$\bar{P} = 2$

$$\underbrace{\begin{bmatrix} y_1[t] \\ \vdots \\ y_N[t] \end{bmatrix}}_{y[t]} = \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{h_1} \underbrace{\overset{?}{s_1[t]}}_{\in \mathbb{C}} + \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{h_2} \underbrace{\overset{?}{s_2[t]}}_{\in \mathbb{C}} + \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{n[t]}$$

$$y[t] \rightarrow \; ? \; \rightarrow \hat{s}_1[t]$$

$$\hat{s}_1[t] = \boxed{w^H \, y[t]}$$

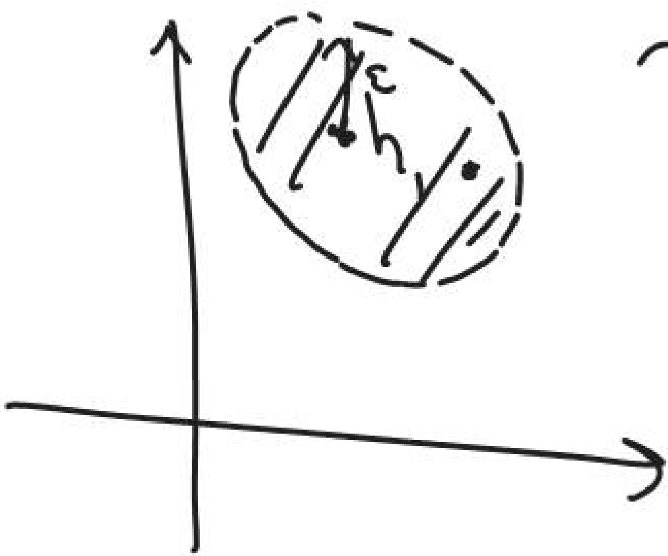
$$= w^H \left(h_1 s_1[t] + h_2 s_2[t] + n[t] \right)$$

$$\boxed{= \overbrace{(w^H h_1)}^{1} s_1[t] + \overbrace{(w^H h_2)}^{0} s_2[t] + w^H n[t]}$$

min. $\mathbb{E}\left(|w^H y[t]|^2\right) = w^H \underbrace{\mathbb{E}\left(y[t]\,y[t]^H\right)}_{R = \frac{1}{T}\sum_{t=1}^{T} y[t]\,y[t]^H} w$

s.t. $w^H h_1 = 1$

w^*_{MVDR}



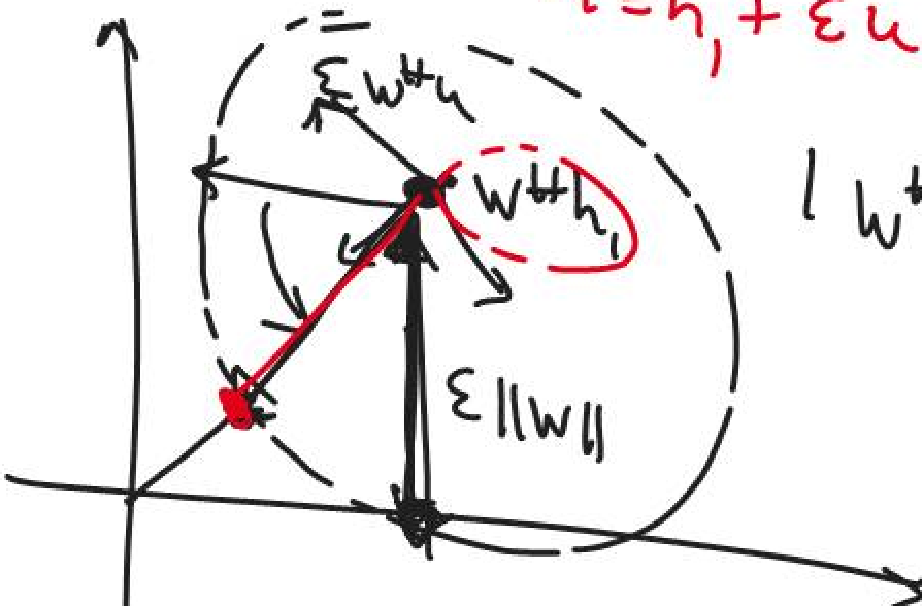
$$\min.\ w^H R w$$

$$\text{s.t. } ``|w^H h| \ge 1 \text{ for all } h \in B_\varepsilon(h_1)"$$

$$\Updownarrow$$

$$\left[\begin{array}{ll}\inf. & |w^H h| \\ \text{s.t.} & h \in B_\varepsilon(h_1)\end{array}\right] \ge 1 \iff |w^H h_1| - \varepsilon \|w\| \ge 1$$

