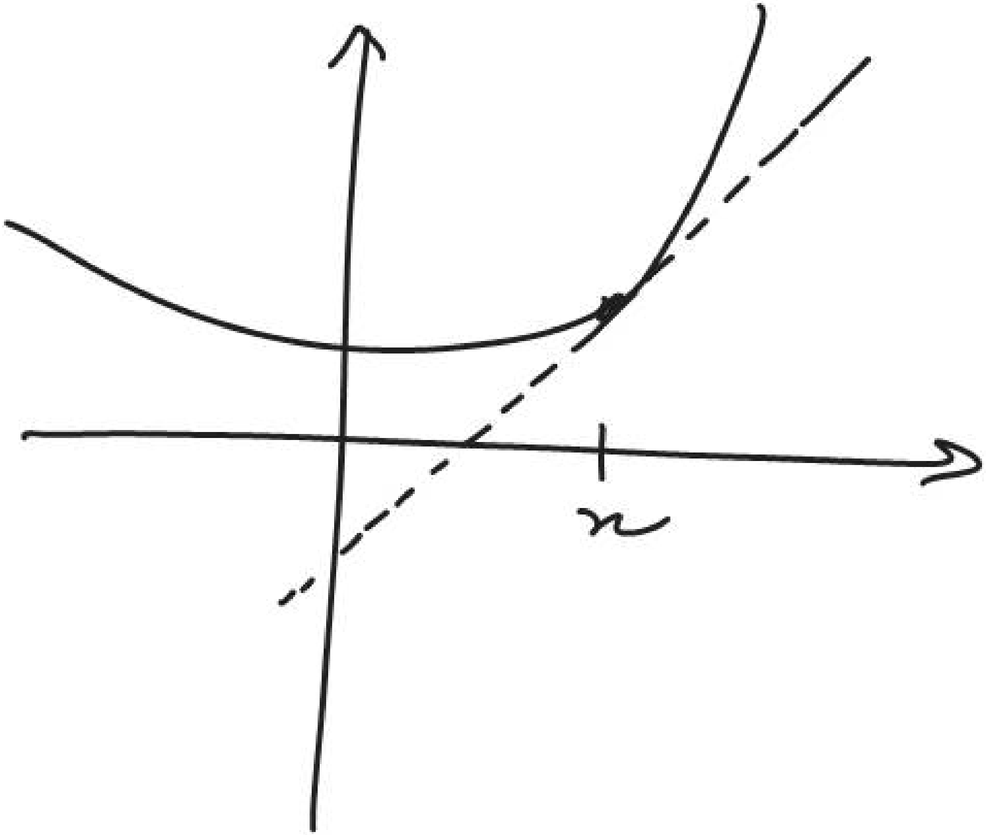
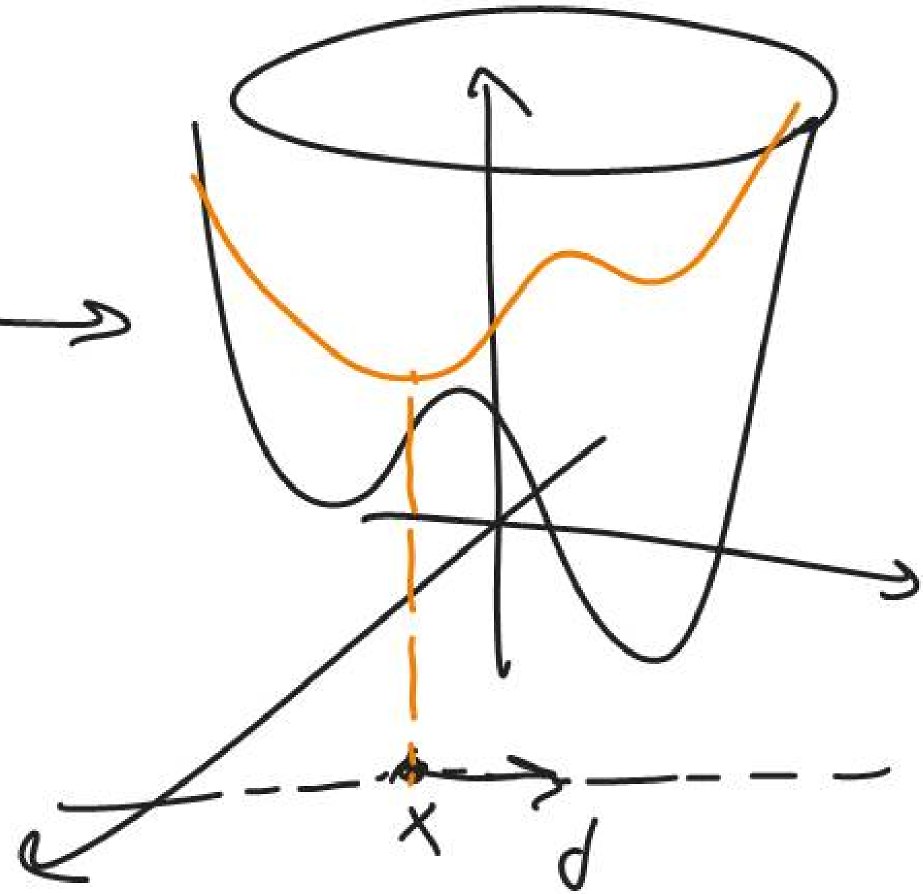
