minimize $\boxed{f(x)}$

subject to $\boxed{\begin{array}{l} g_i(x) \leq 0 \\ h_i(x) = 0 \end{array}}$

$$x^* : f(x^*) \leq f(y)$$

$$\forall y \in \mathrm{Dom}$$

$$\mathrm{Dom} = \left\{ y \in V : \begin{array}{l} g_i(y) \le 0 \\ h_i(y) = 0, \\ i = 1, \dots, k \end{array} \right\}$$

$$\triangleq \quad \text{minimize } e^{x} \quad \text{subject to } x \in \mathbb{R} \quad = p$$

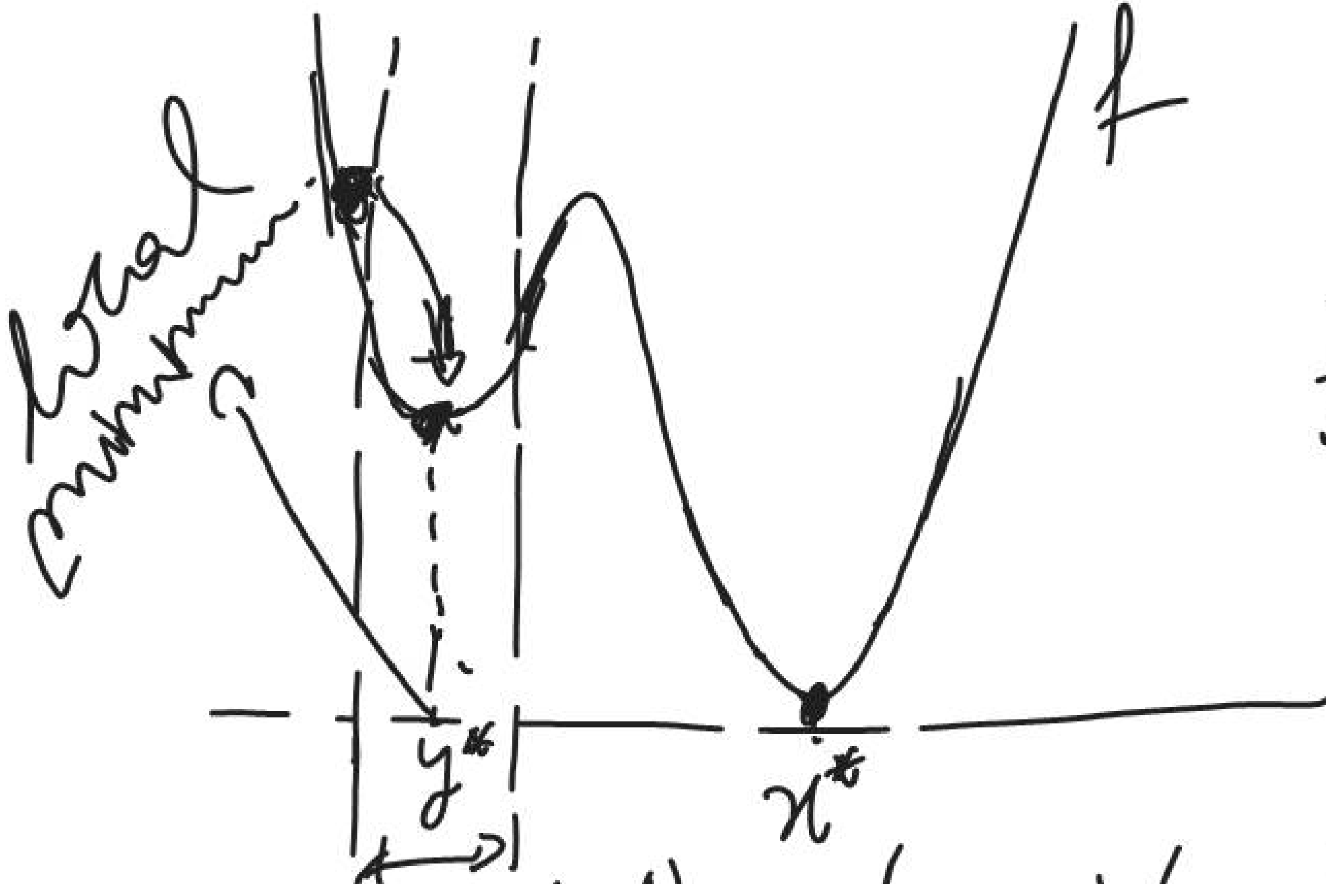
$$x^{*} : e^{x^{*}} \leq e^{y}$$

$$y \in \mathrm{dom} = \mathbb{R}$$

$$\inf\{ e^{x} : x \in \mathbb{R} \} = 0$$

$$x^{*} : e^{x^{*}} = 0$$

Local minimum



$$f(x^*) \leq f(y)$$

$$y \in \mathrm{Dom}$$

$$B_R(y^*) = \{ x \in V : |x - y^*| \leq R \}$$

$$f(y^*) \leq f(y) \quad \forall y \in B_R(y^*)$$

minimize $f(x)$

subject to $g_i(x) \leq 0$

$h_i(x) = 0$

$\boxed{x \in B_R(y^*)}$

$\Updownarrow$

$\|x - y^*\| \leq R$

y^* local

minimum

minimize $f(x)$ $\qquad V \equiv \mathbb{R}^n$

sub. to $\boxed{\begin{array}{c} g_i(x) \le 0 \\ h_i(x) = 0 \end{array}}$

$$\mathrm{Dom} = \left\{ x \in V : g_i(x) \le 0,\ \boxed{h_i(x) = 0},\ i = 1, \dots, k \right\}$$

