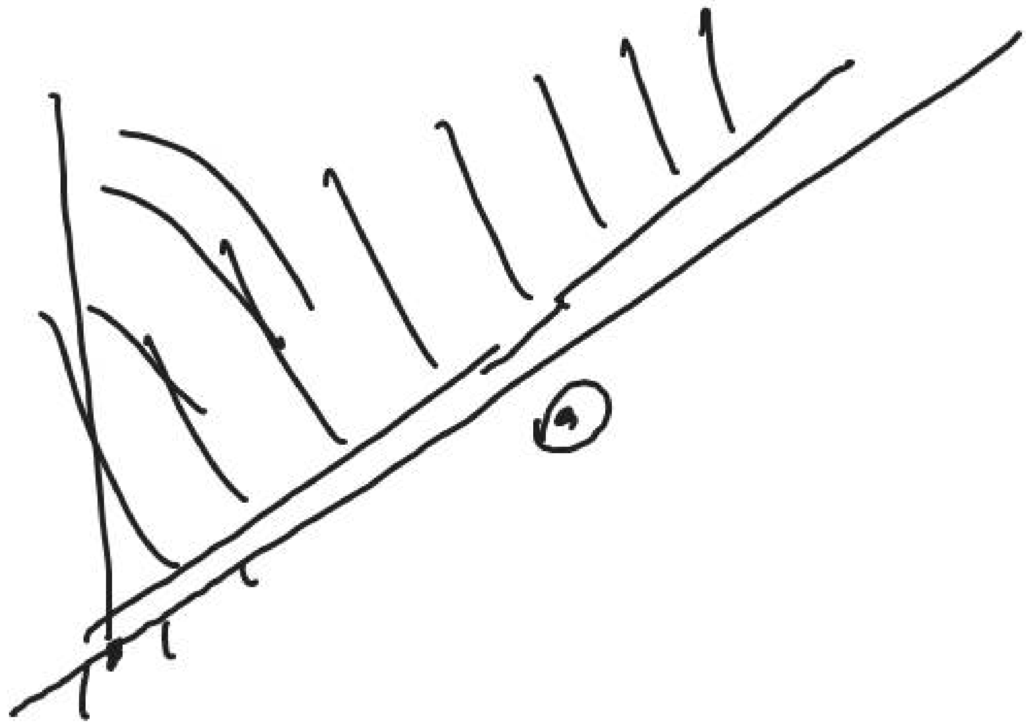
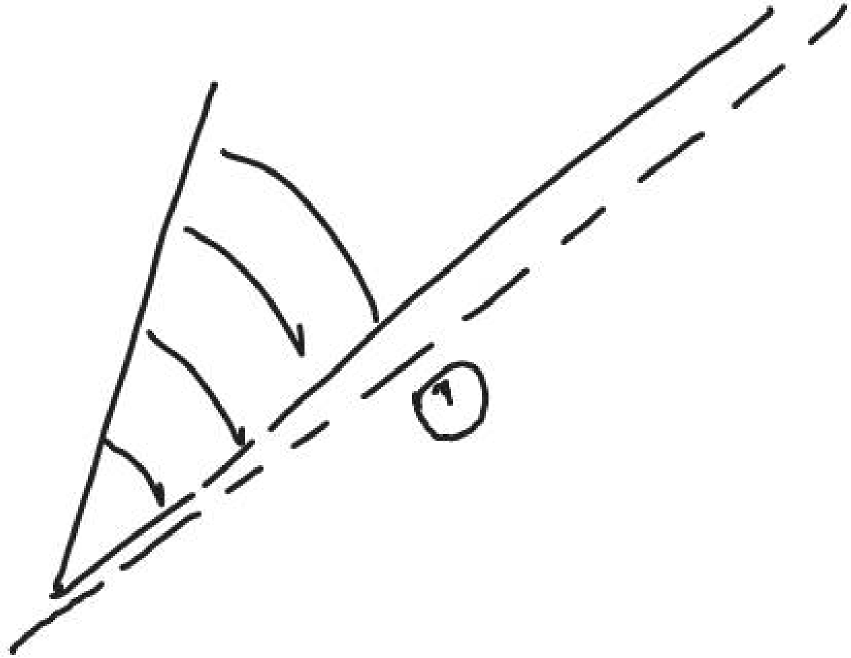