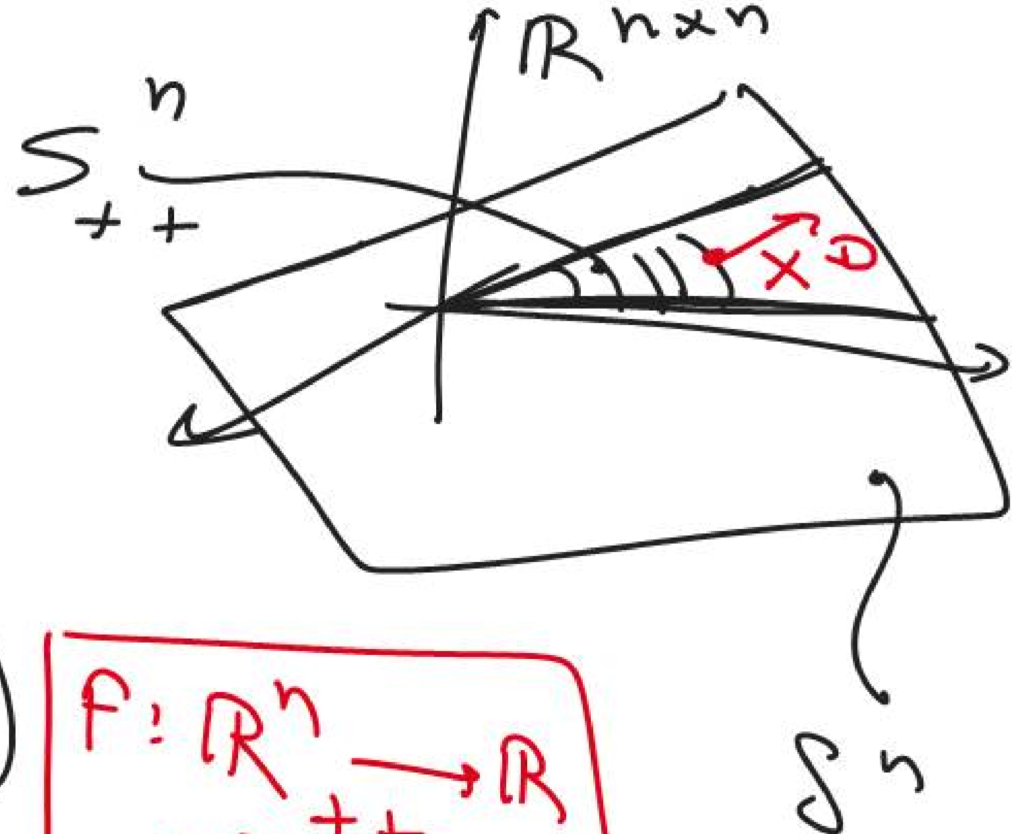

