{X}}) = 1,$$

$$\boldsymbol{f} \, \widehat{\boldsymbol{Y}} \, \boldsymbol{f}^H = 1, \widehat{\boldsymbol{Y}} = \widehat{\boldsymbol{Y}}^H, \widehat{\boldsymbol{Y}} \succeq \mathbf{0}, \mathsf{rank}(\widehat{\boldsymbol{Y}}) = 1$$

and $\widetilde{\boldsymbol{X}} = \mathsf{vec}(\boldsymbol{X}_k)\mathsf{vec}^H(\boldsymbol{X}_k), b^2 = 1$, $\widehat{\boldsymbol{Y}} = \boldsymbol{z}\boldsymbol{z}^H$, $\boldsymbol{z} = \left[\mathsf{vec}^T(\widetilde{\boldsymbol{X}_k}) \; b\right]^T$, $\widetilde{\boldsymbol{X}_k} = \boldsymbol{I}_N \otimes \boldsymbol{X}_k$.

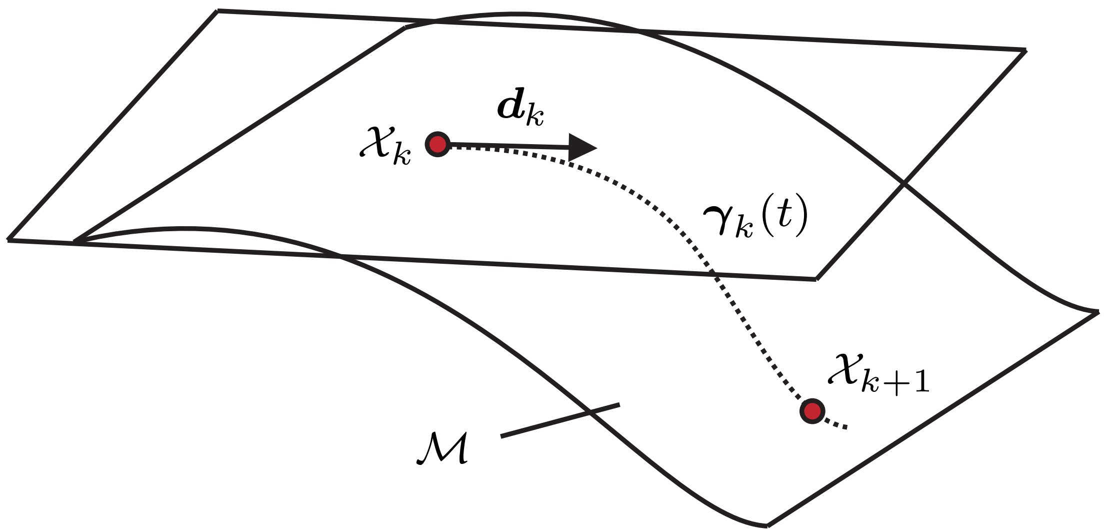
- The matrices $\boldsymbol{M}$, $\boldsymbol{Z}_i$ — linear in $\widehat{\boldsymbol{Y}}$
- The matrices $\boldsymbol{N}_i$, $\boldsymbol{P}_i$, $\boldsymbol{K}$, $\boldsymbol{f}$, $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ — constants, some depend on $\boldsymbol{\Upsilon}$

Codebook Construction - Phase 1

- ▷ Design of the codewords: high-dimensional difficult nonlinear optimization problem (rank condition in (4))
- ▷ Relaxing the rank constraint leads to an SDP [7]
- ▷ The $k^{\underline{th}}$ codeword is extracted from the output variable $\widetilde{\boldsymbol{X}}$ with a technique similar to [8]
- ▷ Initialization $\boldsymbol{X}_1^*$: randomly generated, filling columns of the matrix with eigenvectors associated to the smallest eigenvalues of the noise covariance matrix,etc.

Codebook Construction - Phase 2

▷ Phase II: optimizes a non-smooth function on a manifold



Codebook Construction - Phase 2

- ▷ Iterative algorithm, called GDA (geodesic descent algorithm)
- ▷ Identify "active" pairs (i,j) that attain minimum
- ▷ Check if there is an ascent direction $d_k \in T_{\mathcal{X}_k}\mathcal{M}$ for all active (i,j) (consists of solving LP)
- ▷ When d_k is found, perform Armijo rule along geodesic $\boldsymbol{\gamma}_k(t)$
- ▷ If no d_k is found, the algorithm stops

Computer Simulations

☐ **Example:**

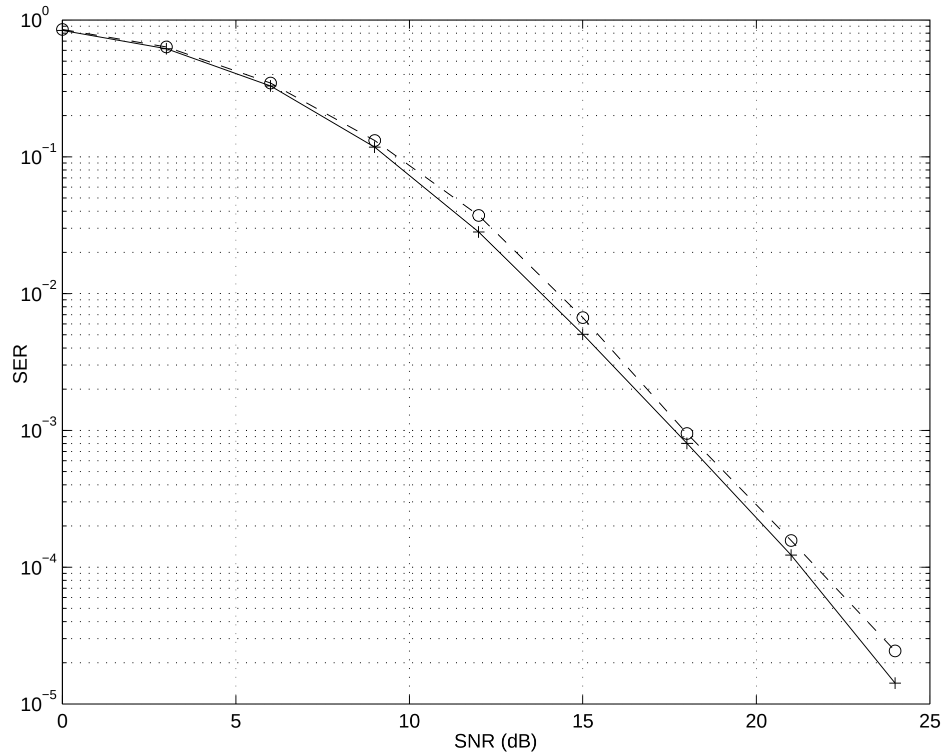


Figure 2: **Category 1 - spatio-temporally white observation noise:** T=8, M=3, N=1, K=256, $\mathbf{\Upsilon}= \boldsymbol{I}_{NT}$. Plus-solid curve-our codes, circle-dashed curve-unitary codes.

□ **Example:**

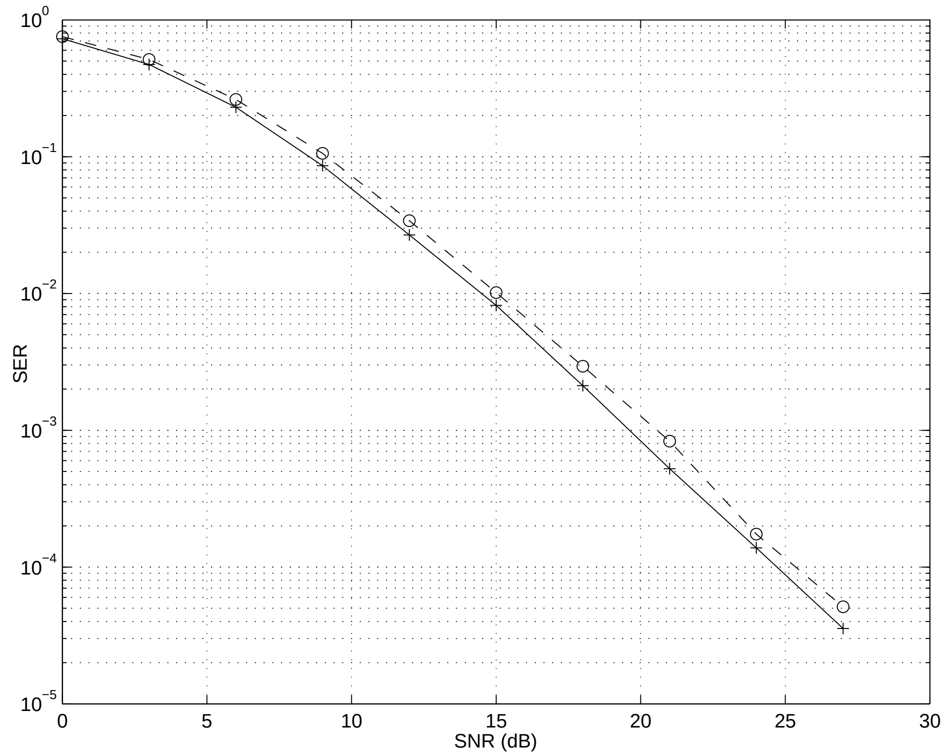


Figure 3: **Category 1 - spatio-temporally white observation noise:** T=8, M=2, N=1, K=256, $\boldsymbol{\Upsilon}= \boldsymbol{I}_{NT}$. Plus-solid curve-our codes, circle-dashed curve-unitary codes.

