Γ

- $L(y; \underbrace{\lambda, \mu}_{\text{fixed}})$: diff. w.r.t. y

- $X = \mathbb{R}^n$

$$\boxed{x \in \operatorname*{argmin}_{y \in \mathbb{R}^n} L(y; \lambda, \mu) \Downarrow \boxed{\nabla_y L(y; \lambda, \mu) = 0}}$$

$$\underbrace{g_1(x)\mu_1}_{\overset{\shortparallel}{0}} + \cdots + \underbrace{g_m(x)\mu_m}_{\overset{\shortparallel}{0}} = 0$$

$$g_i(x) \le 0$$
$$\mu_i \ge 0$$

$$\Updownarrow$$

$$\left\{\begin{array}{l} g_1(x)\mu_1 = 0 \\ g_2(x)\mu_2 = 0 \\ \vdots \\ g_m(x)\mu_m = 0 \end{array}\right. \qquad \Longleftrightarrow \qquad \underline{g_1(x) = 0 \text{ or } \mu_1 = 0}$$

$$\vdots$$

$\min. \quad f(x)$ $\qquad\qquad f(x)$: $\underline{\text{cvx}}$, $\underline{\text{diff}}$

s.t. $\quad \cancel{x \in \mathbb{R}^n} \quad x \in \mathbb{R}^n$

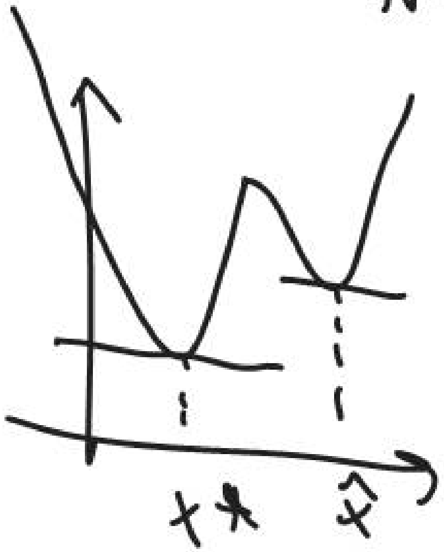
$\cancel{h(x) = 0}$

$\cancel{g(x) \le 0}$

"KKT"

$$x^* \in \underset{x \in \mathbb{R}^n}{\operatorname{argmin}} f(x) \iff \boxed{\nabla f(x^*) = 0}$$

cvx

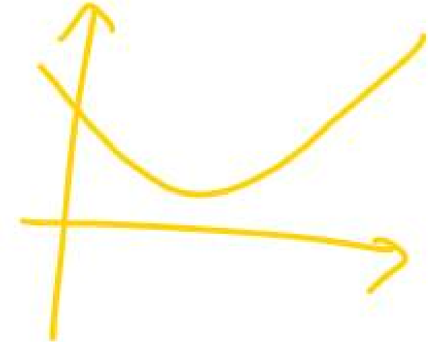


$$\min.\ f(x) \quad \text{strictly convex}$$

$$\text{s.t. } Ax = b \longleftarrow \lambda^*$$

$$g_i(x) \le 0 \quad i = 1, \dots, m \longleftarrow \mu^*$$

$$x \in X$$



not SC →

1) Solve the dual $\longrightarrow (\lambda^*, \mu^*)$

2) • $X^*(\lambda^*, \mu^*) = \arg\min_{x \in X} L(x; \lambda^*, \mu^*)$

$$= \boxed{\arg\min_{x \in X} \underbrace{f(x)}_{SC} + \underbrace{\lambda^{*T}(Ax - b) + \sum_i \mu_i^* g_i(x)}_{C}}$$