x


x
d

$f : S^n_{++} \longrightarrow \mathbb{R} \quad f(X) = \mathrm{tr}(X^{-1})$

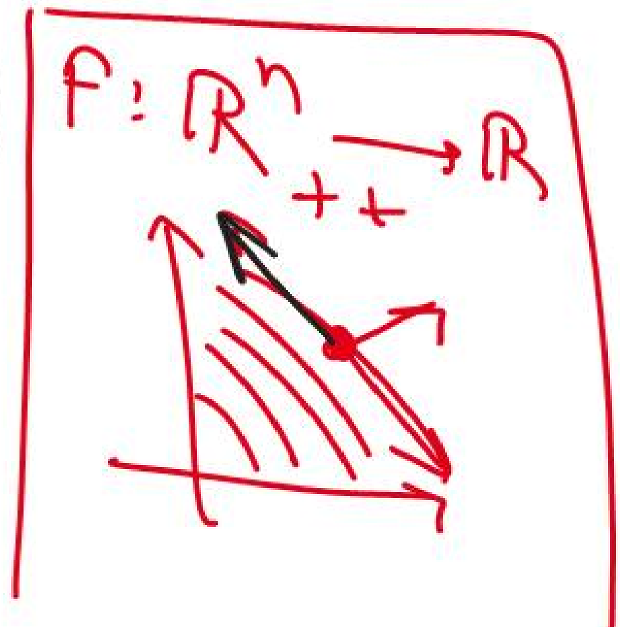


1) $X \in S^n_{++}$

$D \in S^n$

2) $\phi(t) = f(X + tD)$

$= \mathrm{tr}\left[(X+tD)^{-1}\right]$



3) $\exists_{S}: \quad X = SS^{T} \qquad D = S\Lambda S^{T}$

$$\mathrm{tr}\left[(X+tD)^{-1}\right] = \mathrm{tr}\left[\left(S[I+t\Lambda]S^{T}\right)^{-1}\right]$$

$$= \mathrm{tr}\left[S^{-T}(I+t\Lambda)^{-1}S^{-1}\right]$$

$\mathrm{tr}(ABC)$

$\mathrm{tr}(BCA)$

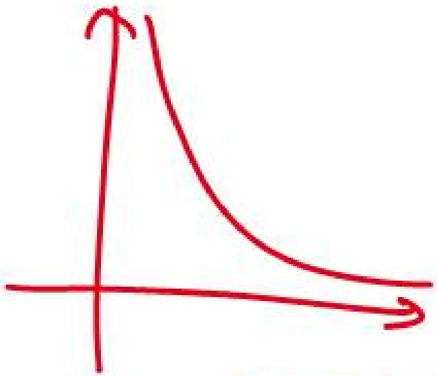
$$= \mathrm{tr}\left[(I+t\Lambda)^{-1}\, S^{-1}S^{-T}\right] - Z$$

$$\operatorname{tr}\left[(I + t\Lambda)^{-1} Z\right] \qquad \Lambda = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$$

$$= \operatorname{tr}\left[\begin{bmatrix} \frac{1}{1+t\lambda_1} & & \\ & \ddots & \\ & & \frac{1}{1+t\lambda_n} \end{bmatrix} \begin{bmatrix} Z_{11} & & * \\ & Z_{22} & \\ * & \ddots & \\ & & Z_{nn} \end{bmatrix} \right]$$