Convex optimization problems


1

minimize $f(x)$ $\qquad V$

subject to $\quad g_i(x) \le 0 \qquad i = 1, \dots, K$

$\qquad\qquad h_i(x) = 0$

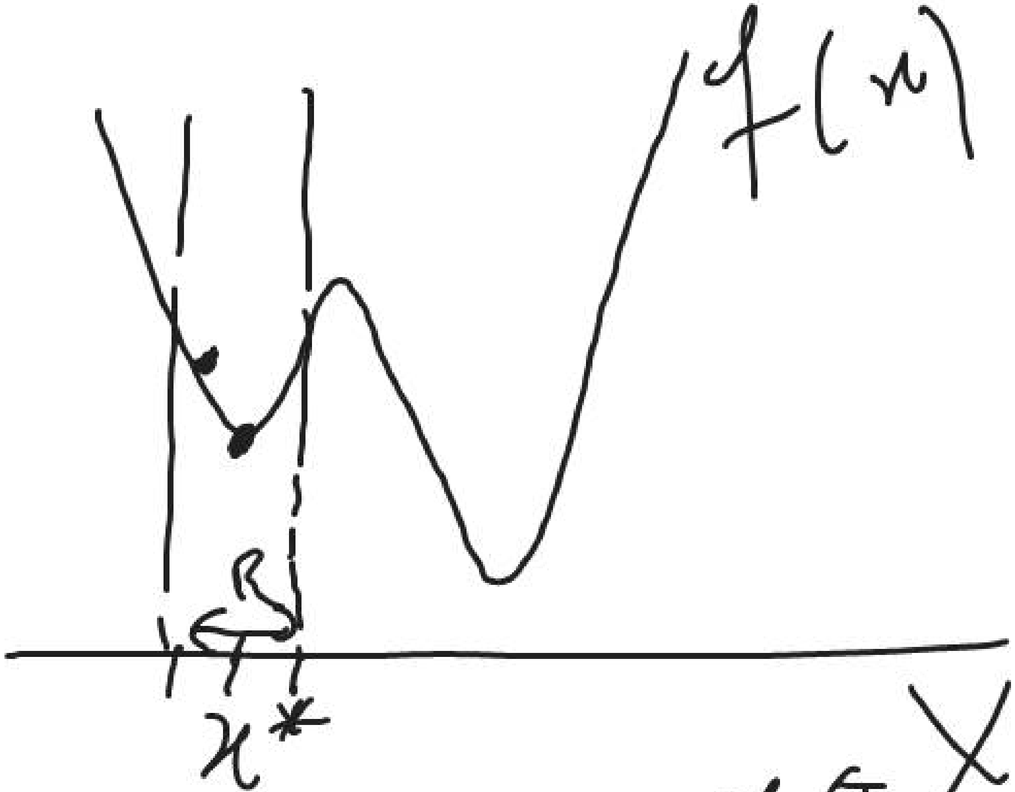
$$x^* \in \mathrm{Dom} = \left\{ x \in V : g_i(x) \le 0;\ h_i(x) = 0 \quad i = 1, \dots, K \right\} =$$

$$= \bigcap_{i=1}^{K} g_i^{-1}\big((-\infty, 0]\big) \cap \bigcap_{j=1}^{K} h_i^{-1}\big(\{0\}\big)$$

$$f(x^*) \leq f(y) \quad \forall y \in \text{Dom}.$$

$$\inf\{ e^{x} : x \in \mathbb{R} \} = 0$$

$$e^{x^*} \leq e^{y} \quad \forall y \in \mathbb{R}$$



$x \in X$

$f(x^*) \le f(y)$ $\quad y \in \boxed{B_R(x^*)}$

$\Updownarrow$

$\boxed{y \text{ ``close'' to } x^*}$

minimize $f(x)$

sub. to

convex $\rightarrow g_i(x) \le 0$

$h_i(x) = 0$

$V = \mathbb{R}^n$

$$\mathrm{Dom} = \{ x \in \mathbb{R}^n : g_i(x) \le 0, \; h_i(x) = 0 \}. \quad i = 1, \dots, k$$