$$\begin{bmatrix} \inf. & |w^H h| \\ \text{s.t.} & \underbrace{h \in B_\varepsilon(h_1)}_{h = h_1 + \varepsilon u \quad \|u\| \le 1} \end{bmatrix} = \inf. \ |w^H h_1 + \varepsilon \underbrace{w^H u}| \qquad \text{s.t.} \ \|u\| \le 1$$



$$|w^H u| \le \|w\| \|u\| \le \|w\|$$

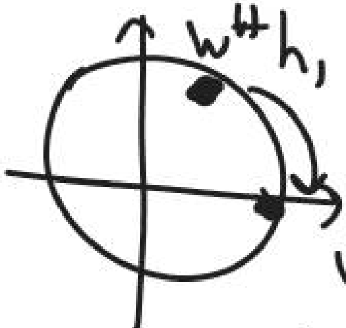
$$u = e^{j\theta} w$$

$$= |w^H h_1| - \varepsilon \|w\|$$

$$\min. \quad w^H R w$$

var: $w \in \mathbb{C}^N = x + jy$

$$\text{s.t.} \quad |w^H h_1| - \varepsilon \|w\| \geq 1$$



$\Updownarrow$

$w' = w^H h_1 e^{i\theta}$

$$\|w\| \leq \frac{1}{\varepsilon}\left(\underbrace{|w^H h_1| - 1}\right)$$

$w' = \underline{w e^{i\theta}}$

✓ $\mathrm{Re}(w^H h_1) \geq 0$

✓ $\mathrm{Im}(w^H h_1) = 0$

$$\min.\ w^H R w$$

$$\text{s.t.}\ \cancel{|}w^H h_1\cancel{|} - \varepsilon\|w\| \geq 1$$

$$\left.\begin{aligned} &\mathrm{Re}(w^H h_1) \geq 0 \\ &\mathrm{Im}(w^H h_1) = 0 \end{aligned}\right\}$$

$$\min.\ w^H R w$$

$$\text{s.t.}\ \boxed{\|w\| \leq \frac{1}{\varepsilon} w^H h_1 - \frac{1}{\varepsilon}}$$

$$\boxed{\mathrm{Re}(w^H h_1) \geq 0}$$

$$\boxed{\mathrm{Im}(w^H h_1) = 0}$$

$$\begin{bmatrix} \mathrm{Re}\, h_1^T & \mathrm{Im}\, h_1^T \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \geq 0$$

$$\begin{bmatrix} \mathrm{Im}\, h_1^T & -\mathrm{Re}\, h_1^T \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\left\| \begin{bmatrix} x \\ y \end{bmatrix} \right\| \leq \frac{1}{\varepsilon} \begin{bmatrix} \mathrm{Re}\, h_1^T & \mathrm{Im}\, h_1^T \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \frac{1}{\varepsilon}$$