		PACKING RADII (DEGREES)		
T	K	MB	JAT	Rankin
2	3	60	60	60
2	4	54.74	54.74	54.74
2	5	45.00	45.00	52.24
2	6	45.00	45.00	50.77
2	7	38.93	38.93	49.80
2	8	37.43	37.41	49.11
2	9	35.26	–	48.59
2	10	33.07	–	48.19
2	11	31.72	–	47.87
2	12	31.72	–	47.61
2	13	28.24	–	47.39
2	14	27.83	–	47.21
2	15	26.67	–	47.05
2	16	25.97	–	46.91

Table 1: PACKING IN COMPLEX PROJECTIVE SPACE: We compare our best configurations (MB) of K points in $\mathbb{P}^{T-1}(\mathbb{C})$ against the Tropp codes (JAT) and Rankin bound [4]. The packing radius of an ensemble is measured as the acute angle between the closest pair of lines. Minus sign symbol (-) means that no packing is available for specific pair (T, K).

		PACKING RADII (DEGREES)		
T	K	MB	JAT	Rankin
3	4	70.53	70.53	70.53
3	5	64.26	64.00	65.91
3	6	63.43	63.43	63.43
3	7	61.87	61.87	61.87
3	8	60.00	60.00	60.79
3	9	60.00	60.00	60.00
3	10	54.74	54.73	59.39
3	11	54.74	54.73	58.91
3	12	54.74	54.73	58.52
3	13	51.38	51.32	58.19
3	14	50.36	50.13	57.92
3	15	49.80	49.53	57.69
3	16	49.60	49.53	57.49
3	17	49.13	49.10	57.31
3	18	48.12	48.07	57.16

Table 2: PACKING IN COMPLEX PROJECTIVE SPACE: We compare our best configurations (MB) of K points in $\mathbb{P}^{T-1}(\mathbb{C})$ against the Tropp codes (JAT) and Rankin bound [4]. The packing radius of an ensemble is measured as the acute angle between the closest pair of lines. Minus sign symbol (-) means that no packing is available for specific pair (T, K).

		PACKING RADII (DEGREES)		
T	K	MB	JAT	Rankin
4	5	75.52	75.52	75.52
4	6	70.89	70.88	71.57
4	7	69.29	69.29	69.30
4	8	67.79	67.78	67.79
4	9	66.31	66.21	66.72
4	10	65.74	65.71	65.91
4	11	64.79	64.64	65.27
4	12	64.68	64.24	64.76
4	13	64.34	64.34	64.34
4	14	63.43	63.43	63.99
4	15	63.43	63.43	63.69
4	16	63.43	63.43	63.43

Table 3: PACKING IN COMPLEX PROJECTIVE SPACE: We compare our best configurations (MB) of K points in $\mathbb{P}^{T-1}(\mathbb{C})$ against the Tropp codes (JAT) and Rankin bound. The packing radius of an ensemble is measured as the acute angle between the closest pair of lines.

		PACKING RADII (DEGREES)		
T	K	MB	JAT	Rankin
5	6	78.46	78.46	78.46
5	7	74.55	74.52	75.04
5	8	72.83	72.81	72.98
5	9	71.33	71.24	71.57
5	10	70.53	70.51	70.53
5	11	69.73	69.71	69.73
5	12	69.04	68.89	69.10
5	13	68.38	68.19	68.58
5	14	67.92	67.66	68.15
5	15	67.48	67.37	67.79
5	16	67.08	66.68	67.48
5	17	66.82	66.53	67.21
5	18	66.57	65.87	66.98
5	19	66.57	65.75	66.77

Table 4: PACKING IN COMPLEX PROJECTIVE SPACE: We compare our best configurations (MB) of K points in $\mathbb{P}^{T-1}(\mathbb{C})$ against the Tropp codes (JAT) and Rankin bound. The packing radius of an ensemble is measured as the acute angle between the closest pair of lines.

		PACKING RADII (DEGREES)	
T	K	MB	Rankin
6	7	80.41	80.41
6	8	77.06	77.40
6	9	75.52	75.52
6	10	74.20	74.21
6	11	73.22	73.22
6	12	72.45	72.45
6	13	71.82	71.83
6	14	71.31	71.32
6	15	70.87	70.89
6	16	70.53	70.53
6	17	70.10	70.21
6	18	69.73	69.94
6	19	69.40	69.70

Table 5: PACKING IN COMPLEX PROJECTIVE SPACE: We compare our best configurations (MB) of K points in $\mathbb{P}^{T-1}(\mathbb{C})$ against Rankin bound. The packing radius of an ensemble is measured as the acute angle between the closest pair of lines.

□ **Example:**

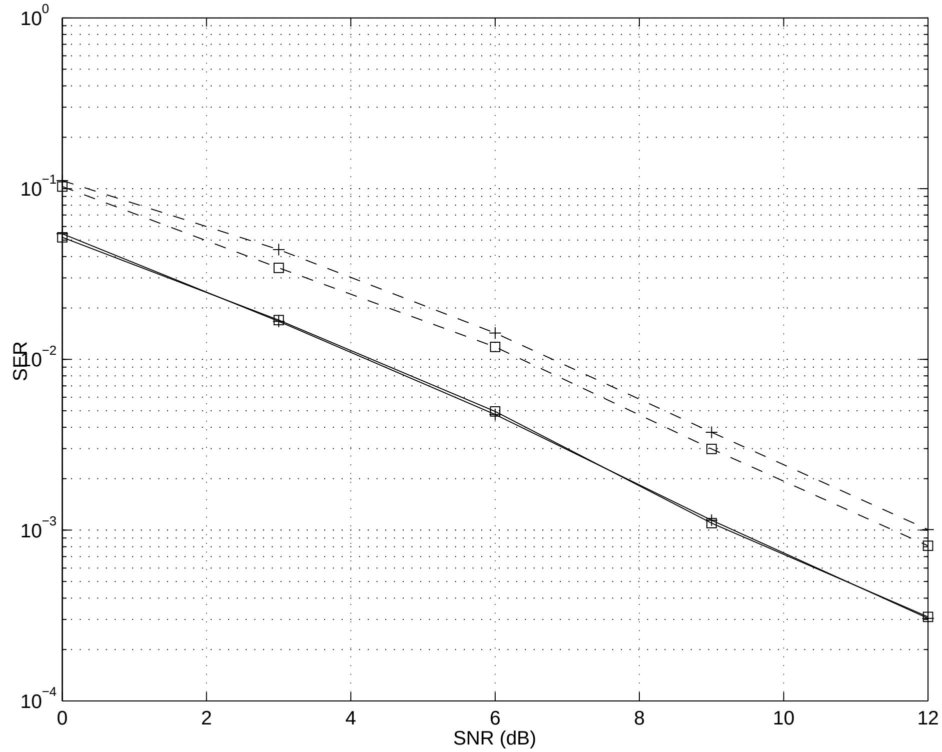


Figure 4: **Category 2 - spatially white - temporally coloured:** T=8, M=2, $N = 1$, K=67, $\boldsymbol{\Upsilon} = \boldsymbol{I}_{NT} \otimes \Sigma(\boldsymbol{\rho})$, $\boldsymbol{\rho}$=[1; 0.85; 0.6; 0.35; 0.1; zeros(3,1)]. Solid curves-our codes, dashed curves-unitary codes, plus signed curves-GLRT receiver, square signed curves-Bayesian receiver.