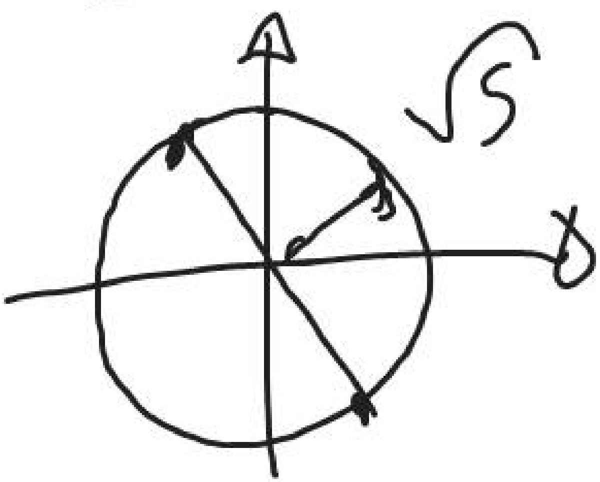
$g_i^{-1}((-\infty, 0])$ cvx

$h_i^{-1}(\{0\})$

$$h_i(x,y) = x^2 + y^2 - 5$$

$$h_i^{-1}(\{0\}) =$$

$$= \left\{ (x,y) \in \mathbb{R}^2 : \underbrace{x^2 + y^2 - 5 = 0}_{\boxed{x^2+y^2=5}} \right\}$$



$$h_i^{-1}(\{0\}) =$$

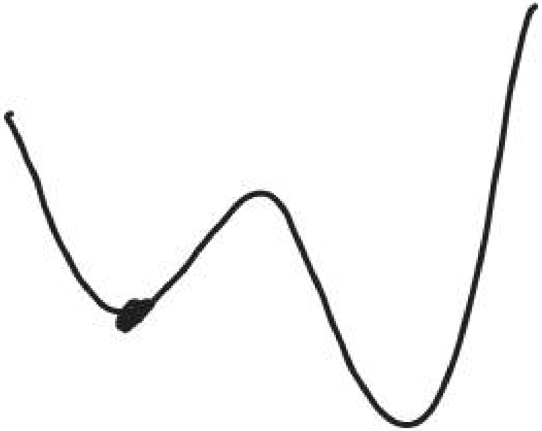
$$V = \mathbb{R}^n$$

$$h_i(x) = a^T x + r$$

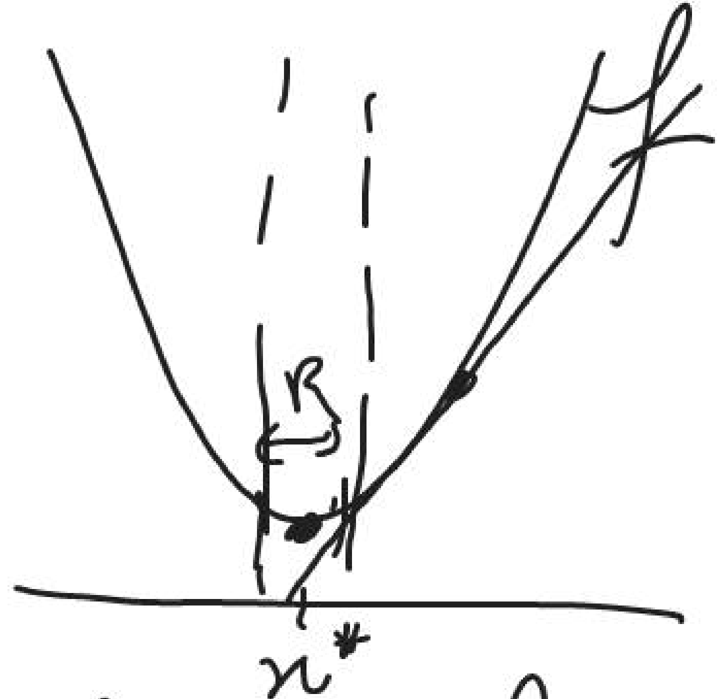
$$a \in \mathbb{R}^n ; \ r \in \mathbb{R}$$

$$h_i^{-1}(\{0\}) = \{ x \in \mathbb{R}^n : a^T x + r = 0 \}$$

$$\underbrace{\left(\bigcap_{j=1}^{k_1} g_j^{-1}((-\infty,0])\right)}_{\text{CVX}} \cap \underbrace{\bigcap_{i=1}^{k_2} h_i(\{0\})}_{\text{CVX}}$$



