DISTRIBUTED COMPRESSED SENSING ALGORITHMS: COMPLETING THE PUZZLE

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ABSTRACT

Reconstructing compressed sensing signals involves solving an optimization problem. An example is *Basis Pursuit* (BP) [1], which is applicable only in noise-free scenarios. In noisy scenarios, either the *Basis Pursuit Denoising* (BPDN) [1] or the Noise-Aware BP (NABP) [2] can be used. Consider a distributed scenario where the dictionary matrix and the vector of observations are spread over the nodes of a network. We solve the following open problem: *design distributed algorithms that solve BPDN with a column partition, i.e., when each node knows only some columns of the dictionary matrix, and that solve NABP with a row partition, i.e., when each node knows only some rows of the dictionary matrix and the corresponding observations.* Our approach manipulates these problems so that a recent general-purpose algorithm for distributed optimization can be applied.

***Index Terms*—** Distributed algorithms, compressed sensing

1. INTRODUCTION AND PROBLEM STATEMENT

The optimization problems BPDN and NABP are, respectively,

$$\text{BPDN:} \qquad \underset{x}{\text{minimize}} \quad \|Ax-b\|^2 + \beta\|x\|_1\,, \tag{1}$$

$$\text{NABP:} \qquad \begin{array}{ll} \underset{x}{\text{minimize}} & \|x\|_1 \\ \text{subject to} & \|Ax-b\| \leq \sigma\,, \end{array} \tag{2}$$

where the dictionary matrix $A \in \mathbb{R}^{m\times n}$, the vector of observations $b \in \mathbb{R}^m$, and the parameters $\beta, \sigma > 0$ are given. We consider a connected network of P nodes, where each node knows only part of the data. Namely, we consider the two cases visualized in Fig. 1: *row partition* (resp. *column partition*), where node p stores a block A_p of m_p rows (resp. n_p columns) of the matrix A. Of course, $m_1 + \cdots + m_P = m$ and $n_1 + \cdots + n_P = n$. Also, in the row partition, the vector b is partitioned the same way as A but, in the column partition, all the nodes know the entire vector b.

Problem statement. While there exist distributed algorithms that solve BPDN with a row partition and NABP with a column partition [3], to the best of our knowledge, there are no algorithms solving the reverse cases, i.e., BPDN with a column partition and NABP with a row partition. Our goal is then to *design a distributed algorithm that solves BPDN with a column partition and NABP with a row partition.* Distributed means that no central node is allowed, no node has access to more than its local data, and each node can communicate only with its neighbors.

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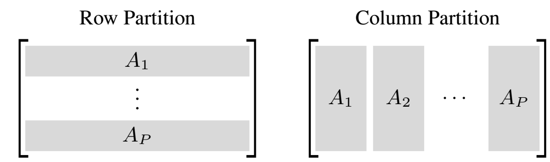

Fig. 1. Row and column partitioning of the dictionary matrix A.

2. OUR APPROACH

We recast (1) (resp. (2)) with a column (resp. row) partition as

$$\begin{array}{ll} \underset{x}{\text{minimize}} & g_1(x) + g_2(x) + \cdots + g_P(x) \\ \text{subject to} & h_1(x) + h_2(x) + \cdots + h_P(x) \leq 0\,, \end{array} \tag{3}$$

where $g_p : \mathbb{R}^n \to \mathbb{R}$ and $h_p : \mathbb{R}^n \to \mathbb{R}^m$ are convex functions, known only by node p. To recover a primal solution, however, we have to assume that each g_p is strictly convex. Reformulating our problems as (3) will enable the use of the recent distributed algorithm proposed in [4], which can solve problems with the format of (3).

Manipulations. BPDN with a column partition is written as the minimization of $(1/2)\|A_1x_1 + \cdots + A_Px_P - b\|^2 + \beta\sum_{p=1}^P \|x_p\|_1$, where $x = (x_1, \ldots, x_P)$ is partitioned according to the columns of A. Introducing a variable $u \in \mathbb{R}^m$, this is equivalent to

$$\begin{array}{ll} \underset{x,u}{\text{minimize}} & \frac{1}{2}\|u\|^2 + \beta\sum_{p=1}^P \|x_p\|_1 \\ \text{subject to} & \sum_{p=1}^P (A_px_p - \frac{1}{P}(u+b)) = 0\,, \end{array}$$

which can readily be written as (3). Regarding NABP with a row partition, it can be written as (3) by setting $g_p(x) = (1/P)\|x\|_1$ and $h_p(x) = \|A_px - b_p\|^2 - \sigma^2/P$. To make the objectives of these problems strictly convex, we can add to them a small perturbation quadratic term, which still allows obtaining good approximations of the solutions of the original problem.

Conclusions. The proposed manipulations enable using the algorithm in [4] to solve the open problem of designing distributed algorithms for BPDN (resp. NABP) with a column (resp. row) partition. Experimental results show the effectiveness of our approach.

3. REFERENCES

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