□ Example:

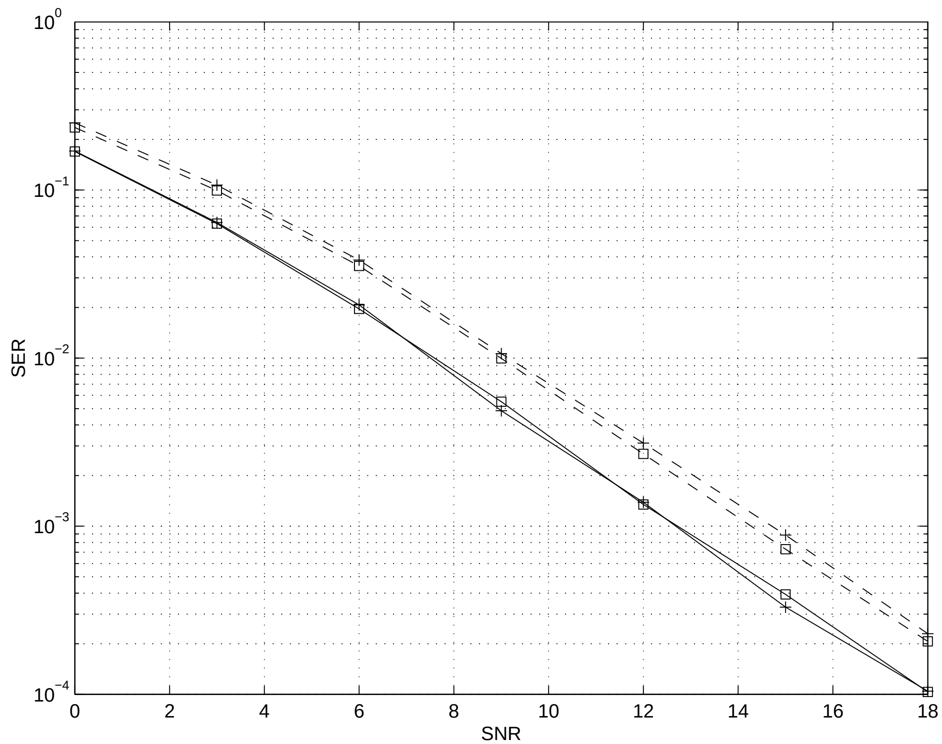


Figure 5: **Category 2 - spatially white - temporally coloured:** T=8, M=2, $N = 1$, K=256, $\boldsymbol{\Upsilon} = \boldsymbol{I}_{NT} \otimes \Sigma(\boldsymbol{\rho})$, $\boldsymbol{\rho}$=[1; 0.8; 0.5; 0.15; zeros(4,1)]. Solid curves-our codes, dashed curves-unitary codes, plus signed curves-GLRT receiver, square signed curves-Bayesian receiver.

□ **Example:**

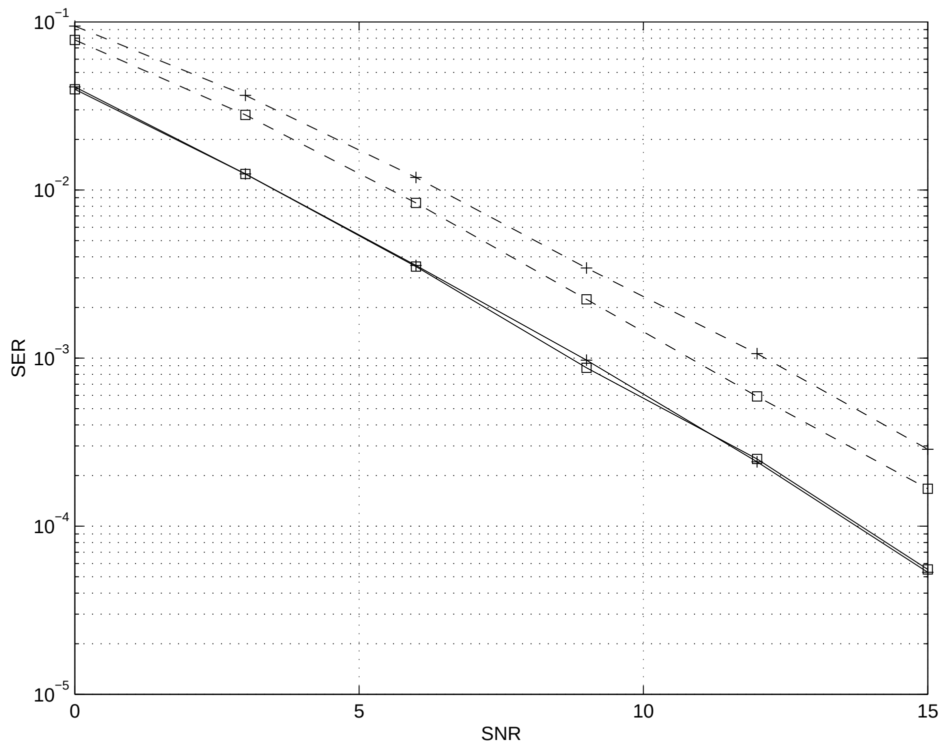


Figure 6: **Category 2 - spatially white - temporally coloured:** T=8, M=2, $N = 1$, K=32, $\boldsymbol{\Upsilon} = \boldsymbol{I}_{NT} \otimes \Sigma(\boldsymbol{\rho})$, $\boldsymbol{\rho}$=[1; 0.8; 0.5; 0.15; zeros(4,1)]. Solid curves-our codes, dashed curves-unitary codes, plus signed curves-GLRT receiver, square signed curves-Bayesian receiver.

☐ Example:

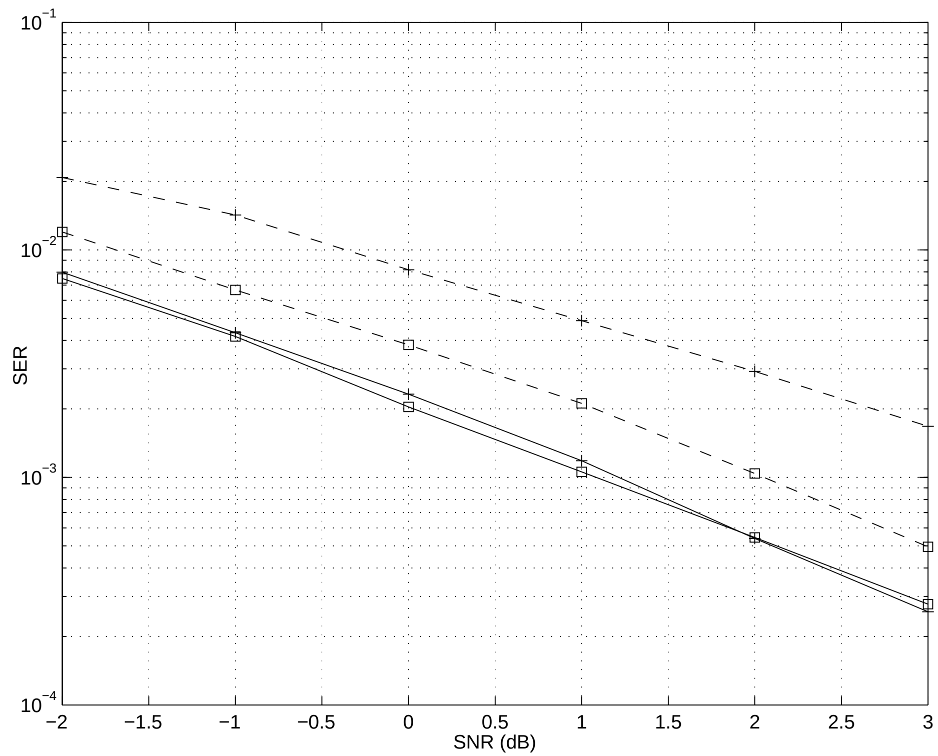


Figure 7: **Category 3 - $\boldsymbol{\Upsilon} = \boldsymbol{\alpha}\boldsymbol{\alpha}^H \otimes \boldsymbol{\Upsilon}_s + \boldsymbol{I}_{NT} \otimes \Sigma(\boldsymbol{\rho})$:** T=8, M=2, $N = 2$, K=32, $\boldsymbol{s}$=[1;0.7;0.4;0.15;zeros(4,1)], $\boldsymbol{\rho}$ = [1;0.8;0.5;0.15;zeros(4,1)], $\boldsymbol{\alpha}$ = [-1.146 + 1.189i;1.191- 0.038i]. Solid curves-our codes, dashed curves-unitary codes, plus signed curves-GLRT receiver, square signed curves-Bayesian receiver.

