$$y = Hs + v$$

$$\begin{bmatrix}1\\1\\1\\1\end{bmatrix} (s^*)$$

$$\boxed{\begin{array}{ll} \text{min.} & \mathrm{tr}(AS) \\ \text{s.t.} & d(S) = 1 \quad \leftarrow \lambda \\ & S \succeq 0 \quad \leftarrow Z \end{array}}$$

$$A = \begin{bmatrix} H^T H & -H^T y \\ -y^T H & 0 \end{bmatrix}$$

$$S^* = ss^T \; ?$$

$$S^* = 1\,1^T \qquad \left(S^* = \begin{bmatrix} s^* \\ 1 \end{bmatrix} [s^{*T} \; 1] \right)$$

$$? \; \lambda, Z$$

KKT

$$\begin{cases} S^* \in \underset{T}{\mathrm{argmin}}\; L(T; \lambda, Z) = \mathrm{tr}(AT) - \lambda^T(\delta(T) - 1) - \mathrm{tr}(TZ) \\ d(S^*) = 1 \quad S^* \succeq 0 \\ Z \succeq 0 \\ \mathrm{tr}(SZ) = 0 \end{cases}$$

$$L(T;\lambda,Z) = tr(AT) - \lambda^T(d(T)-1) - tr(TZ)$$

$$= tr\left((A - \underbrace{D(\lambda)}_{} - Z)\cdot T\right) + 1^T\lambda$$

$$tr(T \cdot D(\lambda))$$

$$D(\lambda) = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

$$\Leftarrow \quad A - D(\lambda) - Z = 0$$

$$S^* \in \underset{T}{\arg\min}\; L(T;\lambda,Z)$$

$$= tr\left(T\,(A - D(\lambda) - Z)\right) + 1^T\lambda$$

$$\langle T, A - D(\lambda) - Z\rangle + 1^T\lambda =$$

$$\langle A,B\rangle = tr(AB)$$

$$\begin{cases} \boxed{A - D(\lambda) - Z = 0} \\ S^* \succeq 0 \quad d(S^*) = 1 \\ Z \succeq 0 \\ \operatorname{tr}(S^* Z) = 0 \end{cases}$$

(λ, Z)

$$\boxed{S^* = 11^T}$$

$S^* = 11^T$

$Z = A - D(\lambda)$

$$\boxed{\frac{a + bs}{c + ds}} \succeq 0$$

$$\begin{cases} A - D(\lambda) \succeq 0 \\ \underline{1^T (A - D(\lambda)) 1 = 0} \end{cases} \Rightarrow \begin{cases} \underline{\quad\quad} \\ \underline{(A - D(\lambda)) 1 = 0} \end{cases} \Rightarrow \begin{cases} A - D(\lambda) \succeq 0 \\ \lambda = (A 1) \end{cases}$$

?

$M \succeq 0, \; \underline{v^T M v = 0} \Rightarrow \underline{Mv = 0}$

$$A = \begin{bmatrix} H^T H & -H^T y \\ -y^T H & 0 \end{bmatrix} = \left[\begin{array}{c|c} H^T H & -H^T(H1+v) \\ \hline -(H1+v)^T H & 0 \end{array}\right]$$

$$y = H1 + v$$

$$\lambda = A1 = \begin{bmatrix} -H^T v \\ -1^T H^T H 1 - v^T H 1 \end{bmatrix}$$

$$0 \preceq \begin{bmatrix} I \\ -1^T \end{bmatrix} \left[H^T H + D(H^T v) \right] \begin{bmatrix} I & 1 \end{bmatrix}$$

$$\Downarrow$$

$$A - D(\lambda) \succeq 0 \rightarrow \begin{bmatrix} H^T H + D(H^T v) & -H^T(H1+v) \\ -(H1+v)^T H & 1^T H^T H 1 + v^T H 1 \end{bmatrix} \succeq 0$$

$$\begin{bmatrix} I \\ \hline -1^T \end{bmatrix} \left[H^T H + D(H^T r) \right] \begin{bmatrix} I & L1 \end{bmatrix} \succeq 0$$

Full column-rank

B^+

$B M B^T (B^+)^T \succeq 0$

Full-column-rank

$\Updownarrow$

$M \succeq 0$

$\lambda_{min}(H^T H > \rho$

$\Downarrow$

ok

> 0

$$\boxed{H^T H + D(H^T r) \succeq 0}$$

$B^+ = (B^T B)^{-1} B^T$

$B^+ B = I$

$$\begin{cases} a - b = c \\ a \geq 0 \\ b \geq 0 \\ a^T b = 0 \end{cases} \longrightarrow \begin{array}{l} a = c^+ = \max\{0, c\} \\ b = c^- = \max\{0, -c\} \end{array}$$

