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Rotation Averaging in a Split Second: A Primal-Dual Method and a Closed-Form for Cycle Graphs

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Introduction

Rotation averaging is a **non-convex optimization problem** which seeks the absolute rotations that optimally explain a set of measured orientations between them. We formulate it as

$$\underset{R_1,\ldots,R_n \in \mathrm{SO}(3)^n}{\text{minimize}} \sum_{i\sim j} \|\tilde{R}_{i,j} - R_i R_j^\top\|_F^2,$$

and make a twofold contribution: a **primal-dual method** and a **closed-form solution for stationary points** in cycle graphs.

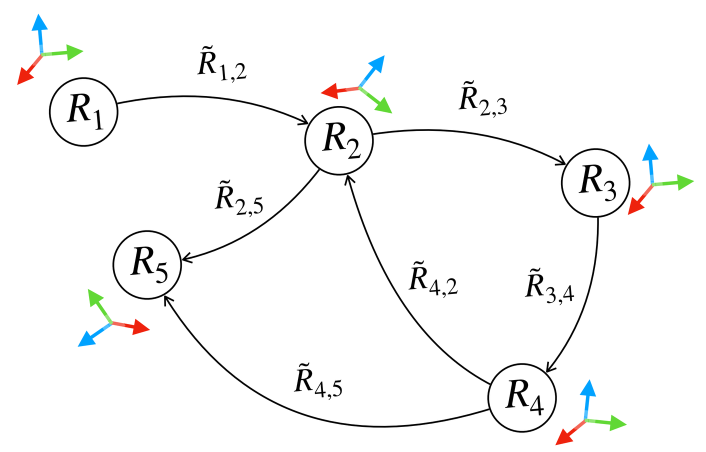


Fig. 1 Rotation averaging graph example.

Primal-dual method

The update rules for our method are derived from the stationarity KKT and the sufficient optimality conditions (Eriksson *et al.*). Respectively,

$$(\Lambda - \tilde{R})R = 0, \qquad \Lambda - \tilde{R} \succeq 0.$$

Measurements matrix

$$\tilde{R} = \begin{bmatrix} I_3 & \tilde{R}_{1,2} & \tilde{R}_{1,3} & 0 & \ldots & \tilde{R}_{1,n} \\ \tilde{R}_{2,1} & I_3 & \tilde{R}_{2,3} & \tilde{R}_{2,4} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{n,1} & 0 & \ldots & \tilde{R}_{n,j} & \ldots & I_3 \end{bmatrix} \in \mathbb{R}^{3n\times 3n}$$

Rotations vector

$$R = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} \in \mathrm{SO}(3)^n$$

Symmetric Lagrange multiplier

$$\Lambda = \begin{bmatrix} \Lambda_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \Lambda_n \end{bmatrix} \in \mathbb{R}^{3n\times 3n}$$

For noise-free measurements $\Lambda_i^* = (1+d_i)I_3$, where d_i is the degree of the i-th node. We set the initialization for the dual variable as

$$\Lambda^0 = (D + I_n) \otimes I_3,$$

where D is the graph degree matrix.

Primal-dual algorithm

Primal

1 Basis for subspace corresponding to 3 smallest eigenvalues of $\Lambda^k - \tilde{R}$

2 Project 3×3 blocks to $\mathrm{SO}(3)$

Dual

3 Update dual variable

$$\Lambda_i^{k+1} = \frac{1}{2}\sum_{i\sim j} \tilde{R}_{i,j} R_j^k R_i^{k^\top} + \frac{1}{2}\sum_{i\sim j} R_i^k \tilde{R}_{i,j}^\top R_j^{k^\top}$$

Closed-forms in cycle graphs

Define the cycle error as $E = \tilde{R}_{1,2}\tilde{R}_{2,3}\ldots\tilde{R}_{n,1}$ and the set of the n-th roots of E as $\{E_0, E_1, \ldots, E_{n-1}\}$ where $\angle(E_k) = \angle(E)/n + 2k\pi/n$. The stationary points verify $R_i^* R_j^{*^\top} = \tilde{R}_{i,j} E_k^\top$ and the spectrum of $\tilde{R}$ is

$$\sigma(\tilde{R}) = \{1 + 2\cos(\angle(E_k)\}_{k=0,\ldots,n-1} \cup \{1 + 2\cos(2k\pi/n)\}_{k=0,\ldots,n-1}$$