$$= \frac{Z_{11}}{1+t\lambda_1} + \cdots + \frac{Z_{nn}}{1+t\lambda_n}$$

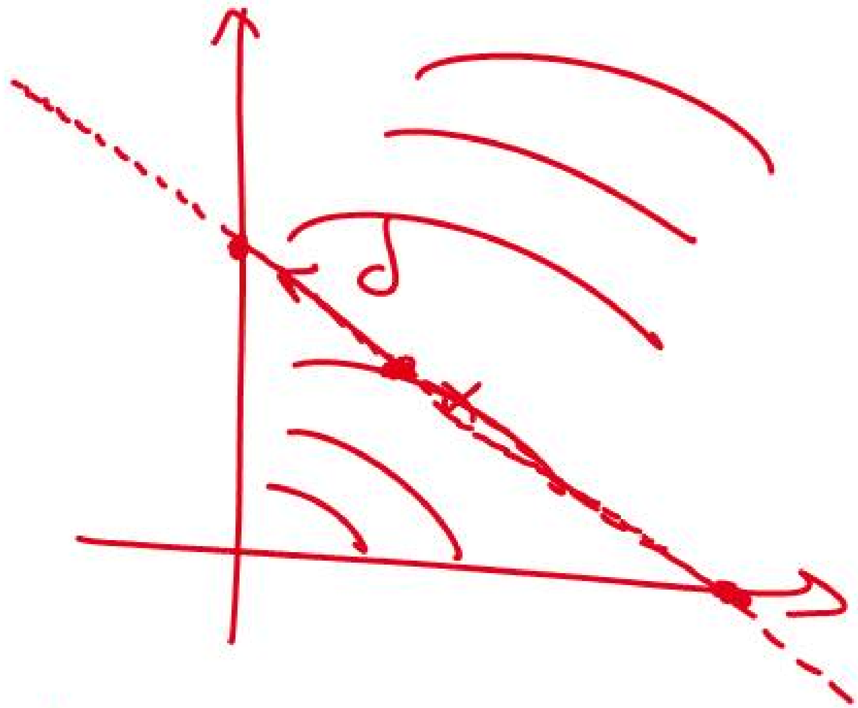
$$\phi(t) = 1/t \quad ; \; t > 0$$



$$\phi(t) = \sum_i \frac{z_{ii}}{1 + t\lambda_i}$$

$$\dot{\phi}(t) = \sum_i \frac{-\lambda_i z_{ii}}{(1 + t\lambda_i)^2}$$

$$\boxed{\ddot{\phi}(t) = \sum_i \frac{2\lambda_i^2 \underbrace{z_{ii}}_{\geq 0}}{(\underbrace{1 + t\lambda_i}_{\geq 0})^3}}$$



$f : \mathbb{R}^n_{++} \to \mathbb{R}$

$\phi(t) = f(x + t\delta)$

$\uparrow$

$I = \{t : x + t\delta \in \mathbb{R}^n_{++}\}$

$$Z = S^{-1} S^{-T} \succeq 0$$

$$AA^T \succeq 0$$

$$\Downarrow$$

$$x^T Z x \geq 0, \quad \forall_{x \in \mathbb{R}^n}$$

$$x = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \leftarrow i \Rightarrow x^T Z x = \boxed{Z_{ii} \geq 0}$$

