Nonlinear optimization

Instituto Superior Técnico–Carnegie Mellon University PhD course
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Main goal

You will learn how to solve optimization problems:

$$\begin{array}{ll} \underset{x}{\text{minimize}} & f(x) \\ \text{subject to} & x \in \Omega \end{array}$$

- $x \in \mathbf{R}^n$ is the optimization variable
- $f : \mathbf{R}^n \to \mathbf{R} \cup \{\infty\}$ is the cost function
- $\Omega \subset \mathbf{R}^n$ is the constraint set

Example: warehouse management

Problem data:

- you can buy two products with costs c_1 and c_2
- you can sell them at prices p_1 and p_2
- demands for the products are d_1 and d_2
- volumes of products are v_1 and v_2
- maximum capacity of your warehouse is volume v

How much should you order from each product?

Decision variables:

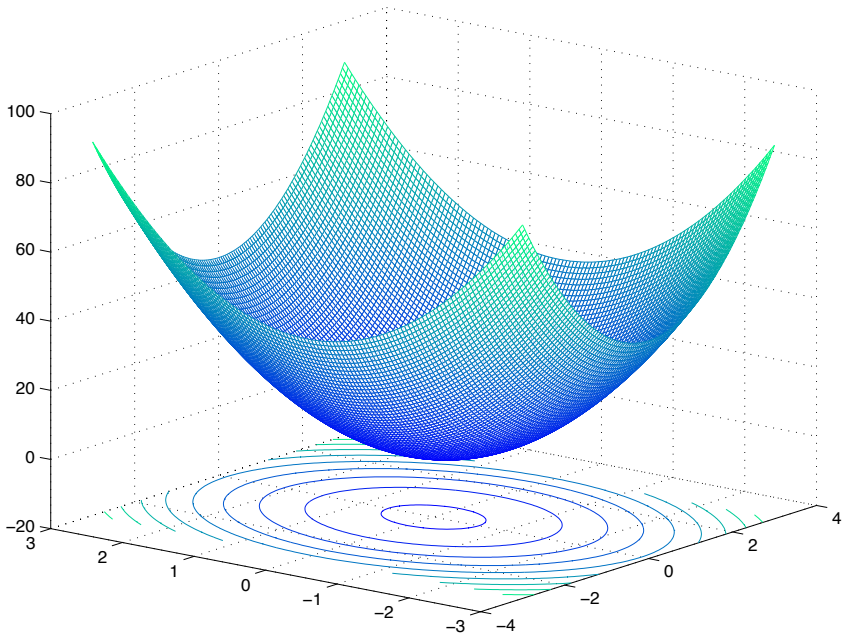
- $x_1 =$ quantity that you order for product 1
- $x_2 =$ quantity that you order for product 2

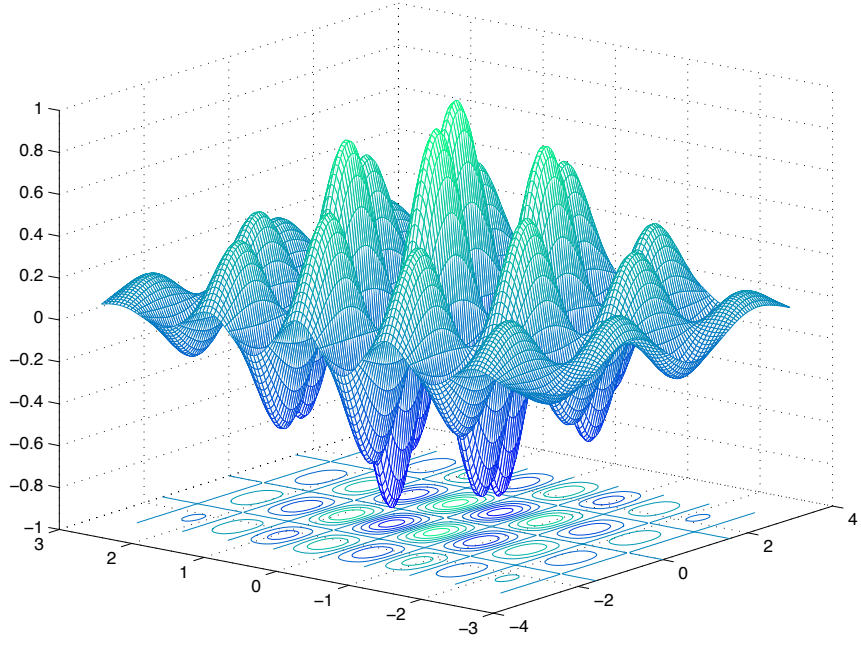
Formulation that minimizes your net cost:

$$\begin{aligned}
&\underset{x_1 \in \mathbf{R}, x_2 \in \mathbf{R}}{\text{minimize}} && \underbrace{\underbrace{c_1 x_1 + c_2 x_2}_{\text{you spend this much}} - \underbrace{(p_1 \min\{x_1, d_1\} + p_2 \min\{x_2, d_2\})}_{\text{you receive this much}}}_{f(x_1,x_2) = \text{net cost for ordering quantities } x_1 \text{ and } x_2} \\
&\text{subject to} && v_1 x_1 + v_2 x_2 \leq v \\
& && x_1 \geq 0 \\
& && x_2 \geq 0
\end{aligned}$$

Key point

Two leagues:





Convex = easy

Nonconvex = hard

We focus on convex optimization: lots of applications, beautiful theory, efficient algorithms

Contents

Bibliography

- *Convex optimization*, S. Boyd and L. Vandenberghe
- *Lectures on modern convex optimization*, Aharon Ben-Tal and Arkadi Nemirovski, 2001, MPS-SIAM Series on Optimization
- *Nonlinear programming*, 2nd ed., Dimitri Bertsekas, 1999, Athena Scientific

Grading

- grade $=$ homework (60%) $+$ 24h take-home exam (40%)
- about 6 homeworks
- 24h take-home exam is on June 4