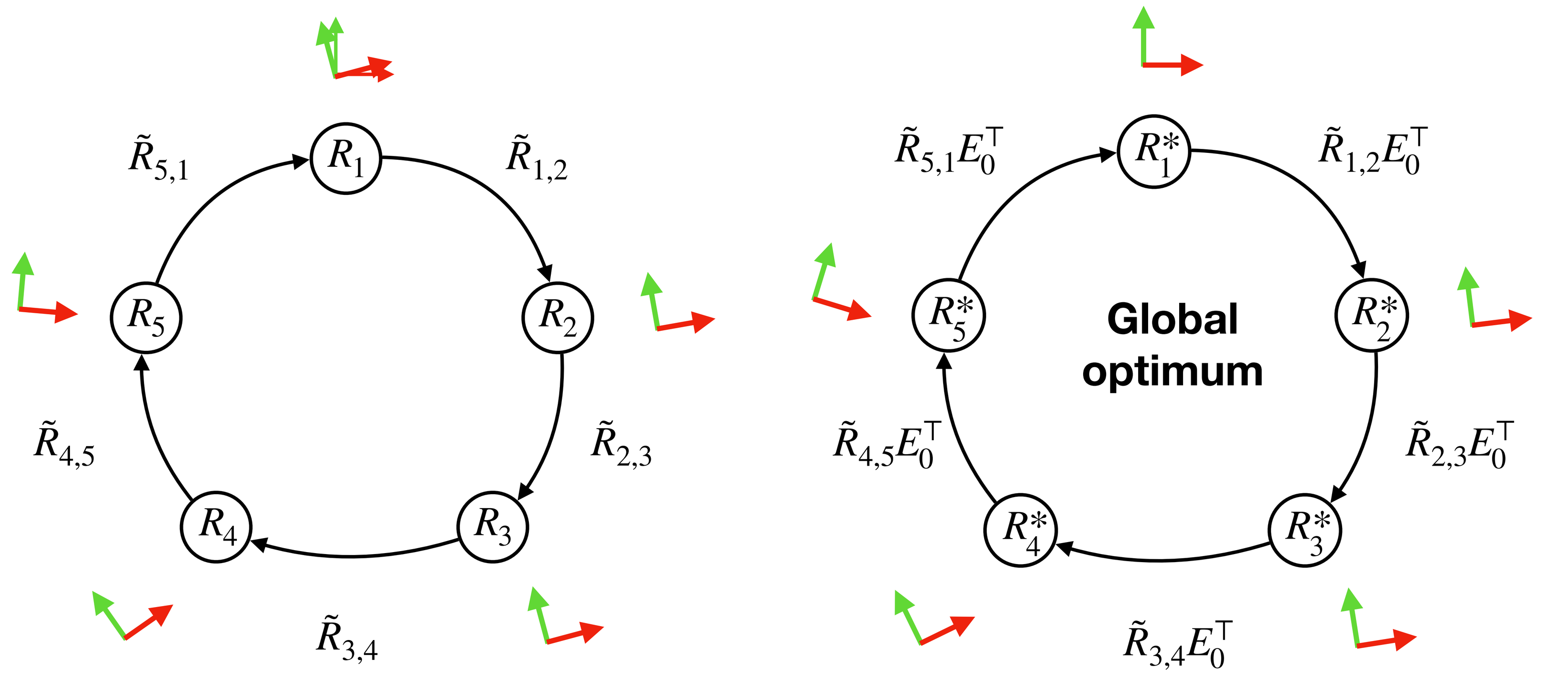


Fig. 2 Global optimum in cycle graphs via redistribution of the cycle error.