□ **Example:**

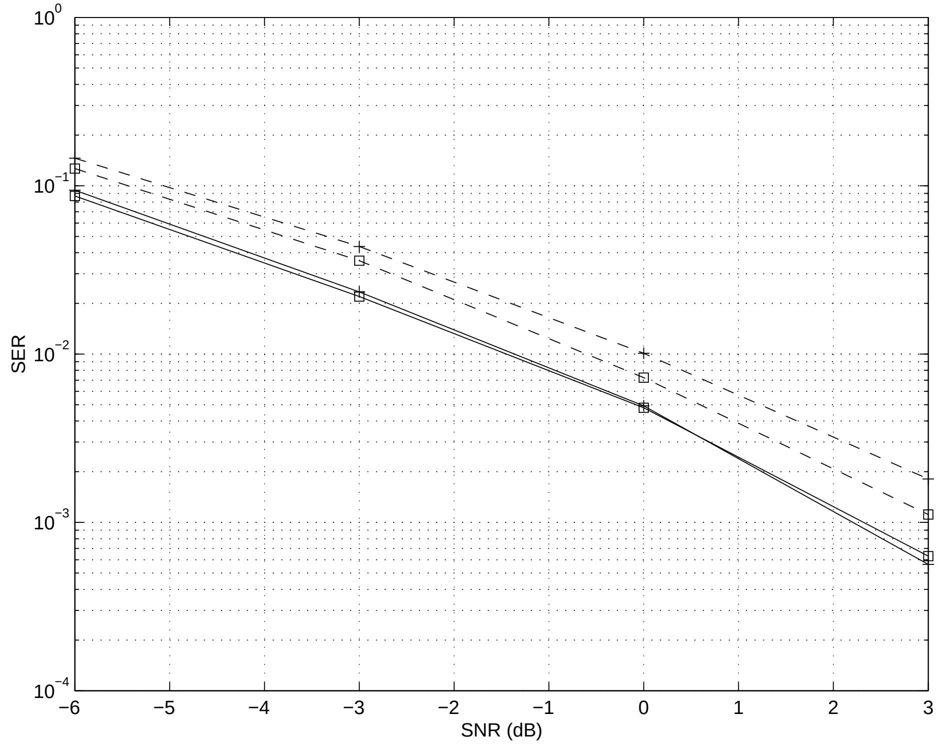


Figure 8: **Category 3 -** $\boldsymbol{\Upsilon} = \boldsymbol{\alpha}\boldsymbol{\alpha}^H \otimes \boldsymbol{\Upsilon}_s + \boldsymbol{I}_{NT} \otimes \Sigma(\boldsymbol{\rho})$**:** T=8, M=2, $N = 2$, K=67, $\boldsymbol{s}$=[1; 0.8; 0.5; 0.15; zeros(4,1)], $\boldsymbol{\rho}$ = [1; 0.7; 0.4; 0.15; zeros(4,1)], $\boldsymbol{\alpha} = [-0.453+0.007i; 0.4869+1.9728i]$. Solid curves-our codes, dashed curves-unitary codes, plus signed curves-GLRT receiver, square signed curves-Bayesian receiver.

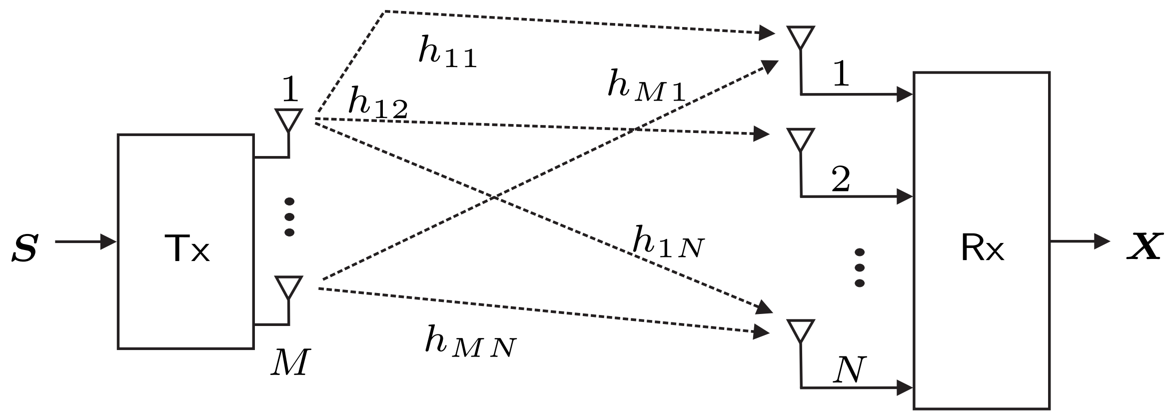
Conclusions

- ▷ Codebook design for noncoherent setup
  - $\boldsymbol{H}$ deterministic, unknown
  - Colored noise: $\text{vec}\,(\boldsymbol{E}) \sim \mathcal{CN}\,(\mathbf{0}, \boldsymbol{\Upsilon})$
- ▷ Results
  - outperform significantly unitary constellations for colored noise case
  - provide good packings for complex projective space $(M = 1)$ (near bound performance)
  - small gain for white noise case
  - for some cases actual Equiangular Tight Frames (ETF's)
- ▷ Publications
  - conference paper published in IEEE ICASSP'2006
  - journal paper submitted to IEEE Transactions on Signal Processing

Part 2: Low SNR regime – random fading channel

Problem Formulation

- Data model: $\boldsymbol{X} = \boldsymbol{S}\boldsymbol{H} + \boldsymbol{E}$



- $\boldsymbol{X}, \boldsymbol{E}$: $T \times N$, $\boldsymbol{S}$: $T \times M$, $\boldsymbol{H}$: $M \times N$
- Contribution: mutual information analysis for on-off and Gaussian signaling when $\boldsymbol{H} = \sqrt{\frac{\rho}{M}} \boldsymbol{K}_t^{\frac{1}{2}} \boldsymbol{H}_w \boldsymbol{K}_r^{\frac{1}{2}}$ and vec $(\boldsymbol{E}) \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Upsilon})$ (colored noise)

Mutual information: on-off signaling

- ▷ The on-off signaling: for any $\epsilon > 1$, $\boldsymbol{S} = \boldsymbol{S}_{on}\rho^{-\frac{\epsilon}{2}}$ w.p. ρ^{ϵ}; $\boldsymbol{S} = \boldsymbol{0}$ w.p. $1 - \rho^{\epsilon}$
- ▷ At sufficiently low SNR

$$I(\boldsymbol{X};\boldsymbol{S}) = \frac{\rho}{M} \operatorname{tr}\left(\boldsymbol{\Upsilon}^{-1}\left(\boldsymbol{K}_r \otimes \boldsymbol{S}_{on}\boldsymbol{K}_t\boldsymbol{S}_{on}^H\right)\right) + o(\rho), \tag{5}$$

- ▷ We maximize $I(\boldsymbol{X};\boldsymbol{S})$ in (5) w.r.t $\boldsymbol{S}_{on}$, $\boldsymbol{K}_t$ and $\boldsymbol{K}_r$
- ▷ The maximum in (5) is attained by

$$\widehat{\boldsymbol{S}}_{on} = \sqrt{TM}\begin{bmatrix}\hat{\boldsymbol{s}} & \boldsymbol{0}_{T\times(M-1)}\end{bmatrix}, \widehat{\boldsymbol{K}}_r = N\hat{\boldsymbol{u}}\hat{\boldsymbol{u}}^H, \widehat{\boldsymbol{K}}_t(i,i) = M\delta_{i1} \tag{6}$$

where

$$(\hat{\boldsymbol{u}},\hat{\boldsymbol{s}}) = \underset{\substack{\boldsymbol{u}\in\mathbb{C}^N, ||\boldsymbol{u}||=1\\ \boldsymbol{s}\in\mathbb{C}^T, ||\boldsymbol{s}||=1}}{\arg\max} (\boldsymbol{u}\otimes\boldsymbol{s})^H\,\boldsymbol{\Upsilon}^{-1}(\boldsymbol{u}\otimes\boldsymbol{s}) \tag{7}$$