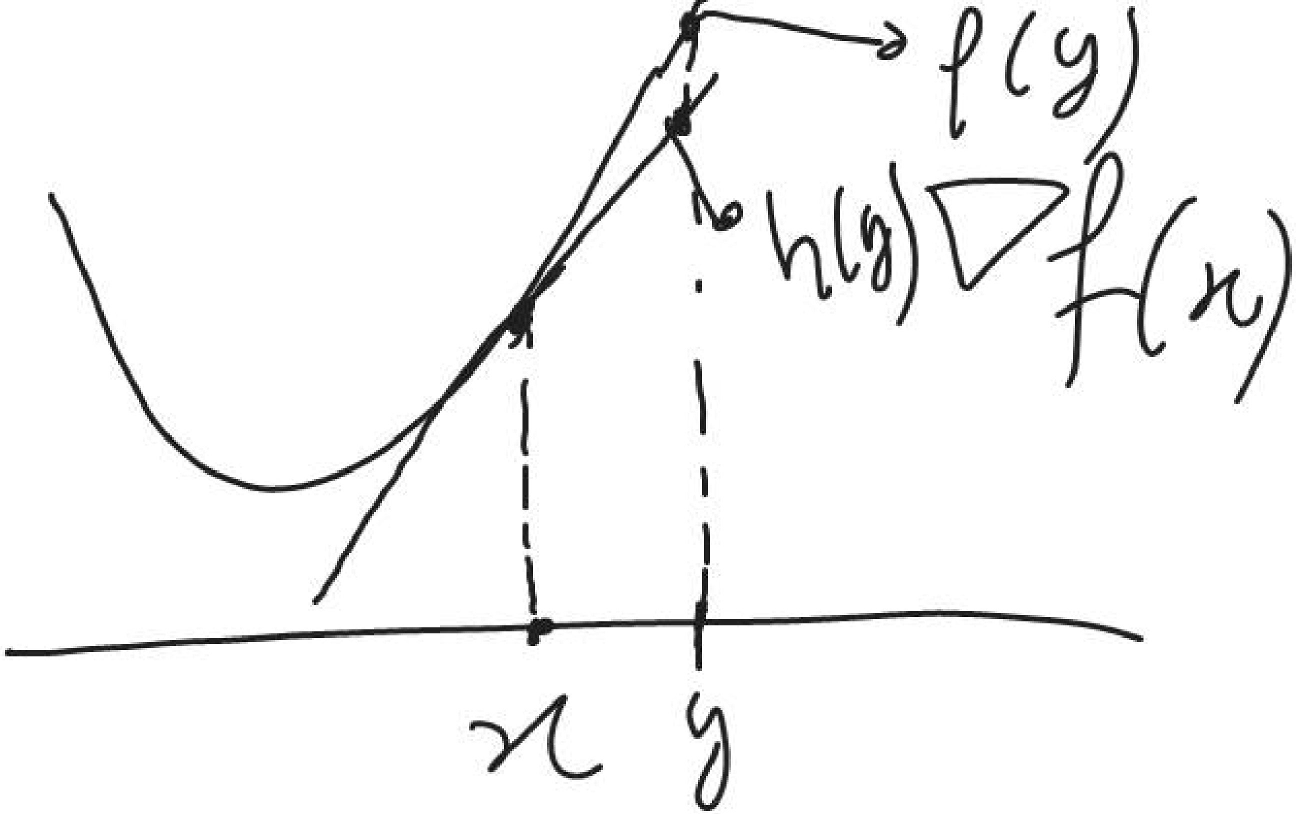
Vs



$$f(y) \geq \underbrace{f(x) + \langle \nabla f(x), y-x \rangle}_{h(y)}$$

$$f(x^*) \leq f(y)$$

$$\forall y \in B_R(x^*)$$



$$f(y)$$

$$\vee\!|$$

$$h(y) = f(x) + \langle$$

$$h(y) = f(x) + \langle \nabla f(x), y - x \rangle$$

$$S_1 \triangleq \{x \in V : g_i(x) \leq \theta\}$$

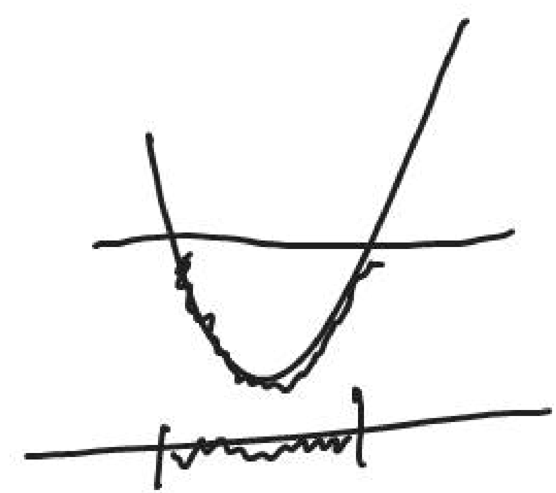
$$S_2 \triangleq \{x \in V : h_i(x) = \theta\}$$

cvx