$$\phi(t) = \mathrm{tr}\left((X + tD)^{-1}\right)$$

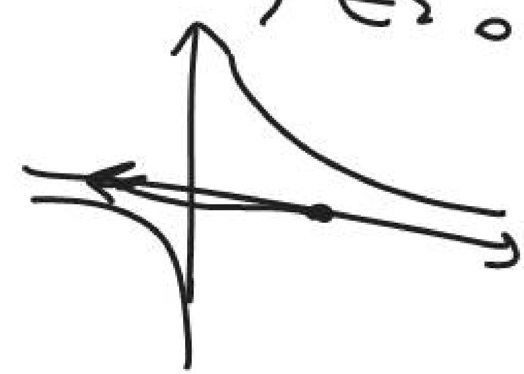
$$\downarrow$$

$$(X, D) = (I, \Lambda)$$

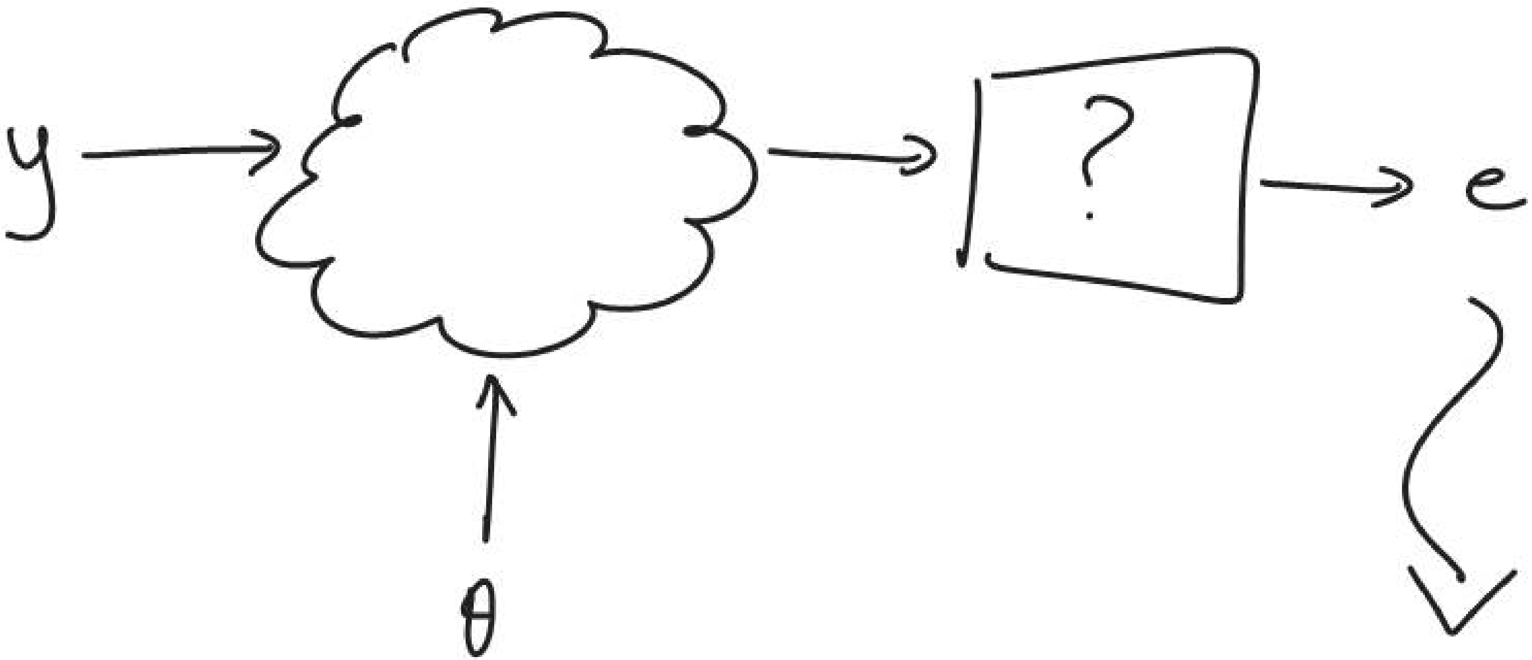
$$\downarrow$$