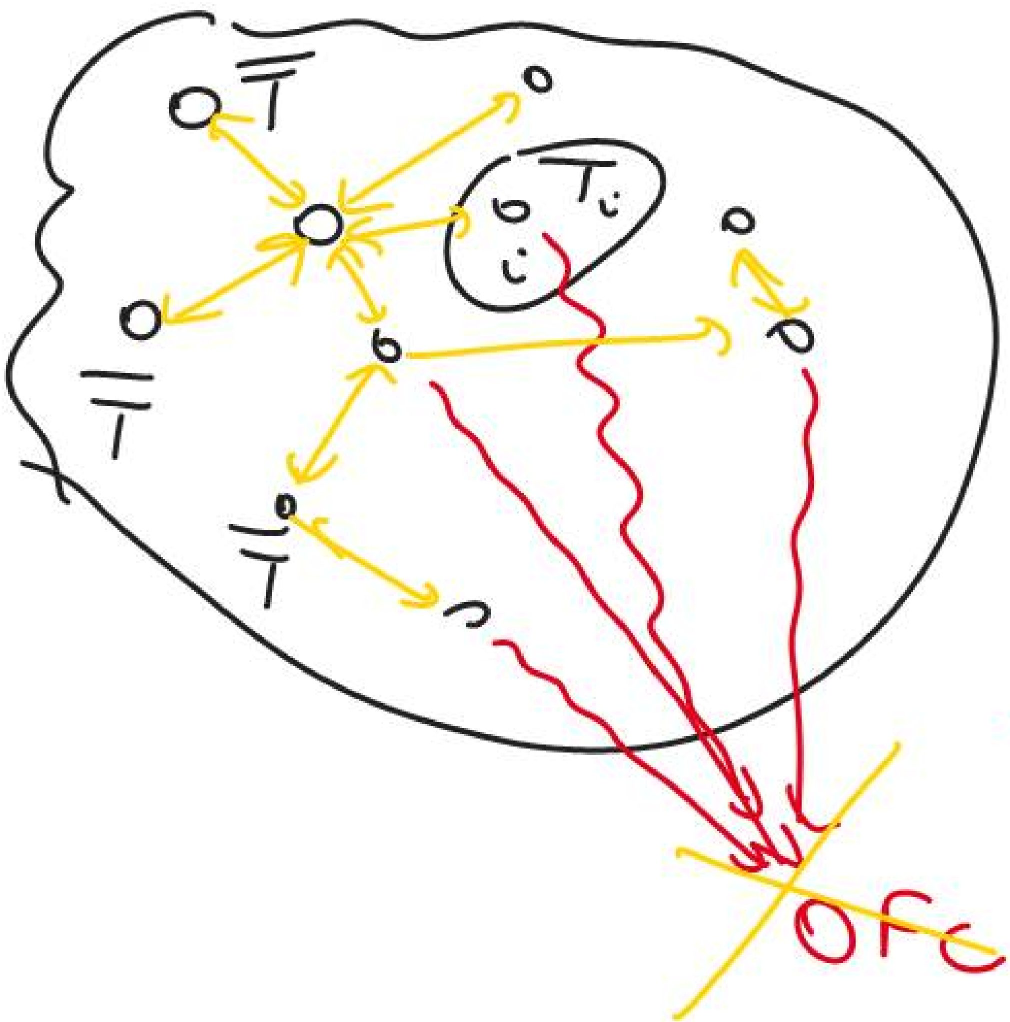
SC

singleton

• $\{x : Ax = b,\ g(x) \le 0\}$

• $\{x : g(x)^T \mu^* = 0\}$

Consensus



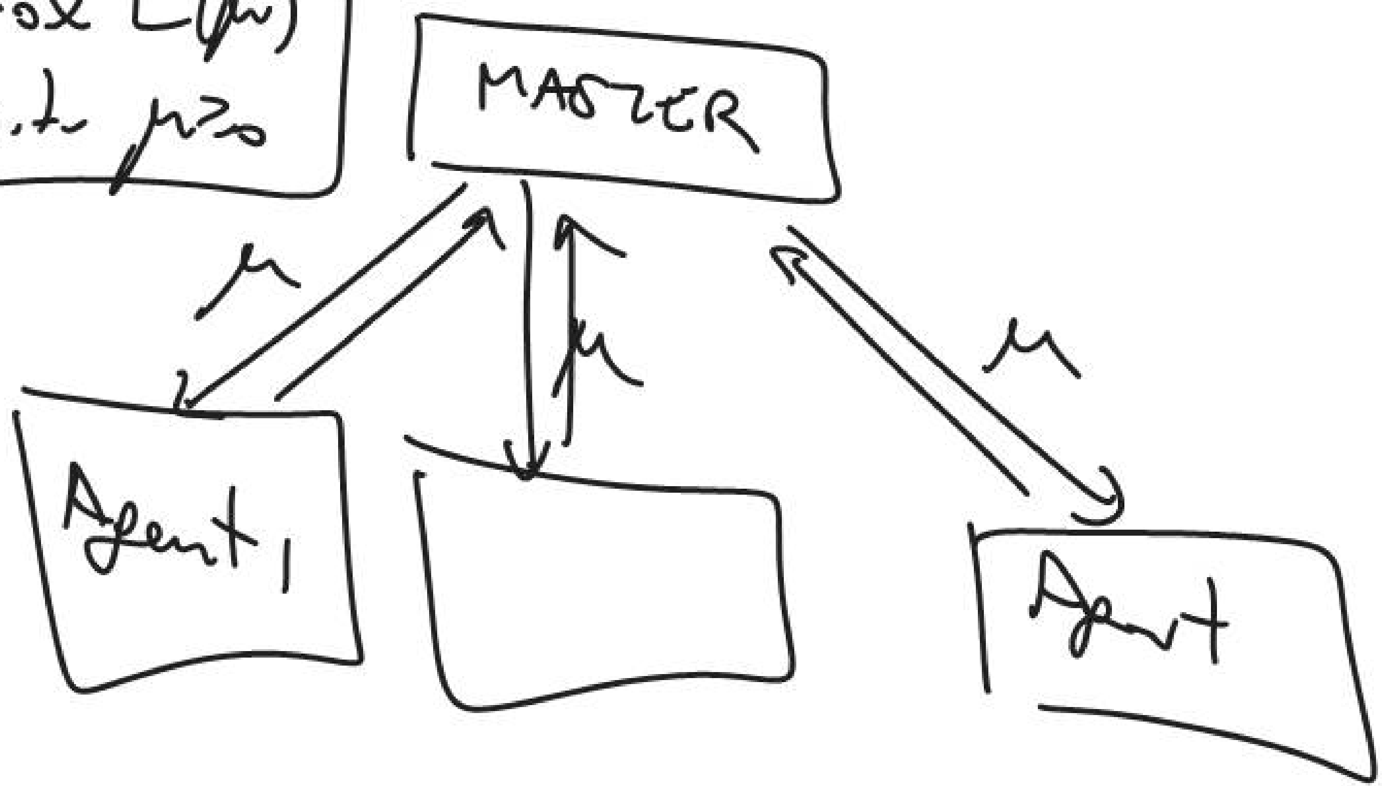
Goal

$$\bar{T} = \frac{1}{n} \sum_{i=1}^{n} T_i$$

$$L(\mu) = \sum_{i=1}^{n} \inf_{x_i \in X_i} f_i(x_i) + \mu g_i(x_i)$$

local to agent i

$$\boxed{\begin{aligned} &\max L(\mu) \\ &\text{s.t. } \mu \geq 0 \end{aligned}}$$



$$\min. \ \tfrac{1}{2}\|x - a\|^2$$

$$\text{s.t.} \quad x \geq 0 \ \leftarrow \ \mu$$

$$1^T x = 1 \ \leftarrow \ \lambda$$

$$x - a - \lambda 1 - \mu = 0$$

$$\Updownarrow$$

$$\nabla_y L(x, \lambda, \mu) = 0$$

$$\Updownarrow$$

KKT

(x, λ, μ)

$$x \in \arg\min_y L(y; \lambda, \mu)$$

$$\tfrac{1}{2}\|y - a\|^2 - \lambda(1^T y - 1) - \mu^T y$$

$$x \geq 0, \quad 1^T x = 1$$

$$\mu \geq 0$$

$$x^T \mu = 0$$

$$\left\{\begin{array}{l} \boxed{\begin{array}{l} x - \mu = q + \lambda \mathbb{1} \\ x \geq 0 \\ \mu \geq 0 \\ x^T \mu = 0 \end{array}} \\ 1^T x = 1 \end{array}\right.$$