$c \in \mathbb{R}^n$ is given

$$\min. \; -\sum_k \log\left(1 + P_k/N_k\right)$$

$$\text{s.t.} \quad P_k \geq 0, \; k = 1, \dots, K \;\longleftarrow\; \mu_k$$

$$P_1 + \dots + P_K = P_0 \;\longleftarrow\; \lambda$$

$$P = (P_1, \dots, P_K)$$

$$\nabla_Q L(P; \lambda, \mu) = 0$$

KKT

(P, μ, λ)

$$P \in \arg\min_Q \; L(Q; \lambda, \mu)$$

$$= -\sum_k \log\left(1 + \frac{Q_k}{N_k}\right) - \sum_k \mu_k Q_k - \lambda(1^T Q - P_0)$$

$P_k \geq 0$ $\quad 1^T P = P_0$

$\mu_k \geq 0$

$\mu_k P_k = 0 \quad \forall k$

$g - \mu = 0$

$P \geq 0$

$\mu \geq 0$

$\mu^T P = 0$

$$L(Q;\lambda,\mu) = -\sum_k \log\left(1+\frac{Q_k}{N_k}\right) - \sum_k \mu_k Q_k$$

$$- \lambda(Q_1 + \cdots + Q_K - P_0)$$

$$\nabla_Q L(P;\lambda,\mu) = \begin{bmatrix} \vdots \\ \dfrac{-1/N_k}{1+P_k/N_k} - \mu_k - \lambda \\ \vdots \end{bmatrix}$$

k^{th} entry

$$\begin{cases} \dfrac{-1/N_k}{1+P_k/N_k} = \mu_k + \lambda \\ P_k \geq 0 \quad 1^T P = P_0 \\ \mu_k \geq 0 \\ \mu_k P_k = 0 \end{cases}$$

$P_k \mu_k = 0$

$$\frac{-1}{P_k+N_k} = \mu_k + \lambda \rightarrow -1 = \mu_k(\cancel{P_k}+N_k) + \lambda(P_k+N_k)$$

$$-1 = \mu_k N_k + \lambda P_k + N_k \lambda$$

<0

$$\lambda = -\frac{1}{\underset{\geq 0}{P_k} + \underset{\geq 0}{N_k}} - \underset{\geq 0}{\mu_k} < 0$$

$$-1 = \mu_k N_k + \lambda P_k + \nu_k \lambda \qquad \eta := 1/\lambda$$

$$\Downarrow$$

$$-\eta = \eta \mu_k N_k + P_k + \nu_k$$

$$\Downarrow$$

$$P_k - \overbrace{(\underbrace{-\eta}_{\geq 0} \mu_k \underbrace{N_k}_{\geq 0})}^{\omega_k} = (-\eta) - \nu_k$$