↓

Convex

h

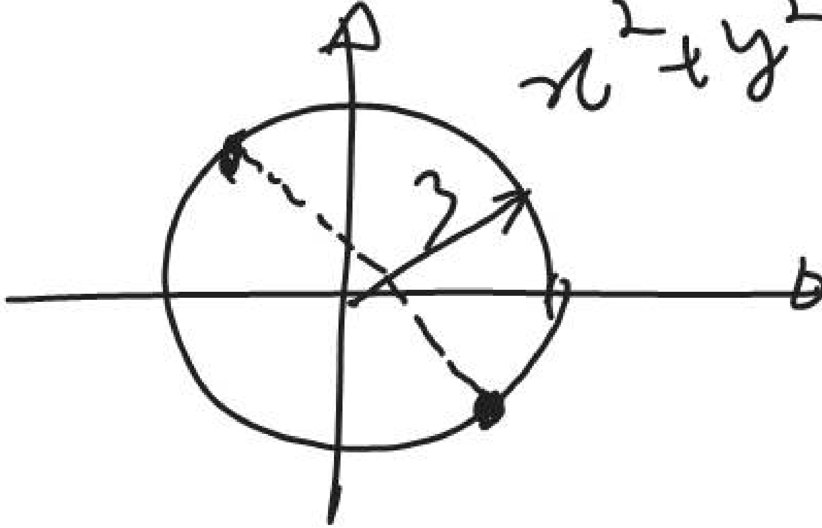
$\{x : h_i(x) = 0\}$ ~~convex~~

convex

$h_i : \mathbb{R}^2 \to \mathbb{R}, \quad h(x,y) = x^2 + y^2 - 9$

$\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 - 9 = 0\} =$

$= \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 9\}$



$x^2+y^2=9$

not CVX

$h_i \rightarrow$ Affine

$$\min f(x)$$
$$\text{s.t. } g_i(x) \leq 0$$
$$h_i(x) = 0$$