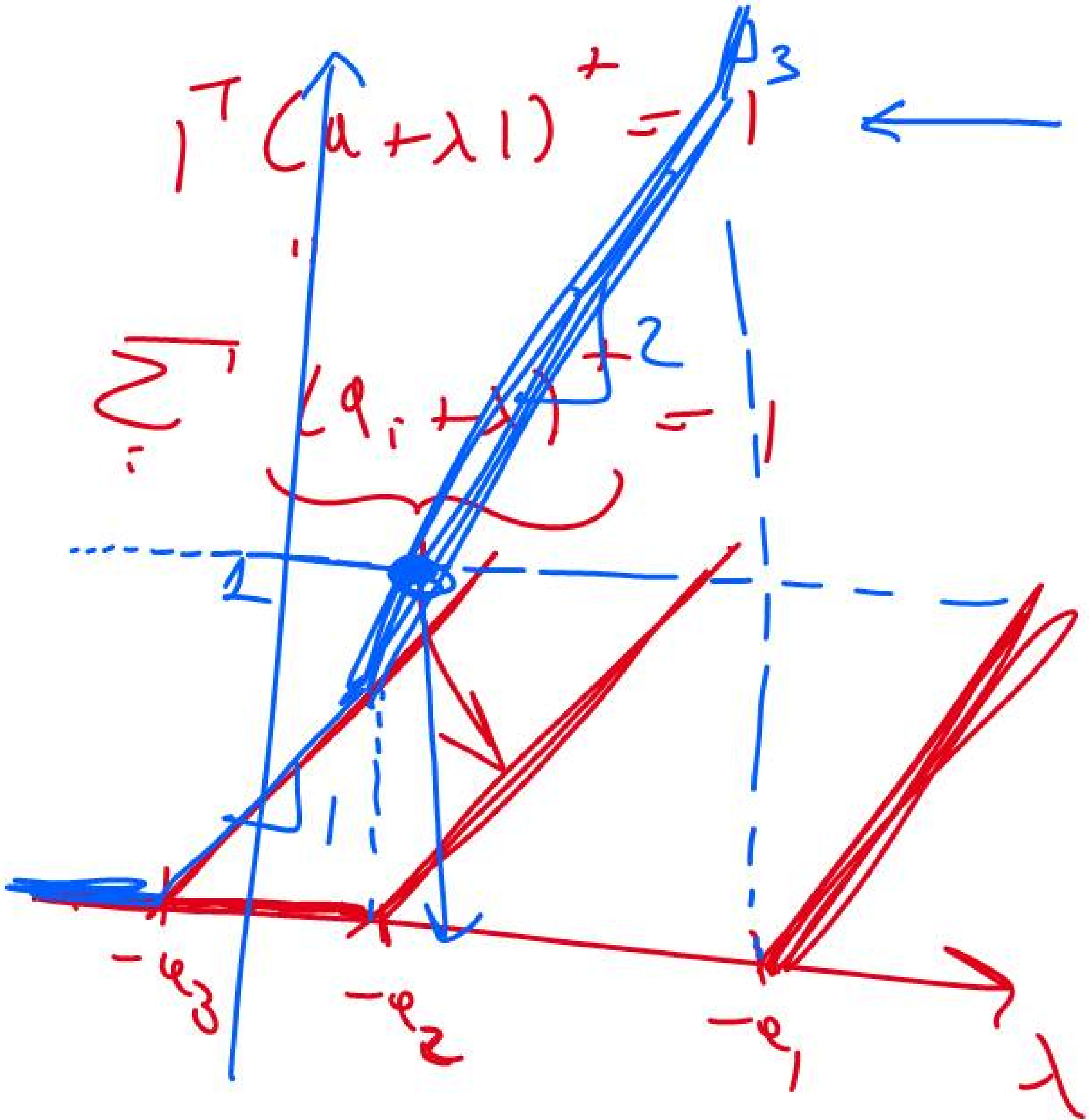
$$x = x(\lambda) = \boxed{(q + \lambda 1)^+}$$

$$\cancel{\mu = \mu(\lambda)}$$

$$1^T x(\lambda) = 1$$

$$1^T (a + \lambda 1)^+ = 1$$

$$\sum_i \underbrace{(a_i + \lambda)^+}_{} = 1$$



$$\boxed{\begin{array}{l} a - b = c \checkmark \\ a \geq 0 \quad \checkmark \quad a_i \geq 0 \\ b \geq 0 \quad \checkmark \quad a_n \geq 0 \\ a^T b = 0 \quad \checkmark \end{array}}$$

$a = c^+ := \max\{c, 0\} = \begin{bmatrix} c_1^+ \\ c_2^+ \\ \vdots \\ c_n^+ \end{bmatrix}$

$b = c^- := \max\{-c, 0\}$

$a_1 b_1 = 0$
$\vdots$
$a_n b_n = 0$

$c = 7$ $\quad$ $\overset{a}{10} - \overset{b}{3} = 7$ $\quad$ $\overset{a}{7} - \overset{b}{0} = 7$ $\quad$ $a = 7^+$ $\quad$ $b = 7^-$

$c = -7$ $\quad$ $\overset{a}{0} - \overset{b}{7} = -7$ $\quad$ $a = (-7)^+$

$b = (-7)^-$

$$\bar{T} = \frac{1}{n}\sum_{i=1}^{n} T_i$$

$$= \underset{T}{\operatorname{argmin}} \sum_{i=1}^{n} (T - T_i)^2$$

$$= \underset{T}{\operatorname{argmin}} (T - T_1)^2 + (T - T_2)^2 + \cdots + (T - T_n)^2$$

$$f_1(x_1)$$

$$= \operatorname{argmin} (x_1 - T_1)^2 + (x_2 - T_2)^2 + \cdots + (x_n - T_n)^2$$

s.t. $x_1 = x_2$

$x_2 = x_3 \cdots x_{n-1} = x_n$

$x_i = x_j \quad \forall i, j$