$$w = x + jy$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^{2N}$$

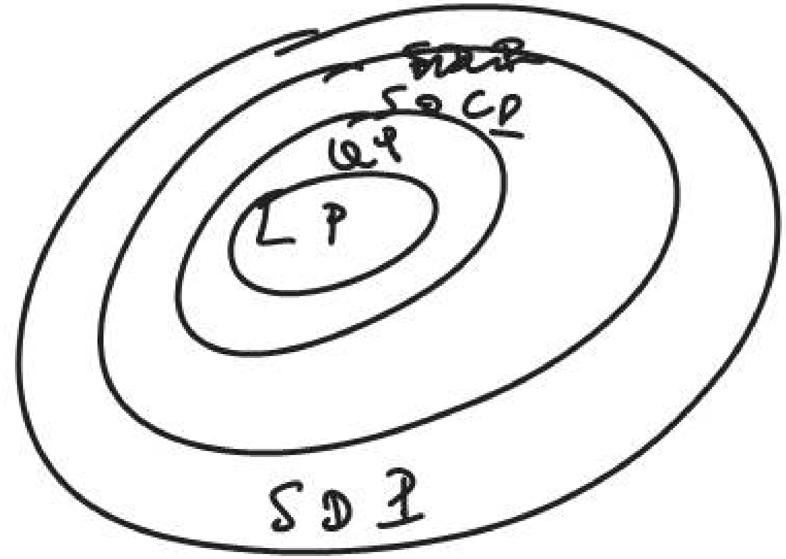
min. t

s.t. $\boxed{\cancel{w^T} R w \le t}$

______ (SOC)

______ (SOC)

______ (SOC)



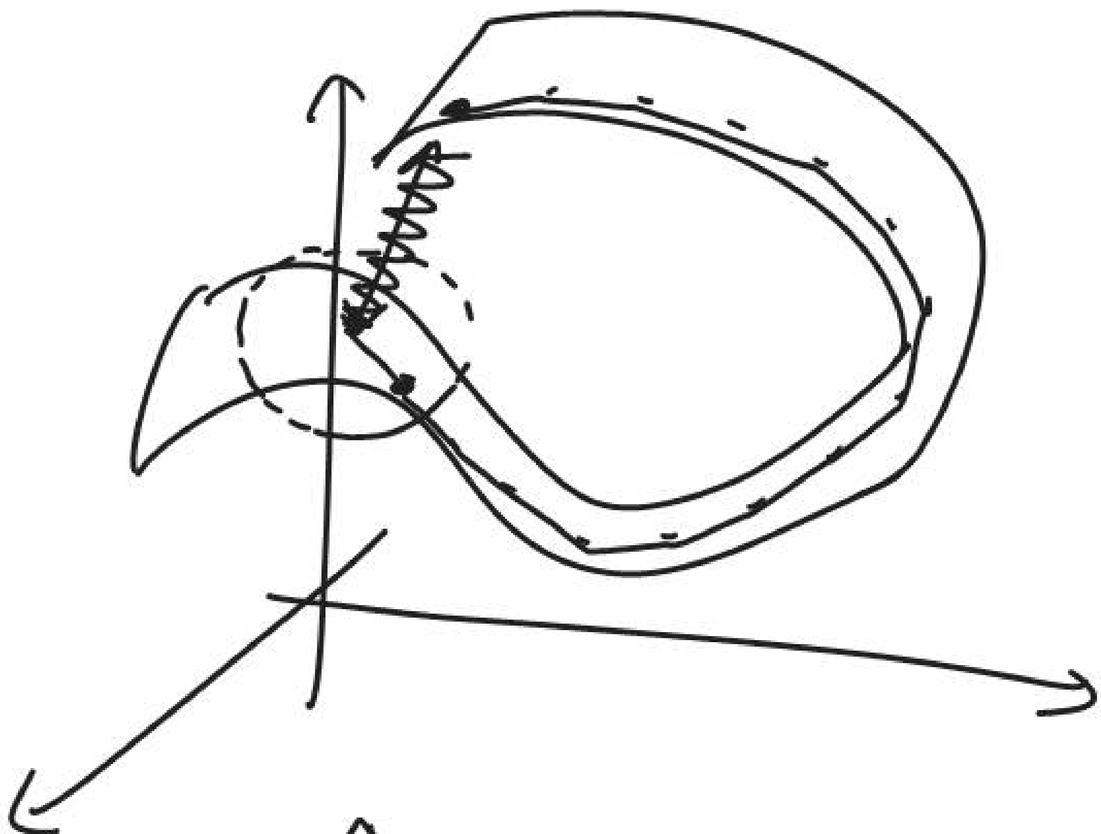
$R \succeq 0 \quad (R \in \mathbb{R}^{n \times n})$

