

Nonlinear optimization

Instituto Superior Técnico–Carnegie Mellon University PhD course
João Xavier
TA: Hung Tuan

February 27, 2018

Main goal

You will learn how to solve optimization problems:

$$\begin{array}{ll} \underset{x}{\text{minimize}} & f(x) \\ \text{subject to} & x \in \Omega \end{array}$$

- $x \in \mathbf{R}^n$ is the optimization variable
- $f : \mathbf{R}^n \rightarrow \mathbf{R} \cup \{\infty\}$ is the cost function
- $\Omega \subset \mathbf{R}^n$ is the constraint set

Example: warehouse management

Problem data:

- you can buy two products with costs c_1 and c_2
- you can sell them at prices p_1 and p_2
- demands for the products are d_1 and d_2
- volumes of products are v_1 and v_2
- maximum capacity of your warehouse is volume v

How much should you order from each product?

Decision variables:

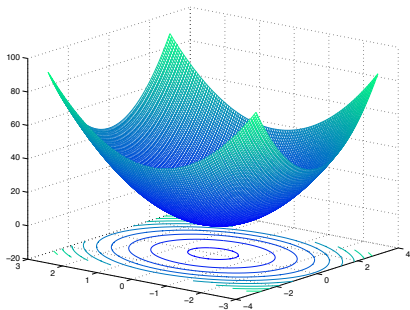
- x_1 = quantity that you order for product 1
- x_2 = quantity that you order for product 2

Formulation that minimizes your net cost:

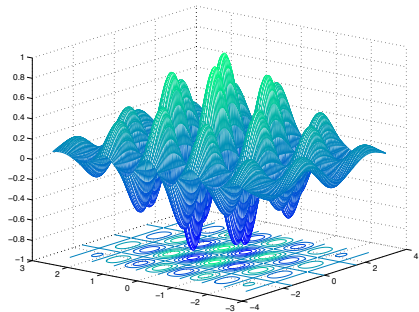
$$\begin{array}{ll} \text{minimize} & \underbrace{c_1x_1 + c_2x_2}_{\text{you spend this much}} - \underbrace{(p_1 \min\{x_1, d_1\} + p_2 \min\{x_2, d_2\})}_{\text{you receive this much}} \\ \text{subject to} & \underbrace{f(x_1, x_2)}_{\text{net cost for ordering quantities } x_1 \text{ and } x_2} \\ & v_1x_1 + v_2x_2 \leq v \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{array}$$

Key point

Two leagues:



Convex = easy



Nonconvex = hard

We focus on convex optimization: lots of applications, beautiful theory, efficient algorithms

Contents

Part 1 Formulation of optimization problems

- convex sets and functions
- the LP, QP, QCQP, SOCP, SDP classes
- CVX software
- applications: learning, control, networks, estimation, communications, finance, . . .

Part 2 Conditions for optimality and duality theory

- KKT conditions
- dual problems
- applications: closed-form solutions or simple finite-step algorithms, provable lower bounds, problem simplification, . . .

Part 3 algorithms

- interior point methods
- large-scale: fast Nesterov gradient method, FISTA, ADMM, proximal methods, . . .

Bibliography

- *Convex optimization*, S. Boyd and L. Vandenberghe
- *Lectures on modern convex optimization*, Aharon Ben-Tal and Arkadi Nemirovski, 2001, MPS-SIAM Series on Optimization
- *Nonlinear programming*, 2nd ed., Dimitri Bertsekas, 1999, Athena Scientific

Grading

- grade = homework (60%) + 24h take-home exam (40%)
- about 6 homeworks
- 24h take-home exam is on June 4