Nonlinear Optimization (18799 B, PP)
IST-CMU PhD course
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The homework is due April 14.

Homework 2

Problem A. (*SVM with regularized hinge loss*) Given a training dataset

$$\{(x_k, y_k) : k = 1, 2, \ldots, K\}$$

where $x_k \in \mathbf{R}^n$ and $y_k \in \{-1, 1\}$ denote the feature vector and the binary label of the kth sample, respectively, the goal of a regularized support vector machine (SVM) with hinge loss is to solve

$$\underset{(s,r)\in\mathbf{R}^n\times\mathbf{R}}{\text{minimize}} \quad \underbrace{\rho \|s\|_1 + \frac{1}{K}\sum_{k=1}^{K}\left(1 - y_k(s^T x_k - r)\right)_+}_{f(s,r)}, \tag{1}$$

where $\rho > 0$ is a given constant. There is an interesting rationale behind the construction of the objective function $f : \mathbf{R}^n \times \mathbf{R} \to \mathbf{R}$ but we will skip it since it is not needed here.

Show that the objective function $f : \mathbf{R}^n \times \mathbf{R} \to \mathbf{R}$ is convex.

Problem B. *Placing a base station (cont.).* Recall problem A from Homework 1: You want to place a base station to serve N wireless users in a given area. The position of user $n = 1, \ldots, N$, is given and is denoted by $u_n \in \mathbf{R}^2$, for $n = 1, ..., N$. The position of the base station is to be decided and is denoted by $b \in \mathbf{R}^2$.

(a) Suppose that if the distance from the base station to a user is less than or equal to a given $R > 0$, then that user is considered to be well-served (the user is well within the range of the base station). In such a case, you pay no penalty to that user. However, whenever the distance exceeds R for a user, you pay a compensation (to that user) that is proportional to the square of the excess; for user n, the constant of proportionality is denoted by $\pi_n > 0$.

The optimal placement of the base station corresponds to solving

$$\underset{b\in\mathbf{R}^2}{\text{minimize}} \quad \underbrace{\sum_{n=1}^{N} \pi_n \left((\|u_n - b\| - R)_+\right)^2}_{f(b)}.$$

Show that the objective function $f : \mathbf{R}^2 \to \mathbf{R}$ is convex.

(b) Suppose now that we can also change the nominal range R of the base station − say, by building a base station with different transmitter power. Assume that the cost of building a base station with nominal range R is ρR^2, where the constant $\rho > 0$ is given. The problem of *jointly* deciding the optimal base station placement and nominal range is

$$\begin{array}{ll} \underset{(b,R)\in\mathbf{R}^2\times\mathbf{R}}{\text{minimize}} & \underbrace{\sum_{n=1}^{N} \pi_n \left((\|u_n - b\| - R)_+\right)^2 + \rho R^2}_{f(b,R)} \\ \text{subject to} & R \geq 0. \end{array}$$

Show that the objective function $f : \mathbf{R}^2 \times \mathbf{R} \to \mathbf{R}$ is convex.