$$= \mathrm{tr}\left[(I + t\Lambda)^{-1}\right]$$

$f(t) = 1/t \; ; \; t > 0$



$$I = \{t : I + t\Lambda > 0\} = \{t : 1 + t\lambda_i > 0, \forall i\}$$

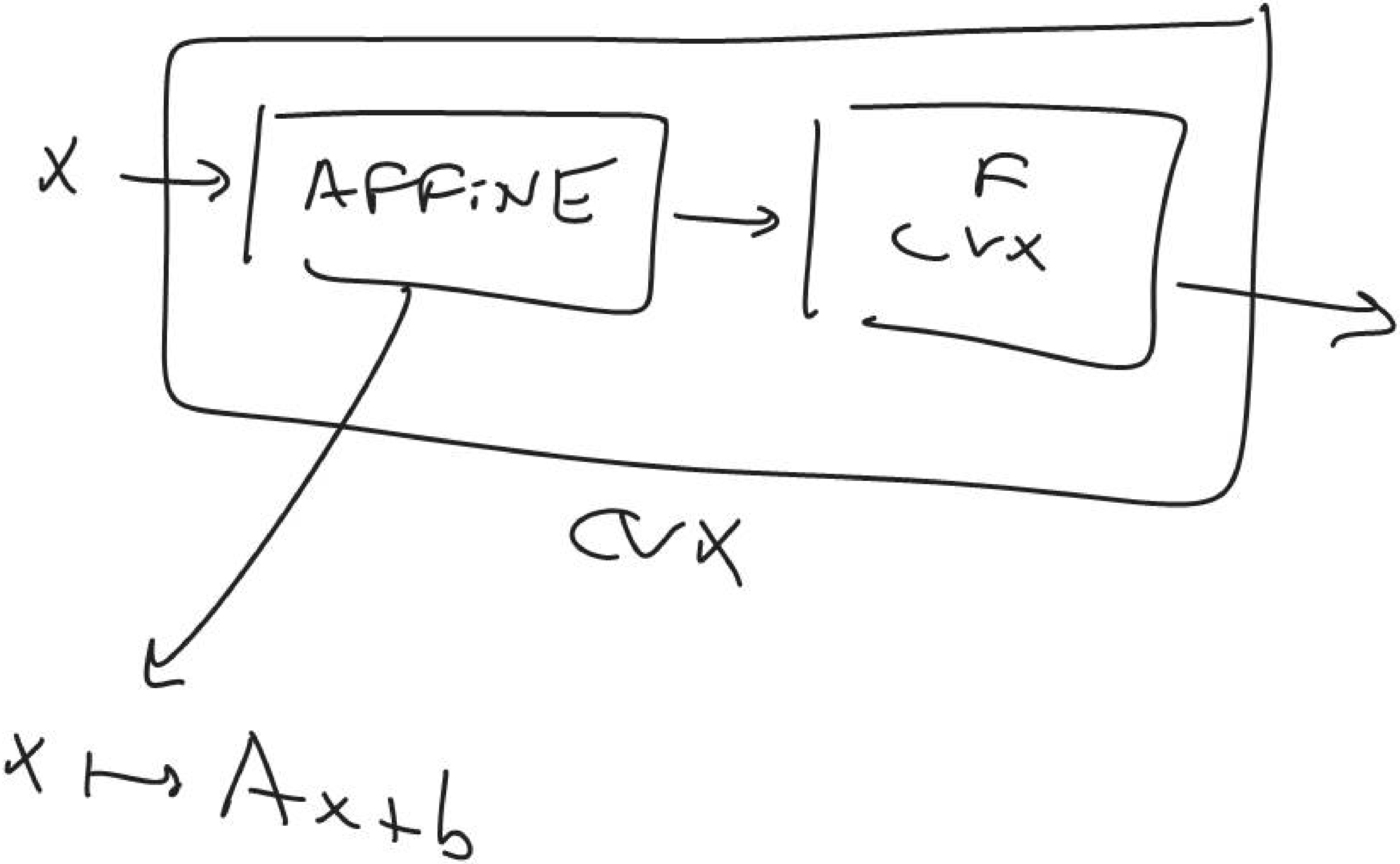