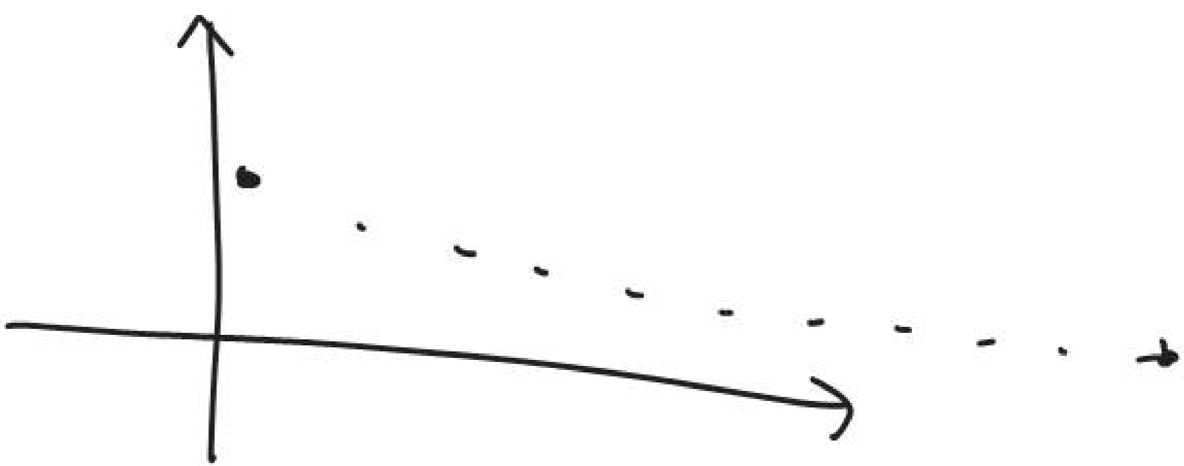
$\quad \| $

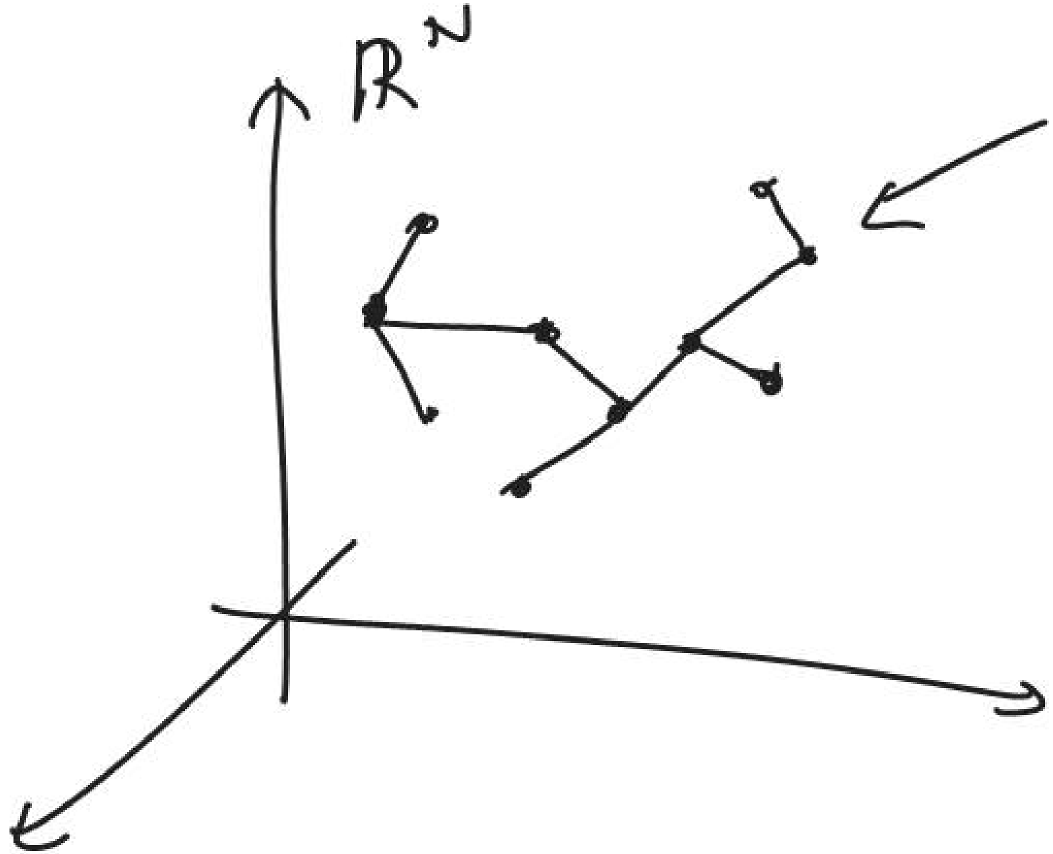
$R^{1/2} R^{1/2}$

$w^T R^{1/2} R^{1/2} w = \| R^{1/2} w \|^2 \le t$

$(R^{1/2} \succeq 0) \quad \boxed{\left\| \begin{bmatrix} 2R^{1/2} w \\ t-1 \end{bmatrix} \right\| \le t+1}$ (SOC)







Neighborhood graph
is
Connected

$$w^H R w = [x^T \; y^T] \, B \begin{bmatrix} x \\ y \end{bmatrix}$$

$$R \succeq 0 \quad (R \in \mathbb{C}^{n \times n})$$

$$\succeq 0 \quad (\mathbb{R}^{n \times n})$$

$$w = x + j y$$