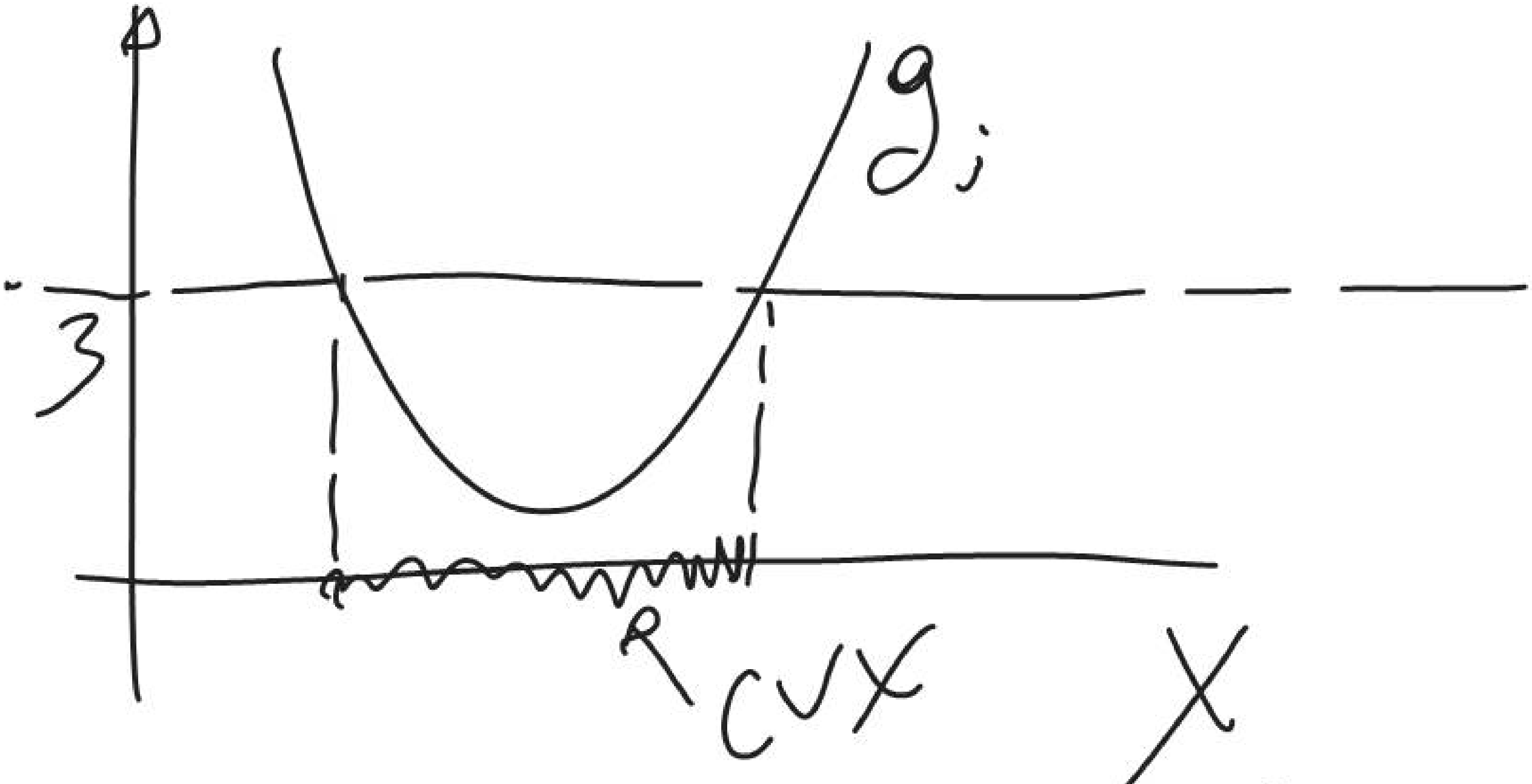
$$S_g \triangleq \{ x \in V : g_i(x) \leq 0 \}$$

CVX $\quad$ CVX?

g_i CVX $\Longrightarrow S_g$ CVX

because

sub-level set of CVX function.



$$\{x \in X : g_i(x) \leq 3\} \text{ cvx}$$

$\{x : g_i(x) \leq 0\}$ cvx

$$\| $$

Because

$$g_i^{-1}\big(\underbrace{(-\infty, 0]}_{\text{cvx}}\big)$$

inverse image of a convex set by a convex function.

