$K \subset \mathbb{R}^n$

(convex cone)

$x \in \mathbb{R}^n$

$x \preceq_K 0$

$\Updownarrow$

$-x \in K$

$\sim , \sim$

$K_1 \subset \mathbb{R}^n$, $K_2 \subset \mathbb{R}^m$

$K := K_1 \times K_2$

$f_1(x) \preceq_{K_1} 0$

$f_2(x) \preceq_{K_2} 0$

$\Longleftrightarrow$

$f(x) = (f_1(x), f_2(x))$

$f(x) \preceq_K 0$

$K = \mathbb{R}^n_+ \qquad K^* = \mathbb{R}^n_+$

SOCP $\longrightarrow$

SDP $\longrightarrow$

max. $x^T A x$

s.t. $x_i^2 = 1 \quad i = 1, \dots, n \leftarrow \lambda$

$\rightarrow x^T E_i x - 1 = 0$

$$E_i = \begin{array}{c} \\ \begin{bmatrix} 0 & & & & \\ & \ddots & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ i \\ \\ \\ \end{array}$$

1) $L(x;\lambda) = x^T A x + \sum_i \lambda_i (1 - x^T E_i x)$

$= x^T (A - D(\lambda)) x + 1^T \lambda$

$$D(\lambda) = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

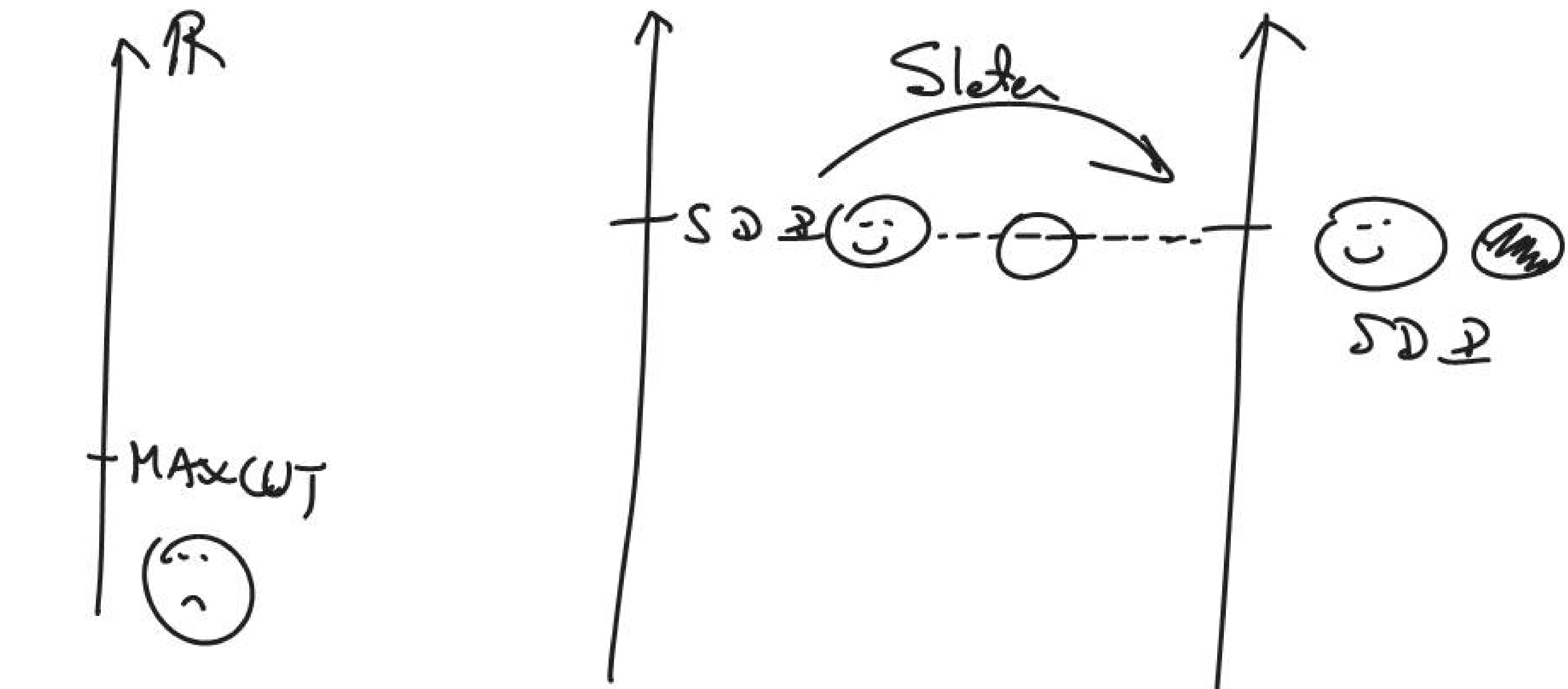
2) $L(\lambda) = \sup_x L(x;\lambda)$

$$= \begin{cases} 1^T \lambda ; & A - D(\lambda) \le 0 \\ +\infty ; & \text{oth.} \end{cases}$$

3) min. $1^T \lambda$

s.t. $A - D(\lambda) \preceq 0$

SDP



Slaterpoint:

$$\left.\begin{array}{l}\lambda = cI \\ c > \lambda_{max}(A)\end{array}\right\} \Longrightarrow \begin{array}{l} A - \mathrm{Diag}(\lambda) \prec 0 \\ A \prec cI \ \checkmark \end{array}$$