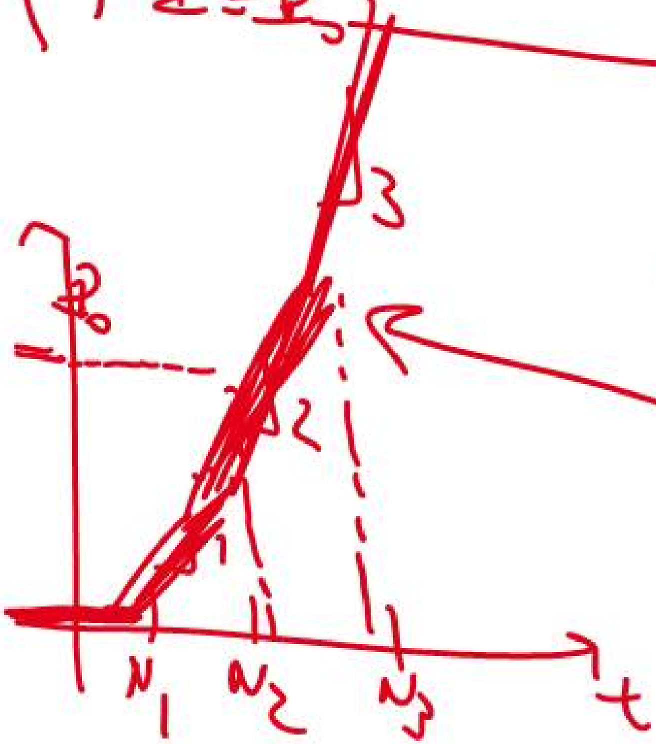
$$P_k - \mu_k = \oplus$$

$$\left\{\begin{array}{l} P_k \geq 0 \\ \mu_k \geq 0 \Leftrightarrow \overbrace{-\eta N_k \mu_k}^{\omega_k} \geq 0 \\ P_k \mu_k = 0 \Leftrightarrow \underbrace{-\eta N_k \mu_k}_{\omega_k} P_k = 0 \\ 1^T P = P_0 \end{array}\right.$$

$$(P, \lambda, \mu) \rightarrow (P, \eta, \mu) \rightarrow (P, \eta, \omega)$$

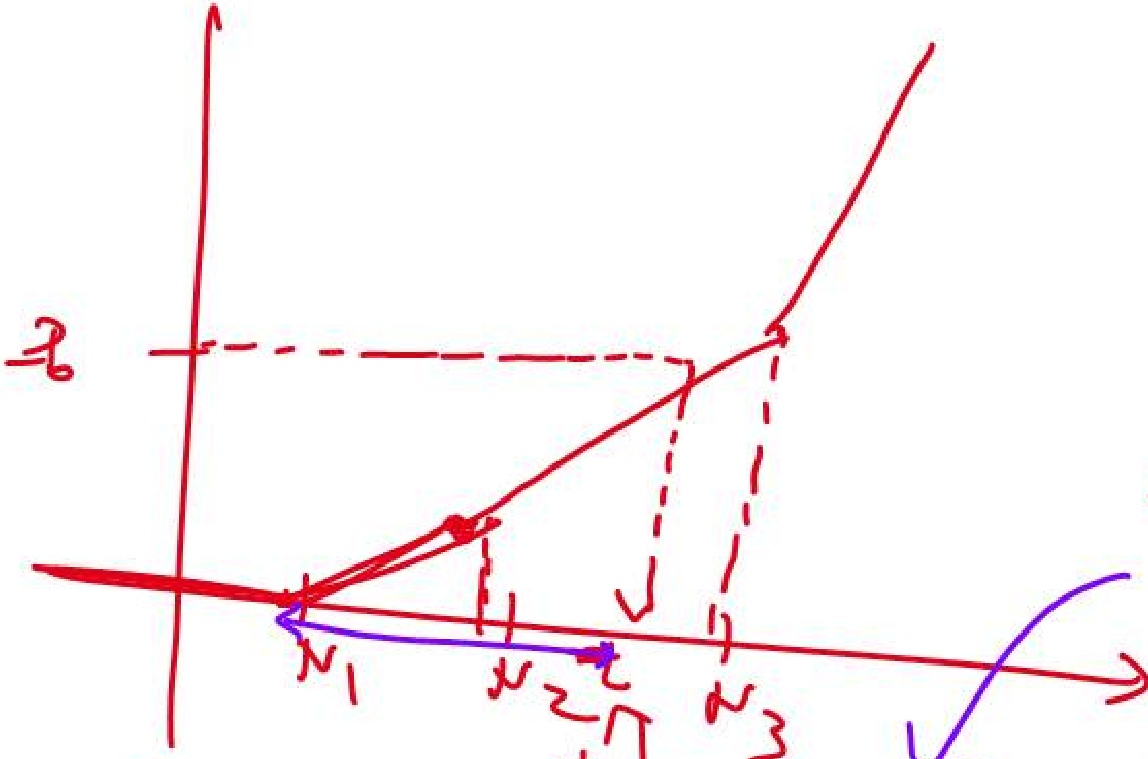
$$\begin{cases} \boxed{\begin{aligned} & p_k - u_k = (-y) - N_k \\ & p_k \geq 0 \\ & u_k \geq 0 \\ & u_k p_k = 0 \end{aligned}} \\ 1^T p = p_0 \end{cases}$$

$$p_k = \left((-y) - N_k\right)^+$$



y: $\sum_k \left((-y) - N_k\right)^+ = p_0$

t: $\boxed{\sum_k (t - N_k)^+} = p_0$



$$P_1 = (t - x_1)^+$$
$$= t - x_1$$
$$P_2 = (t - x_2)^+$$
$$= t - x_2$$
$$P_3 = (t - x_3)^+ = 0$$

$$P_k = (t - x_k)^+$$