$$\{x : \underset{\text{cvx}}{h_i}(x) = 0\} \ \cancel{\text{cvx}}$$

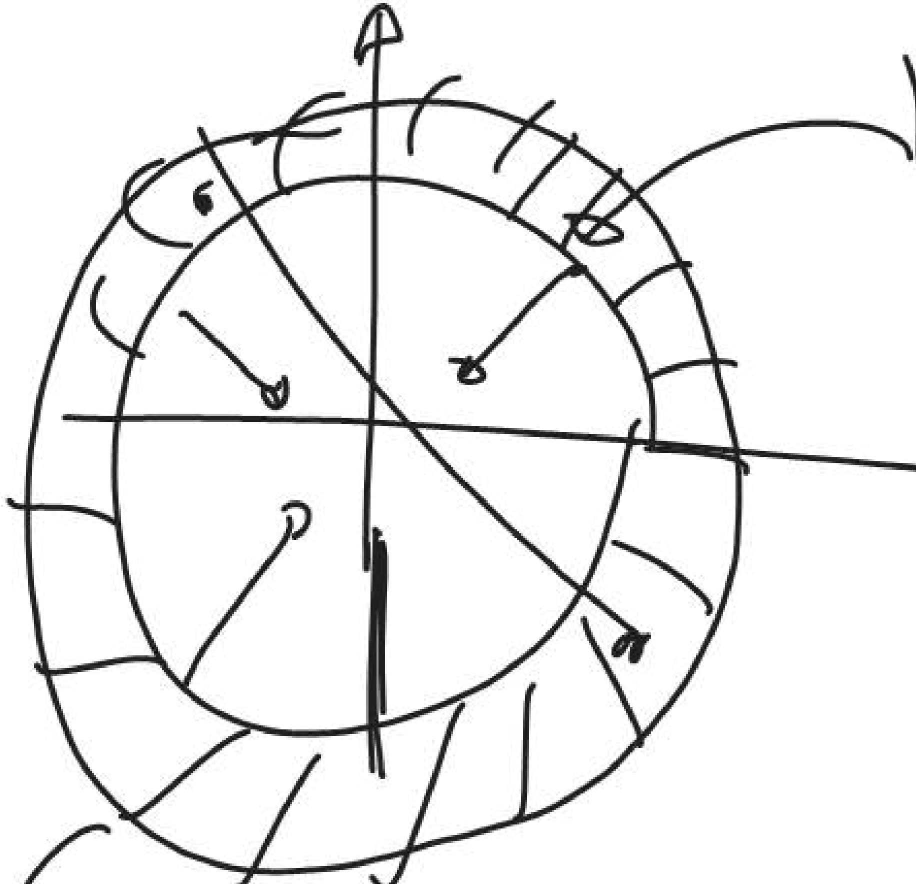
$$h_i(x,y) = x^2 + y^2 - 9$$

$$\|$$

$$h_i^{-1}\Big(\underbrace{\{0\}}_{\text{cvx}}\Big)$$

$$\{x : g_i(x) \leq 0\} =$$

$$= \underbrace{g_i^{-1}\Big(\underbrace{(-\infty, 0]}_{\text{cvx}}\Big)}_{\substack{\text{sub-level set} \\ \text{at level '0'}}}$$

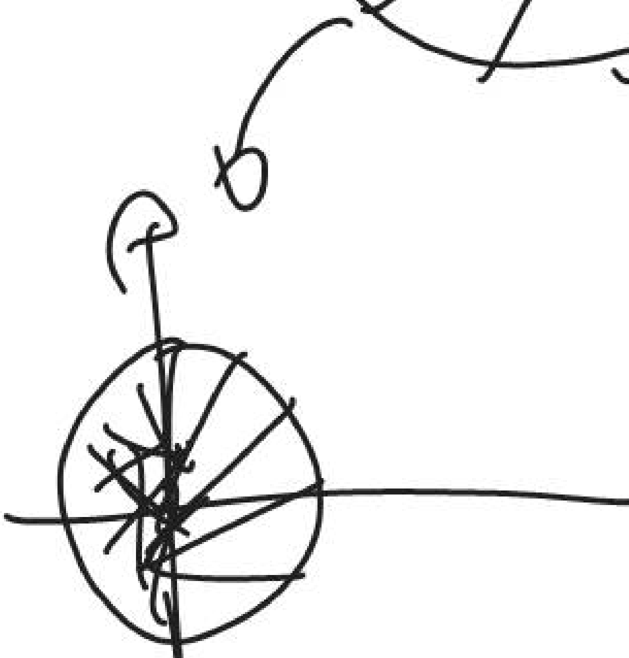


$h_i^{-1}(\{0\})$

$h_i^{-1}([0,3])$

$h_i^{-1}((-\infty,0])$

special.



$$f(x) = a^T x + b$$

$$x \in \mathbb{R}^n$$

$$\exists_{a \in \mathbb{R}^n}, \; b \in \mathbb{R}$$

$$f(x) = a^T x + b$$

$f($

f linear

$$f(a_1 x + a_2 y) = a_1 f(x) + a_2 f(y) \quad \forall x, y \in V = \boxed{\mathbb{R}^n}$$

$$\exists\, A \in \mathbb{R}^{m \times n} : f(x) = Ax$$

f linear

$f(a_1 x + a_2 y) = a_1 f(x) + a_2 f(y)$

$\forall x, y \in V = \boxed{\mathbb{R}^n}$

$\exists A \in \mathbb{R}^{m \times n} : f(x) = Ax$

f : Affine $\quad f: \mathbb{R}^n \to \mathbb{R}$

$$f(a_1 x + a_2 y) = a_1 f(x) + a_2 f(y)$$

$$\forall_{x, y \in \mathbb{R}^n}, \quad \boxed{a_1 + a_2 = 1}$$

f : Affine $\Longrightarrow \exists_{A \in \mathbb{R}^{n \times m},\, b \in \mathbb{R}^m}$