$$R = \mathbb{E}(ee^T)$$

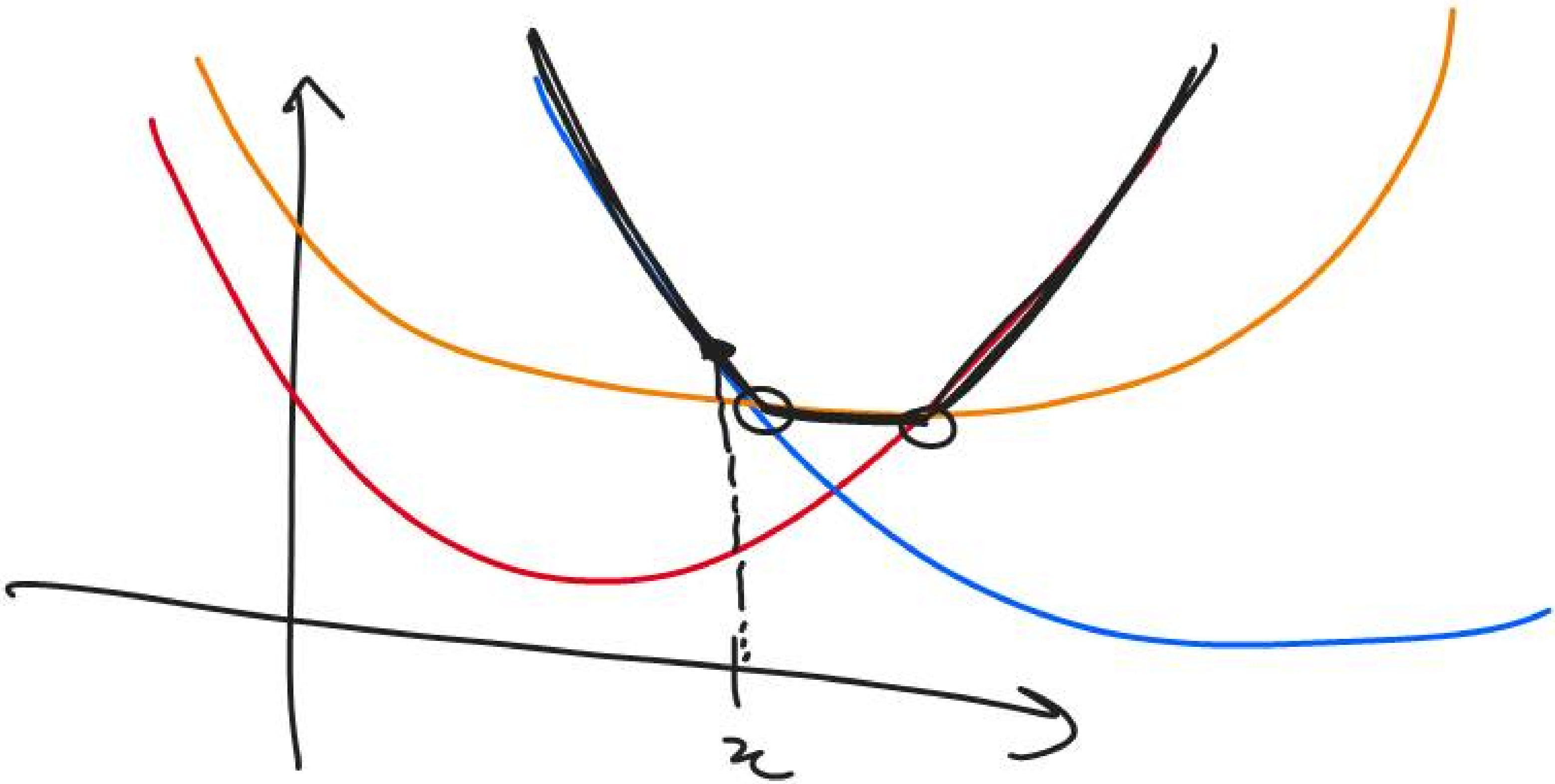
$$x \sim N(\mu, R)$$

$$p(x) = \frac{1}{(2\pi)^{n/2}|R|^{1/2}} e^{-\frac{1}{2}(x-\mu)^T R^{-1}(x-\mu)}$$