Mutual information: on-off signaling

- The optimization problem in (7) always admits a solution (maximization of a continuous function over a compact set)
- For the choice in (6), the maximal mutual information (p.c.u) is equal to

$$\frac{1}{T} I(\boldsymbol{X};\boldsymbol{S}) = \rho\, N\, M \hat{\lambda} + o(\rho).$$

where $\hat{\lambda} = (\hat{\boldsymbol{u}} \otimes \hat{\boldsymbol{s}})^H \, \boldsymbol{\Upsilon}^{-1} \, (\hat{\boldsymbol{u}} \otimes \hat{\boldsymbol{s}})$

- Conclusions:
  - From (6) we see that both $\boldsymbol{K}_t$ and $\boldsymbol{K}_r$ should be of rank one
  - Correlated Rayleigh fading channel is beneficial from capacity viewpoint. Gain of order M with respect to uncorrelated Rayleigh fading channel
  - On-off signaling attains the known channel capacity
  - Correlation in noise is beneficial too, $\hat{\lambda} \geq 1$

Mutual information: Gaussian modulation

- ▷ On-off signaling is unpracticable due to large peakiness of the input signal
- ▷ Let $\boldsymbol{s} = \text{vec}(\boldsymbol{S}) \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{P})$. At sufficiently low SNR

$$I(\boldsymbol{X};\boldsymbol{S}) = \frac{\rho^2}{2M^2} \operatorname{tr}\left(\mathsf{E}[\boldsymbol{Z}^2] - (\mathsf{E}[\boldsymbol{Z}])^2\right) + o(\rho^2) \tag{8}$$

where $\boldsymbol{Z} = \boldsymbol{\Upsilon}^{-\frac{1}{2}} \left(\boldsymbol{K}_r \otimes \boldsymbol{S}\boldsymbol{K}_t\boldsymbol{S}^H\right) \boldsymbol{\Upsilon}^{-\frac{1}{2}}$

- ▷ We maximize $I(\boldsymbol{X};\boldsymbol{S})$ in (8) w.r.t $\boldsymbol{P}$, $\boldsymbol{K}_t$ and $\boldsymbol{K}_r$
- ▷ The maximum in (8) is attained by

$$\widehat{\boldsymbol{P}} = TM\boldsymbol{F}_1 \otimes \hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^H,\ \widehat{\boldsymbol{K}}_r = N\hat{\boldsymbol{u}}\hat{\boldsymbol{u}}^H,\ \widehat{\boldsymbol{K}}_t(i,i) = M\delta_{i1} \tag{9}$$

where

$$(\hat{\boldsymbol{u}}, \hat{\boldsymbol{s}}) = \underset{\substack{\boldsymbol{u} \in \mathbb{C}^N, ||\boldsymbol{u}|| = 1 \\ \boldsymbol{s} \in \mathbb{C}^T, ||\boldsymbol{s}|| = 1}}{\arg\max} (\boldsymbol{u} \otimes \boldsymbol{s})^H \boldsymbol{\Upsilon}^{-1} (\boldsymbol{u} \otimes \boldsymbol{s})$$

Mutual information: Gaussian modulation

- ▷ The $M \times M$ matrix $\boldsymbol{F}_1$ has all the entries equal to zero except the entry (1,1) which is one
- ▷ For the choice in (9), the maximal mutual information (p.c.u) is equal to

$$\frac{1}{T} I(\boldsymbol{X};\boldsymbol{S}) = \frac{\rho^2}{2}\, N^2\, T\, M^2\, \hat{\lambda}^2 + o(\rho^2).$$

- ▷ Conclusions:
  - – From (9) we see that both $\boldsymbol{K}_t$ and $\boldsymbol{K}_r$ should be of rank one
  - – Correlated Rayleigh fading channel is beneficial from capacity viewpoint. Gain of order $M^2 N$ with respect to uncorrelated Rayleigh fading channel
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Part 2: Low SNR regime – deterministic fading channel

Problem Formulation

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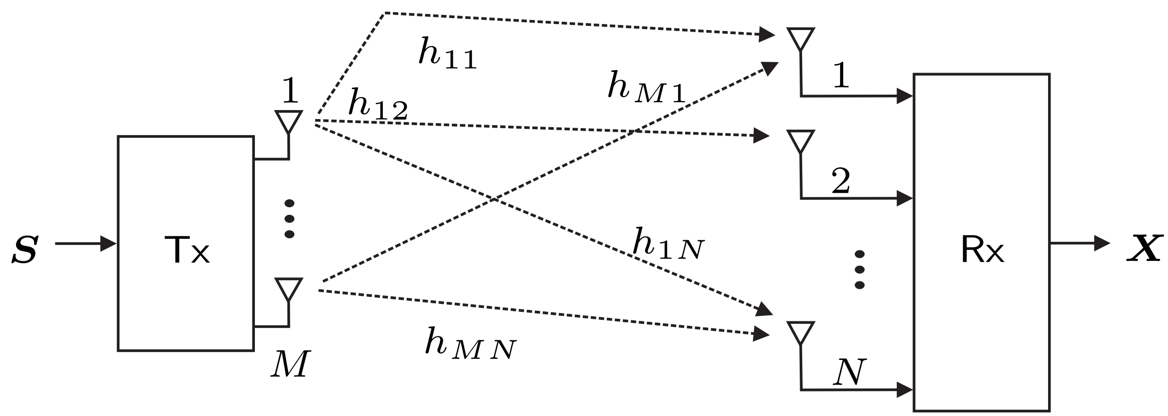

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▷ Contribution: design codebook when $\boldsymbol{H}$ deterministic, unknown and $\mathrm{vec}\,(\boldsymbol{E}) \sim \mathcal{CN}\,(\mathbf{0}, \boldsymbol{\Upsilon})$ (colored noise)

▷ GLRT receiver:

$$\widehat{k} = \underset{k=1,2,\ldots,K}{\text{argmax}} \; p(\boldsymbol{x}|\boldsymbol{S}_k, \widehat{\boldsymbol{g}}_k)$$
$$= \underset{k=1,2,\ldots,K}{\text{argmin}} \; ||\boldsymbol{x} - \widetilde{\boldsymbol{S}_k}\widehat{\boldsymbol{g}}_k||^2_{\boldsymbol{\Upsilon}^{-1}}$$

$\widetilde{\boldsymbol{S}_k} = \boldsymbol{I}_N \otimes \boldsymbol{S}_k$, $\widehat{\boldsymbol{g}}_k = (\widehat{\boldsymbol{S}_k}^H \widehat{\boldsymbol{S}_k})^{-1} \widehat{\boldsymbol{S}_k}^H \boldsymbol{\Upsilon}^{-\frac{1}{2}} \boldsymbol{y}$ (ML channel estimate),
$\widehat{\boldsymbol{S}_k} = \boldsymbol{\Upsilon}^{-\frac{1}{2}} \widetilde{\boldsymbol{S}_k}$, $||\boldsymbol{z}||^2_{\boldsymbol{A}} = \boldsymbol{z}^H \boldsymbol{A} \boldsymbol{z}$, $\boldsymbol{x} = \text{vec}\,(\boldsymbol{X})$

▷ PEP analysis: it can be shown that at low SNR and $T \geq 2M$

$$P_{\boldsymbol{S}_i \to \boldsymbol{S}_j} \approx \text{Prob}\left(Y > \boldsymbol{g}^H \, \boldsymbol{L}_{ij} \boldsymbol{g}\right), \tag{10}$$

with $\boldsymbol{L}_{ij} = \widehat{\boldsymbol{S}_i}^H \boldsymbol{\Pi}_j^{\perp} \widehat{\boldsymbol{S}_i}$, $\boldsymbol{\Pi}_j^{\perp} = \boldsymbol{I}_{TN} - \widehat{\boldsymbol{S}_j} \left(\widehat{\boldsymbol{S}_j}^H \widehat{\boldsymbol{S}_j}\right)^{-1} \widehat{\boldsymbol{S}_j}^H$, and
$Y = \sum_{m=1}^{MN} \sin \alpha_m \left(|a_m|^2 - |b_m|^2\right)$ where a_m, $b_m \overset{iid}{\sim} \mathcal{CN}\,(0,1)$ for $m = 1, \ldots, MN$. The angles α_m are the *principal angles* between the subspaces spanned by $\widehat{\boldsymbol{S}_i} \left(\widehat{\boldsymbol{S}_i}^H \widehat{\boldsymbol{S}_i}\right)^{-\frac{1}{2}}$ and $\widehat{\boldsymbol{S}_j} \left(\widehat{\boldsymbol{S}_j}^H \widehat{\boldsymbol{S}_j}\right)^{-\frac{1}{2}}$