$f(x) = Ax + b$

minimize $f(x)$

sub. to $g_j(x) \leq 0$

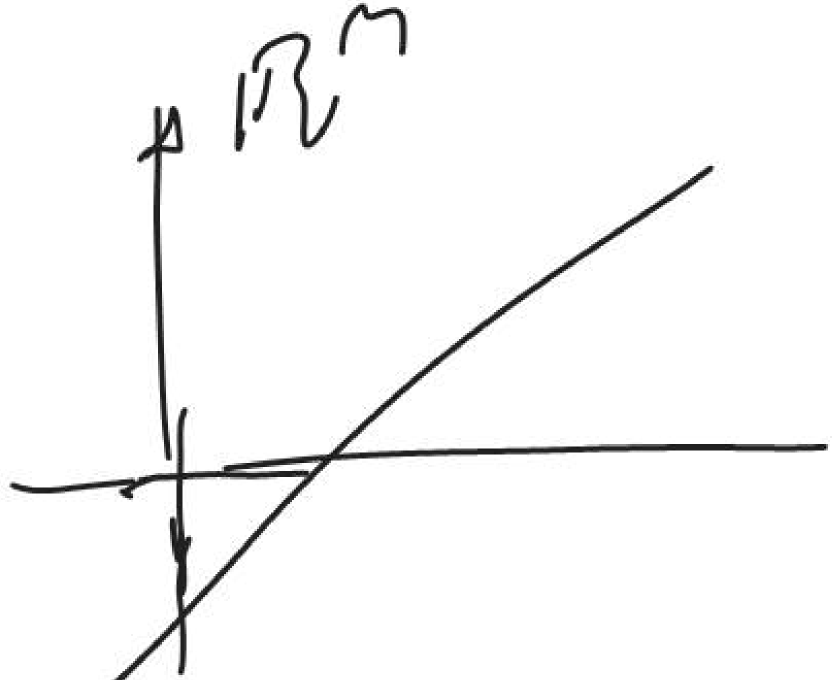
$h_i(x) = 0$

Affine

$$\{x : h_i(x) = 0\} =$$

affine

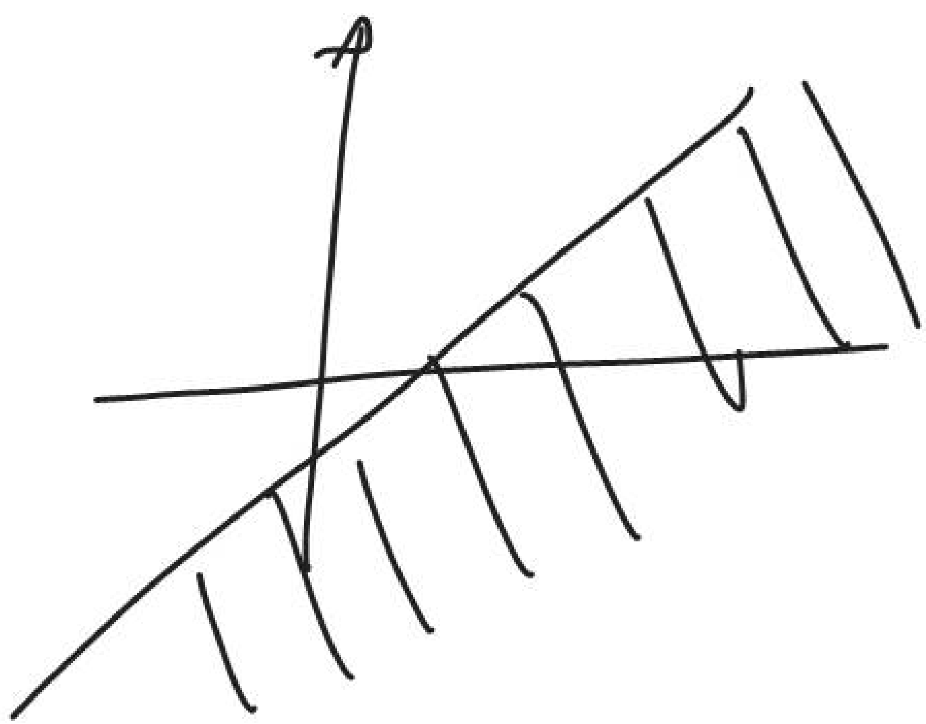
$$= \{x : a_i^T x + b_i = 0\}$$



$\{ x : g_i(x) \leq 0 \}$ for each i

$\Downarrow$

$\{ x : a_i^T x + b_i \leq 0 \}$



subject to $g_i(x) \leq 0$ $\quad i = 1, \dots, K$

$h_j(x) = 0$ $\quad j = 1, \dots, m$

half spaces

$$\{x : g_1(x) \leq 0\} \cap \{x : g_2(x) \leq 0\} \cap \dots \cap \{x : g_K(x) \leq 0\} \cap \{x : h_1(x) = 0\} \cap \dots \cap \{x : h_m(x) = 0\}$$

hyperplanes.

$\bigcap_j g_j^{-1}(x)$