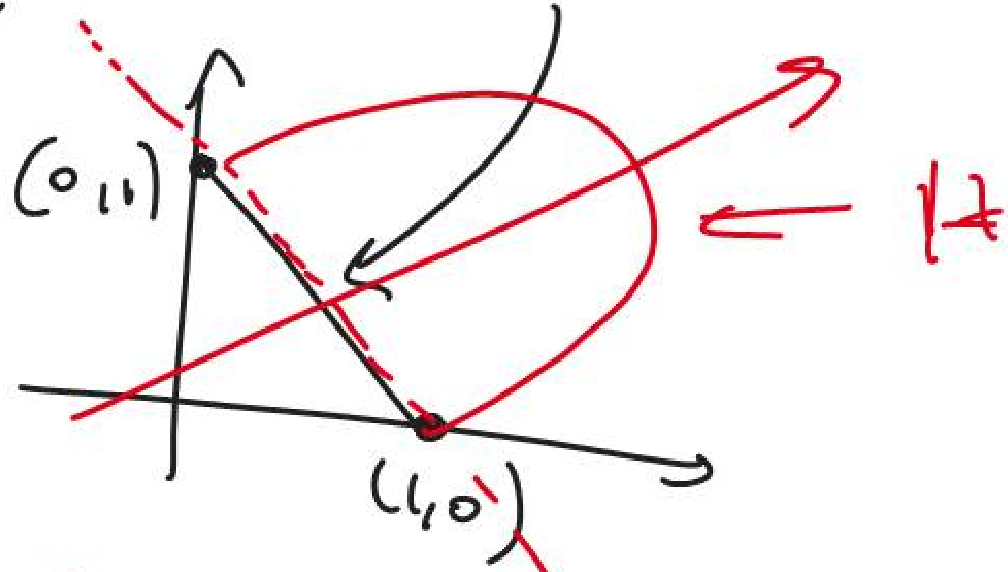
$$R = R(\theta) \succ 0$$

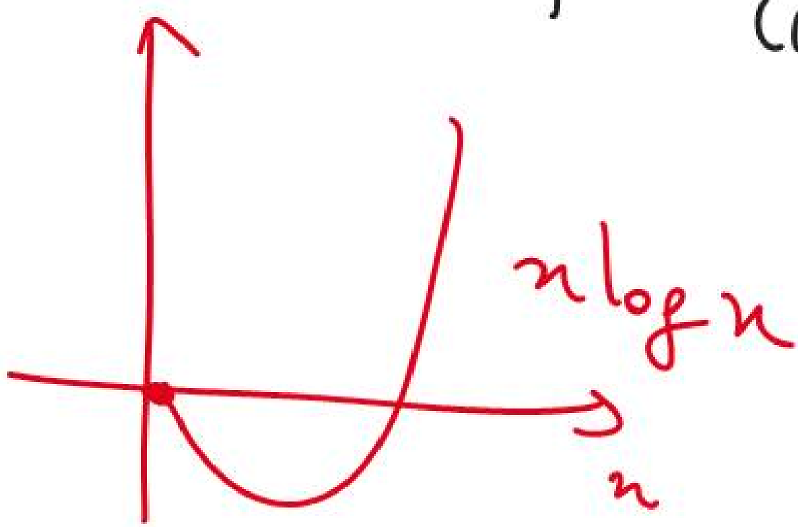

x
AFFINE
F
cvx
cvx
$x \mapsto Ax+b$

$$\left.\begin{array}{l} J = \{1, 2, 3\} \\ f_1, f_2, f_3 : \text{cvx} \end{array}\right\} \Rightarrow \max\{f_1, f_2, f_3\} \text{ is cvx}$$



$$\Delta_2 = \{ (x_1, x_2) : x_i \geq 0, x_1 + x_2 = 1 \}$$





$$H(p) = \boxed{-\sum_{i=1}^{n} p_i \log p_i} \;:\text{cvx} \quad p=(p_1,\dots,p_n)$$

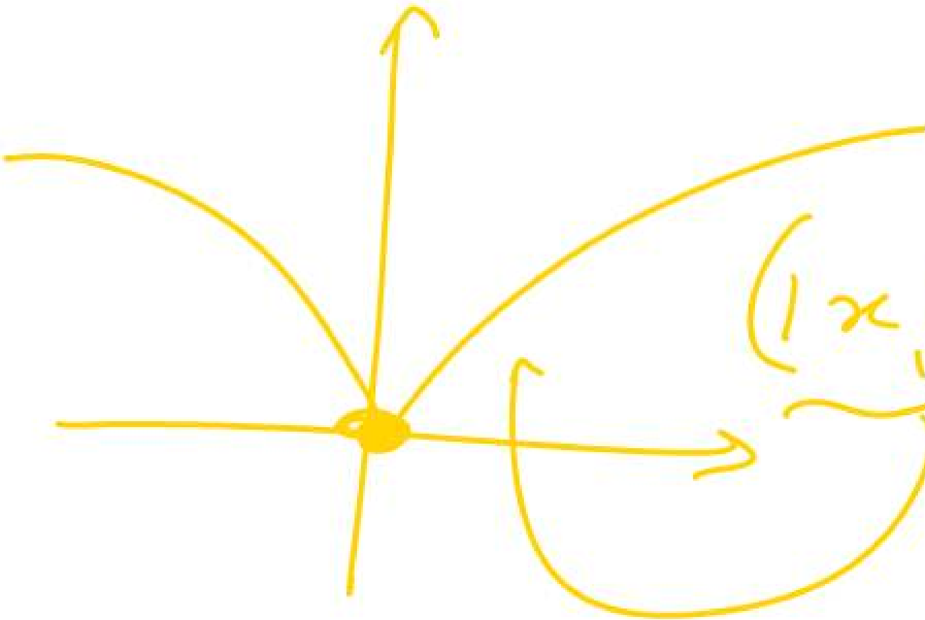
$$\|$$

$$H_1(p) + \underbrace{H_i(p)}_{\text{cvx}} + H_n(p)$$

$$p=(p_1,\dots,p_n) \to \underbrace{\boxed{\text{Affine}}}_{e_i^T p} \xrightarrow{p_i} \boxed{\phi} \to p_i \log p_i$$

$$e_i = \begin{array}{c c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \begin{array}{c} \\ \leftarrow i \\ \\ \end{array} \end{array}$$

$$\phi(t) = t\log t \;\text{cvx}$$



$p = 1/2$

$\left(|x_1|^{1/2} + |x_2|^{1/2}\right)^2$