Problem Formulation

- ▷ PEP analysis: for $M = 1$ and $\boldsymbol{\Upsilon} = \boldsymbol{I}_{TN}$, (10) becomes

$$P_{\boldsymbol{s}_i \rightarrow \boldsymbol{s}_j} = P\left(\sum_{n=1}^{N} \left(|a_n|^2 - |b_n|^2\right) > ||\boldsymbol{h}||^2 \sin \alpha_{ij}\right) \tag{11}$$

where $a_n, b_n \overset{iid}{\sim} \mathcal{CN}(0, 1)$ and the angle α_{ij} is the acute angle between the codewords $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$

- ▷ In our work [5] the expression for the PEP in the high SNR regime, $M = 1$ and $\boldsymbol{\Upsilon} = \boldsymbol{I}_{TN}$ is given by

$$P_{\boldsymbol{s}_i \rightarrow \boldsymbol{s}_j} = \mathcal{Q}\left(\frac{1}{\sqrt{2}} ||\boldsymbol{h}|| \sin \alpha_{ij}\right) \tag{12}$$

where $\mathcal{Q}(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$

- ▷ Equations (11)-(12) confirm that the codewords $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$ should be constructed as separate as possible
- ▷ The problem of constructing good codes corresponds to the very well known packing problem in the complex projective space [4]

Problem Formulation

▷ From (10), an upper bound on the PEP is readily found

$$P_{\boldsymbol{S}_i \rightarrow \boldsymbol{S}_j} \leq \text{Prob}\left(Z > ||\boldsymbol{g}||^2 \lambda_{\min}\left(\boldsymbol{L}_{ij}\right)\right), \tag{13}$$

where $Z = \sum_{m=1}^{MN} |a_m|^2$, $a_m \overset{iid}{\sim} \mathcal{CN}(0,1)$

▷ The codebook design criterion in (13) is equivalent to the one for the high SNR regime

$$\mathcal{S}^* = \underset{\mathcal{S} \in \mathcal{M}}{\arg\max} \ \min\{\lambda_{\min}(\boldsymbol{L}_{ij}(\mathcal{X})) : 1 \leq i \neq j \leq K\}$$

Computer Simulations

□ **Category 1 - spatio-temporally white observation noise: Constellations with equal priors**

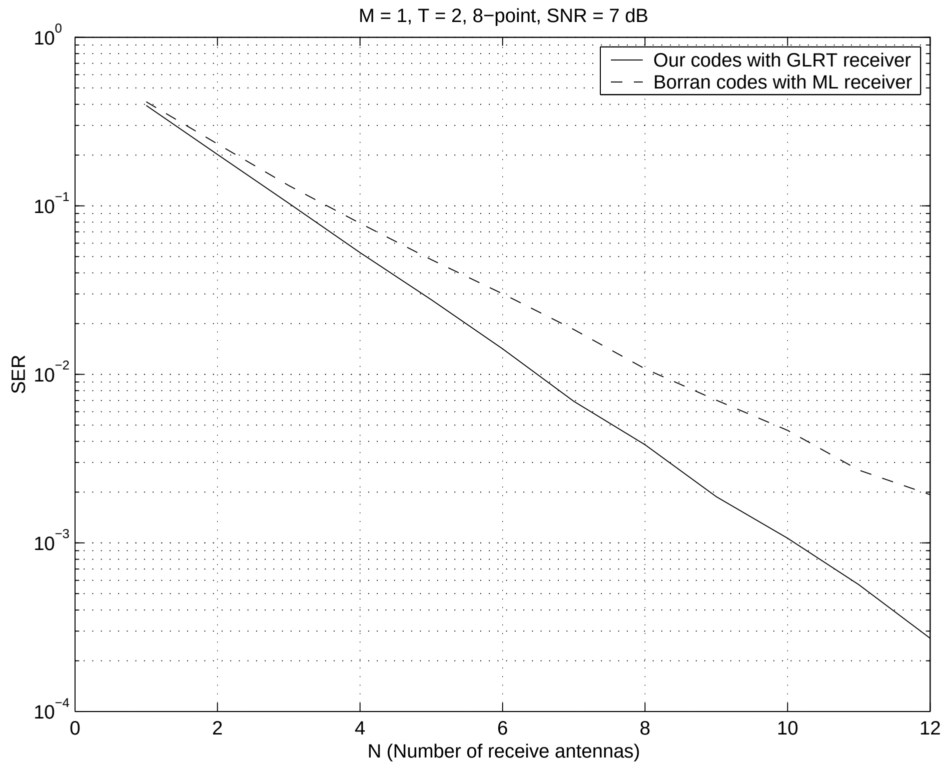

Figure 9: M=1, T=2, K=8, SNR = 7 dB. Solid curve:our codes with our GLRT receiver. Dashed curve:Borran codes designed for SNR = 7dB with ML receiver [1].

☐ **Category 1 - spatio-temporally white observation noise: Constellations with equal priors**

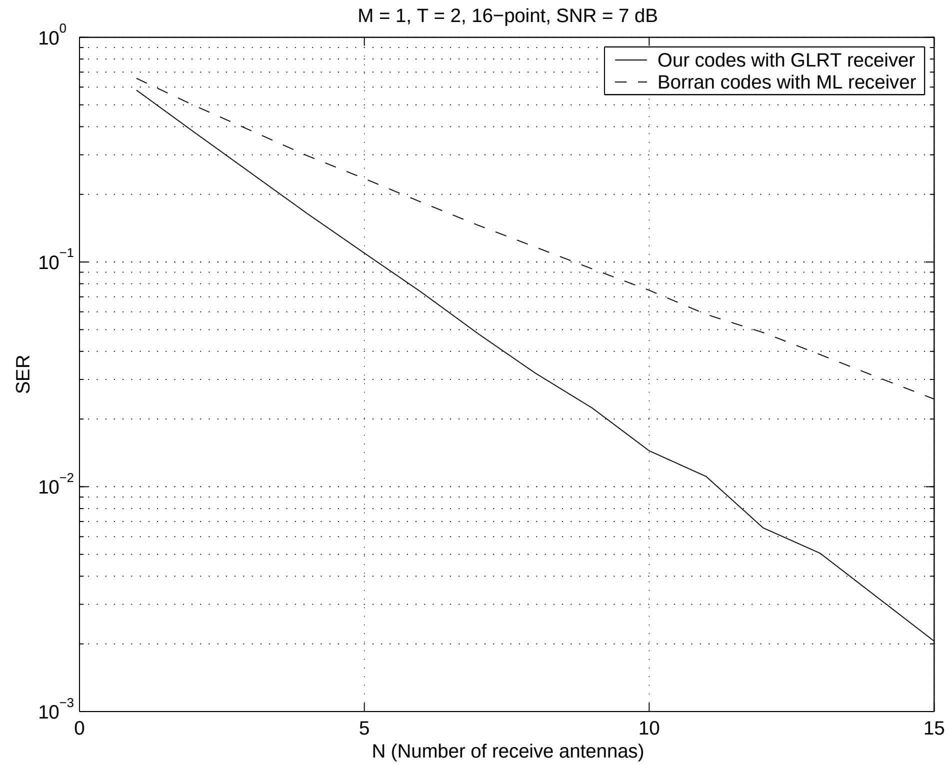

Figure 10: M=1, T=2, K=16, SNR = 7 dB. Solid curve:our codes with our GLRT receiver. Dashed curve:Borran codes designed for SNR = 7dB with ML receiver [1].