S_1 cvx

$\|$

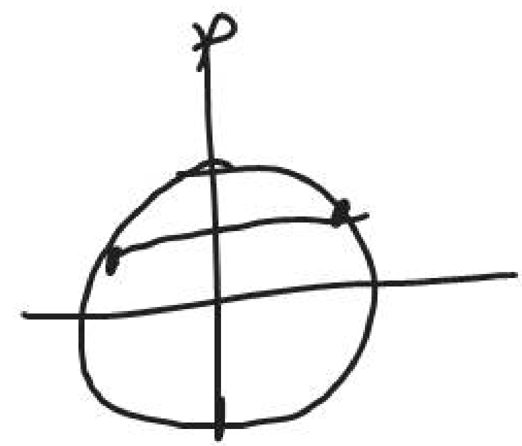
$$g_i^{-1}((-\infty, \theta])$$

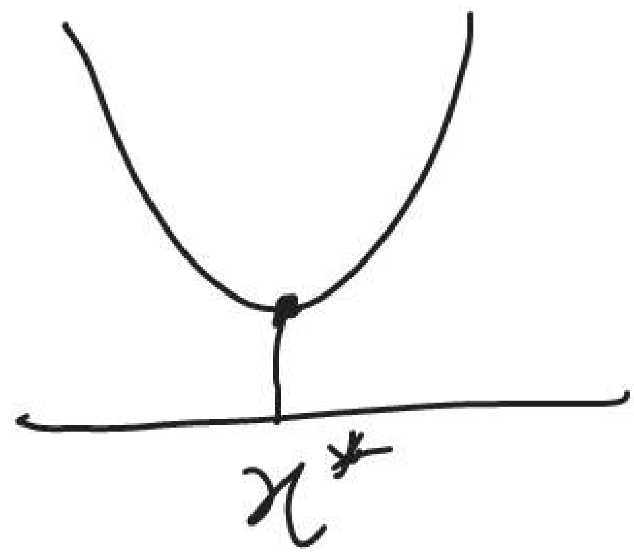


$$h_i(x,y) = x^2 + y^2 - 5$$

$$S_2 = \{x \in V : h_i(x,y) = 0\} =$$

$$=$$





$$\boxed{\nabla f(x^*) = 0}$$

$$f(y) \geq f(x^*) + \langle \underbrace{\nabla f(x^*)}_{=0}, y - x \rangle$$

$$f(x^*) \leq f(y) \quad \forall y \in \mathrm{Dom}$$

CONVEX

Necessary conditions

$\Downarrow$

SUFFICIENT