$\min.\ 1^T\lambda$

s.t. $\underbrace{A - D(\lambda)}_{n\times n} \succeq 0 \longleftarrow Z$

1) $L(\lambda; Z) = 1^T\lambda + \underbrace{\mathrm{tr}(Z(A - D(\lambda)))}_{\langle Z, A - D(\lambda)\rangle}$

$\langle C, D\rangle = \mathrm{tr}(CD)$

$= \lambda^T(1 - d(Z)) + \mathrm{tr}(ZA)$

$\mathrm{tr}(Z\,D(\lambda))$

$\mathrm{tr}\left(Z \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}\right) = z_{11}\lambda_1 + \dots + z_{nn}\lambda_n = \lambda^T d(Z) := \begin{bmatrix} z_{11} \\ z_{22} \\ \vdots \\ z_{nn} \end{bmatrix}$

$$K^* = \{ y : \langle x, y \rangle \geq 0, \ \forall x \in K \}$$

$K = \mathbb{R}^n_+$ $\qquad$ $K^* = \mathbb{R}^n_+$

SOC $\qquad$ "

SDP $\qquad$ "

$\langle x, y \rangle = x^T y$ $\qquad$ $\langle X, Y \rangle = \mathrm{tr}(XY)$

$$L(\lambda; Z) = \lambda^T\underline{(1 - d(Z))} + tr(ZA)$$

2) $$L(Z) = \inf_{\lambda} L(\lambda; Z) = \begin{cases} tr(ZA); & \underline{d(Z) = 1} \\ -\infty & \text{o.th.} \end{cases}$$

3) $$\boxed{\begin{array}{l} \max.\ tr(ZA) \\ \text{s.t. } d(Z) = 1 \iff \begin{cases} Z_{11} = 1 \\ \vdots \\ Z_{nn} = 1 \end{cases} \\ \rightarrow Z \succeq 0 \end{array}}$$

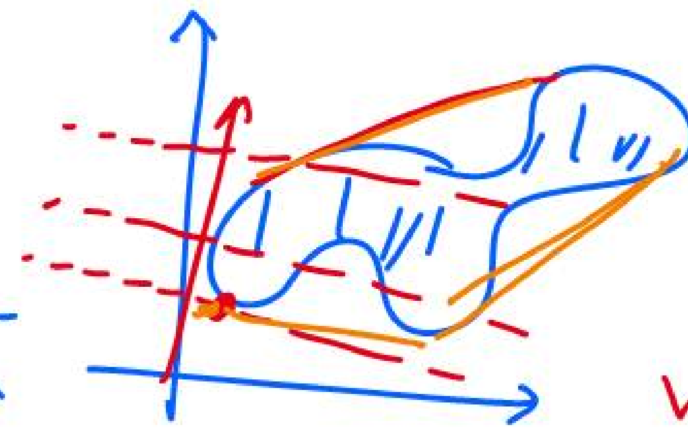
$$\begin{array}{ll} \min. & x^3 \\ \text{s.t.} & x = 0 \end{array} \iff \begin{array}{ll} \min. & \cancel{\ } \\ \text{s.t.} & x = 0 \\ & \text{cvx} \end{array}$$

$\downarrow$

$$L(x;\lambda) = x^3 + \lambda x$$

$$L(\lambda) = \inf_x \, x^3 + \lambda x = -\infty \rightarrow \begin{array}{ll} \max. & L(\lambda) = -\infty \\ \text{s.t.} & \lambda \end{array}$$

$$\min.\quad f(x)$$
$$\text{s.t.}\quad x \in X$$



$$\min\ c^T z$$
$$\text{s.t.}\ z \in Z$$

$\Updownarrow$

$$\min\ c^T z$$
$$\text{s.t.}\ z \in \mathrm{co}\, Z$$

$\Updownarrow$

$$\min.\quad t$$ "linear function of (x,t)"

$$\text{s.t.}\quad x \in X,\ f(x) \le t \qquad x \in X,\ (x,t) \in \mathrm{epi}\, f$$

$\Updownarrow$

$$\min$$

$(x,t) \in$ "some set"

min. $f(Q)$

s.t. $Q^T Q = I$

H. Wolcowitz

(QAP)

$Q: n \times n$

min. $f(Q)$

s.t. $Q^T Q = I$

$Q Q^T = I$

∂

$D, G = 0$!

$D, G \geq 0$

$I \otimes \oplus$

$\oplus \otimes I$

AL

$$\min_x \; f(x) \quad \text{s.t.} \quad h(x) = 0$$

$$\min_x \; f(x) + \lambda^T h(x) + \frac{c}{2}\|h(x)\|^2$$

$\xrightarrow{D}$

$$L(x;\lambda) = f(x) + h(x)^T \lambda$$

$$\min \; f(x) + \frac{c}{2}\|h(x)\|^2 \quad \text{s.t.} \quad h(x) = 0$$

$\xrightarrow{D}$

$$\min.\quad x^T A_0 x + b_0^T x \Longleftrightarrow \operatorname{tr}\left(\begin{bmatrix} A_0 & b_0/2 \\ b_0^T/2 & 0 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} \begin{bmatrix} x^T & 1 \end{bmatrix} \right)$$

$$\text{s.t.}\quad Cx \le d \rightarrow d - Cx \ge 0$$

$$(d-Cx)(d-Cx)^T \ge 0$$

$$x^T A_i x + b_i^T x \le c_i \qquad i = 1, \dots, p$$

$$\operatorname{tr}\left(\begin{bmatrix} A_i & b_i/2 \\ b_i^T/2 & 0 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} \begin{bmatrix} x^T & 1 \end{bmatrix} \right) \le c_i$$

QCQP

SHERALI

Reformulation Linearization Technique (RLT)