☐ Category 1 - spatio-temporally white observation noise: Constellations with unequal priors

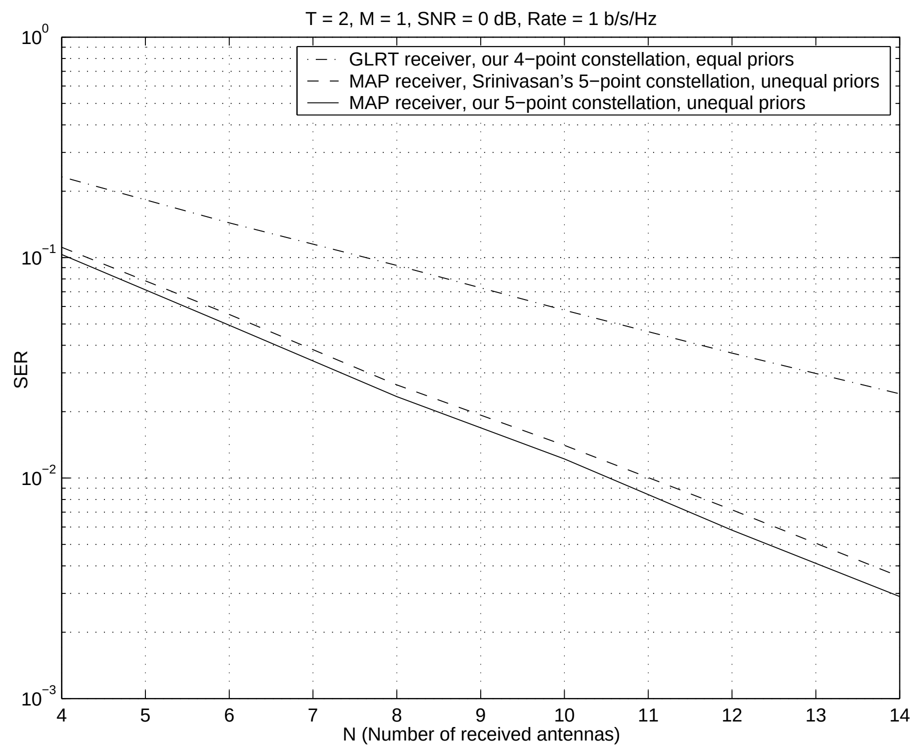

Figure 11: T=2, M=1, SNR = 0 dB, Rate = 1 b/s/Hz. Solid curve-our 5 point constellation with unequal priors, dashed curve-Srinivasan's 5 point constellation with unequal priors [2], dash-dotted curve-our 4 point constellation with equal priors. Our and Srinivasan's 5 point constellations use *maximum a-posteriori* (MAP) receiver, our 4 point constellation uses GLRT receiver.

□ **Category 1 - spatio-temporally white observation noise: Constellations with equal priors and $M \geq 1$**

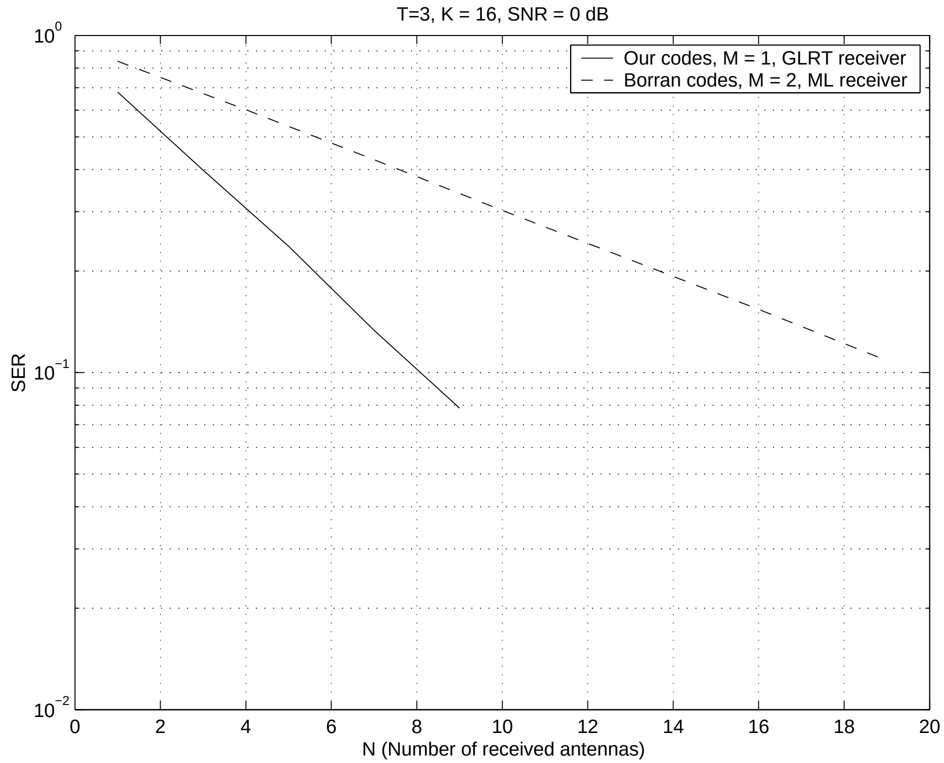

Figure 12: Solid curve-our codes for $K = 16$, $T = 3$, $M = 1$, dashed curve-Borran codes for $K = 16$, $T = 3$, $M = 2$.

□ **Category 1 - spatio-temporally white observation noise: Constellations with equal priors and $M \geq 1$**

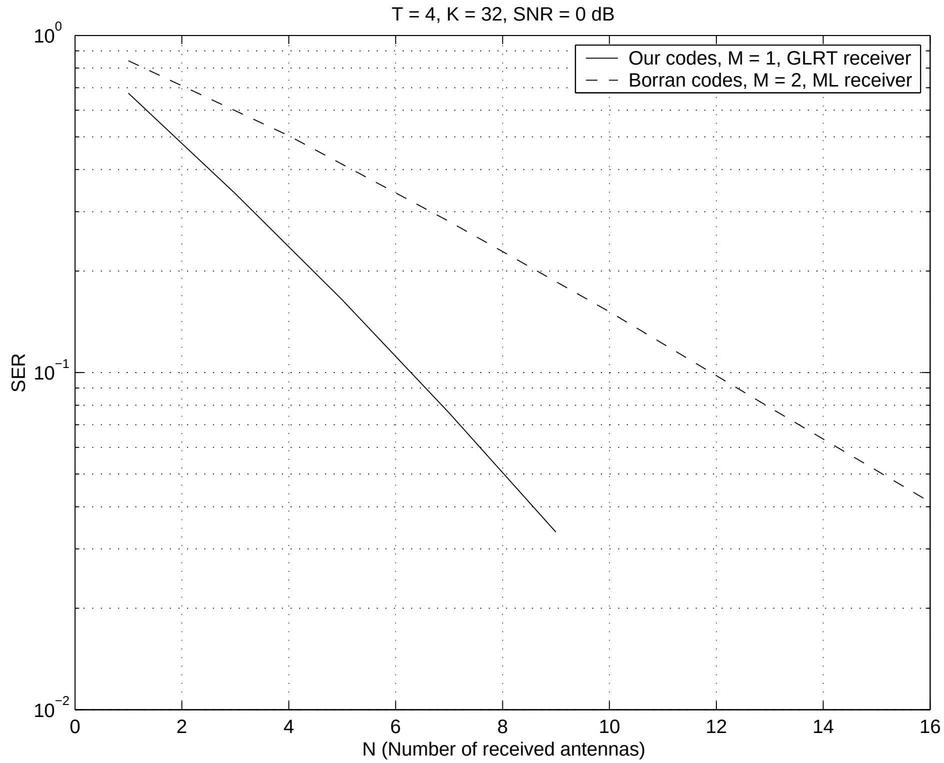

Figure 13: Solid curve-our codes for $K = 32$, $T = 4$, $M = 1$, dashed curve-Borran codes for $K = 32$, $T = 4$, $M = 2$.