Experiments

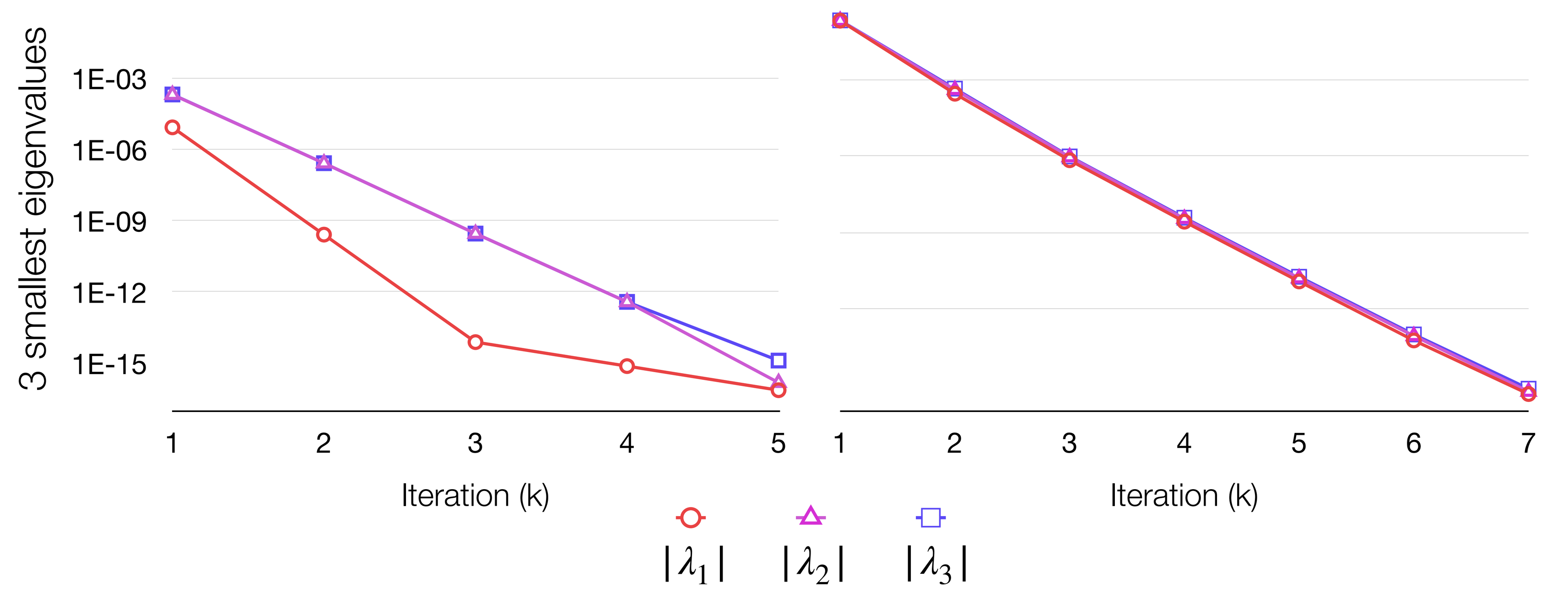
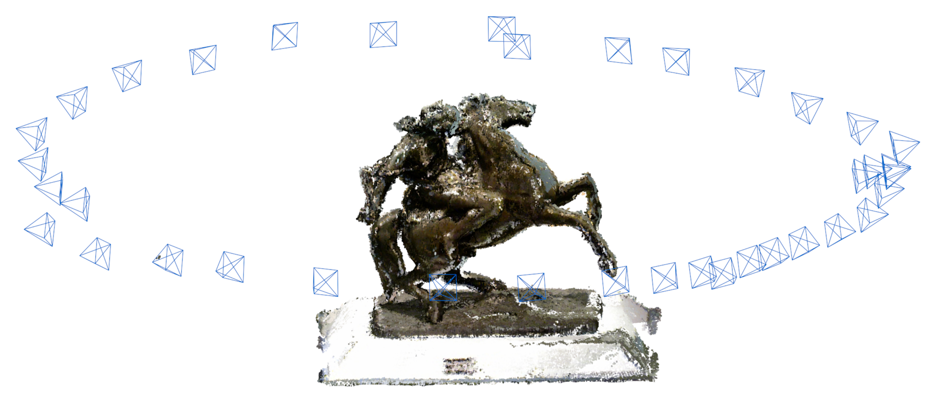


Fig. 3 Convergence of the 3 smallest eigenvalues of $\Lambda^k - \tilde{R}$ to machine-precision, for two pose graph optimization datasets, Cubicle and Sphere, taking a total of 0.46s and 0.36s, respectively.

Fig. 4 3D reconstruction using the closed-form solution for cycle graphs computed in $70\mu s$.



Conclusion

Primal-dual

Faster than SoA

Globally optimal for moderate noise levels

Cycle graphs

Closed-form **stationary points** for the sum of squared chordal errors

Spectral characterization of the measurements matrix $\tilde{R}$

github.com/gabmoreira/maks

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