□ **Category 1 - spatio-temporally white observation noise: Constellations with equal priors and $M \geq 1$**

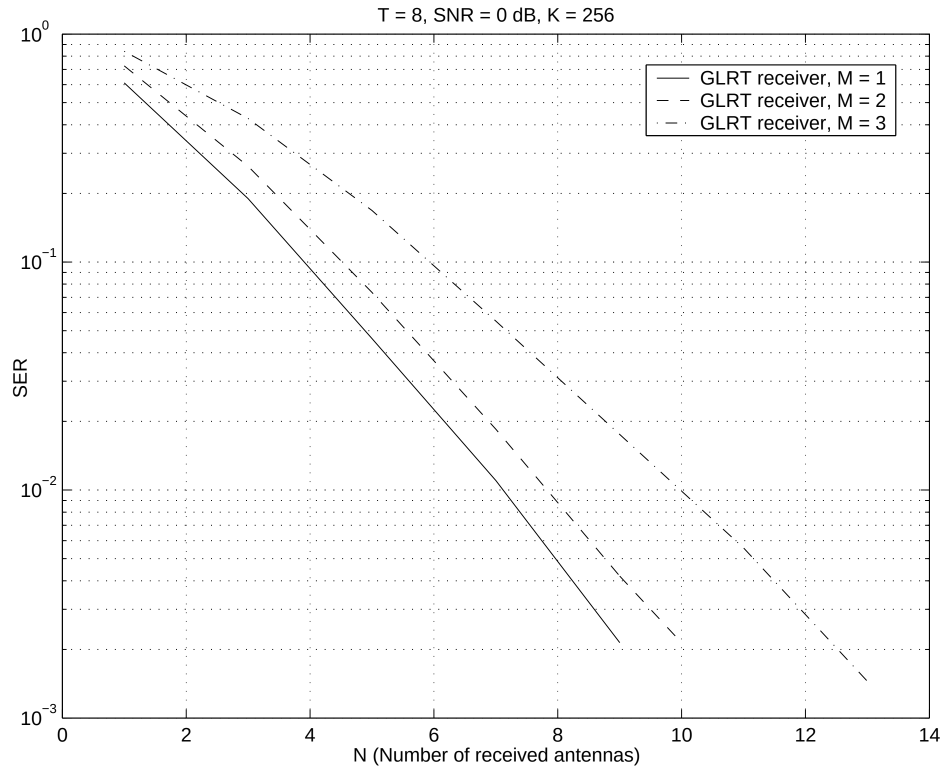

Figure 14: T=8, K=256, SNR = 0 dB. Solid curve-our codes for M = 1, dashed curve-our codes for M = 2, dash-dotted curve-our codes for M = 3. All codes use GLRT receiver.

□ **Category 2 - spatially white - temporally colored:** $\boldsymbol{\Upsilon} = \boldsymbol{I}_{NT} \otimes \Sigma(\boldsymbol{\rho})$

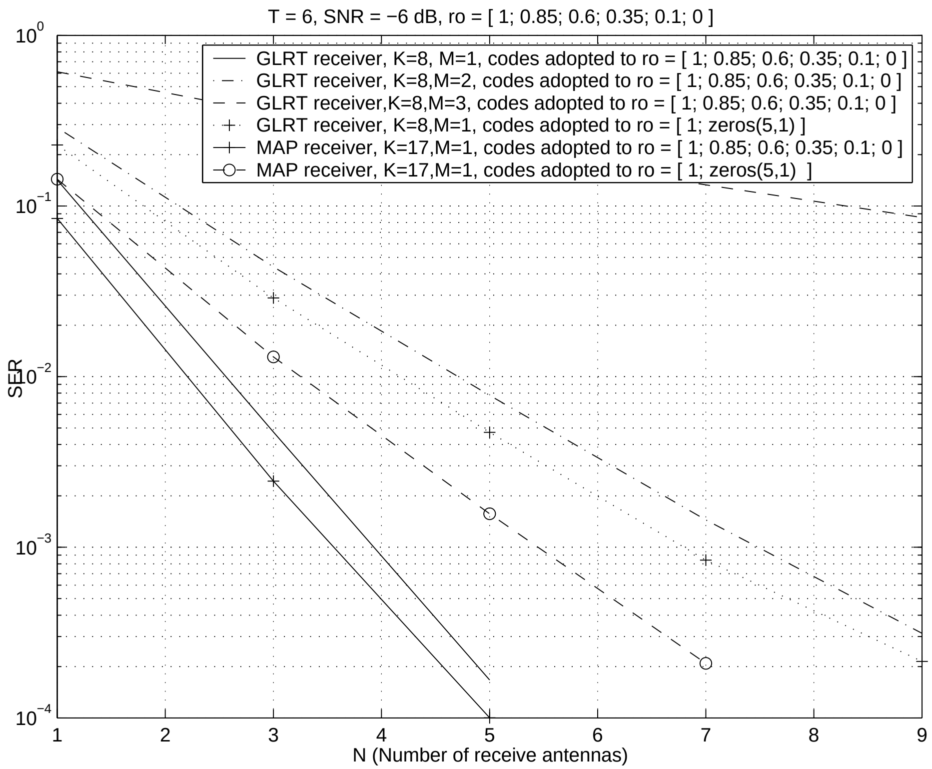

Figure 15: T=6, SNR=-6dB, $\boldsymbol{\rho}$=[1; 0.85; 0.6; 0.35; 0.1; 0].

□ Category 3 - $\boldsymbol{E} = \boldsymbol{s}\,\boldsymbol{\alpha}^T + \boldsymbol{E}_{\text{temp}}$

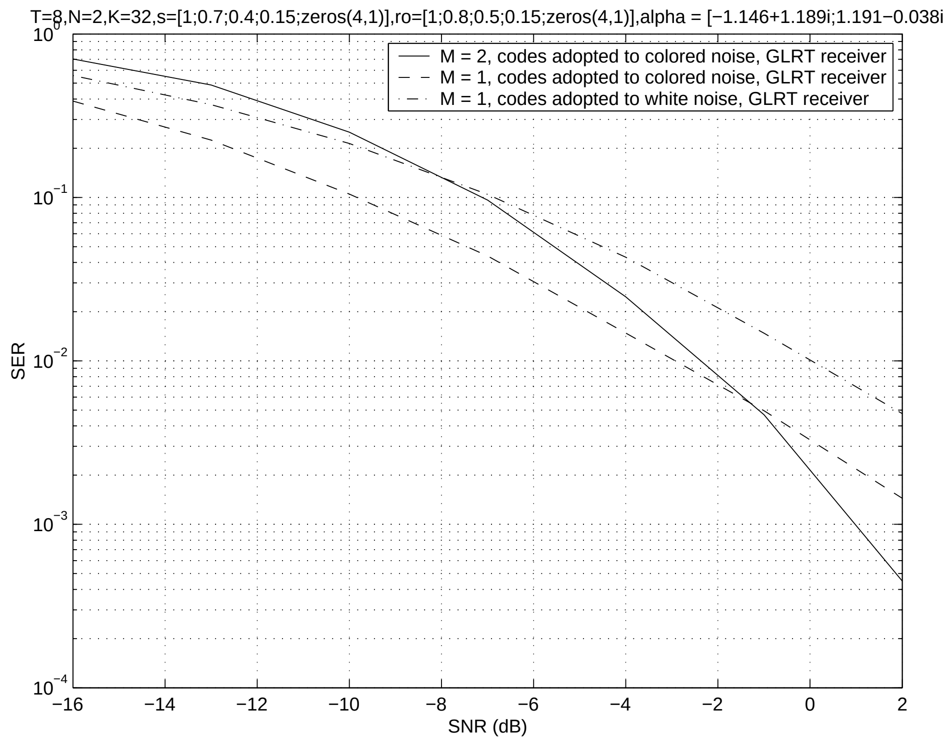

Figure 16: T=8, $N = 2$, K=32, $\boldsymbol{s}$=[1;0.7;0.4;0.15;zeros(4,1)], $\boldsymbol{\rho}$ = [1;0.8;0.5;0.15;zeros(4,1)], $\boldsymbol{\alpha}$ = [-1.146 + 1.189i;1.191- 0.038i].

Conclusions

▷ PEP analysis and codebook design in low SNR regime when $\boldsymbol{H}$ is deterministic and unknown

▷ Results

- outperform significantly state-of-art known solutions which assume equal prior probabilities
- also of interest for the constellations with unequal priors

▷ Publications

- conference paper published in IEEE SPAWC'2006
- conference paper submitted to IEEE ICASSP'2007
- journal paper in preparation for IEEE Trans. on Signal Processing

Part 3: Future work

- ▷ Influence of unperfect estimate of noise covariance matrix on the error performance
- ▷ Cooperative diversity
- ▷ Space-frequency signaling in MIMO-OFDM systems (frequency-selective fading)
- ▷ ETF's
- ▷ Study of double scattering MIMO channels in the low SNR regime

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THANK YOU

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