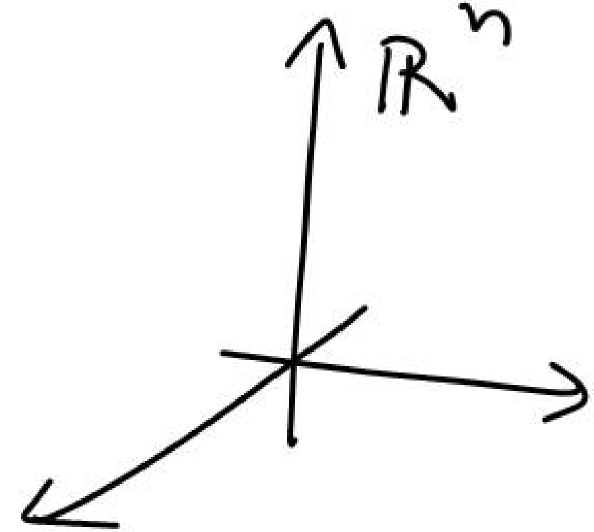
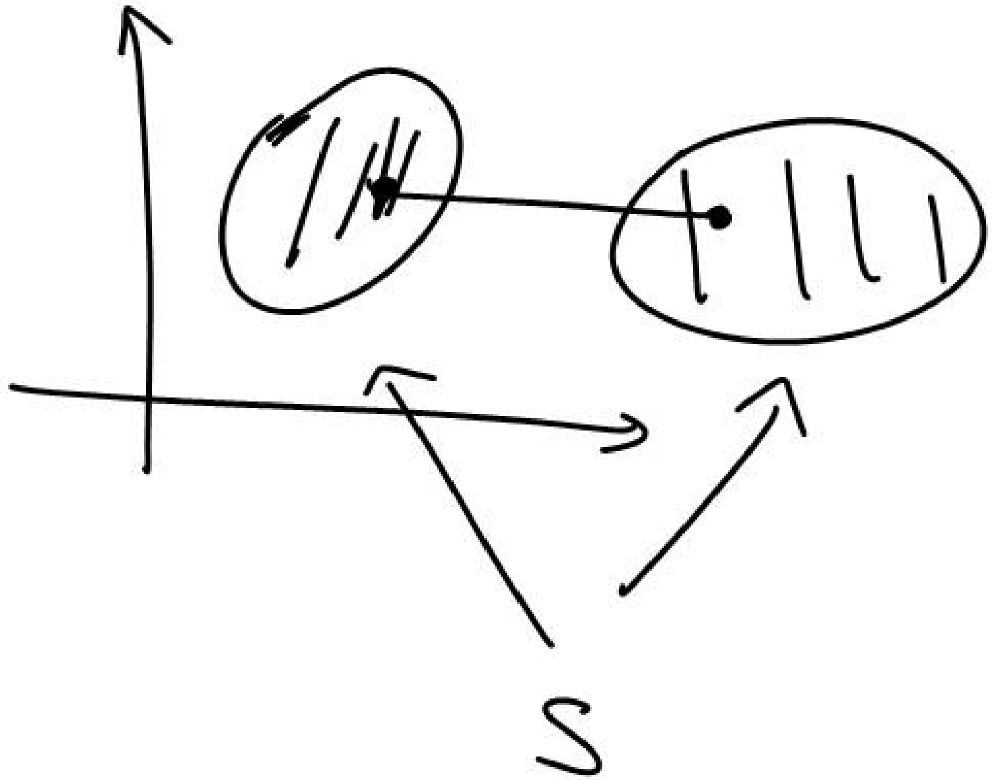
$$\min. \quad f(x)$$

$$\text{s.t.} \quad x \in \Omega$$



$$[x, y] = \{(1-\alpha)x + \alpha y : 0 \leq \alpha \leq 1\}$$



$\underline{\mathbb{R}^n}$ $\quad \langle x, y \rangle = x_1 y_1 + \cdots + x_n y_n = x^T y$

$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad H_{s,r} = \{ x \in \mathbb{R}^n : s^T x = r \}$

$$H_{s,r} = \{ x \in V : \langle s, x \rangle = r \}$$

1) Pick $x, y \in H_{s,r}$; $\alpha \in [0,1]$

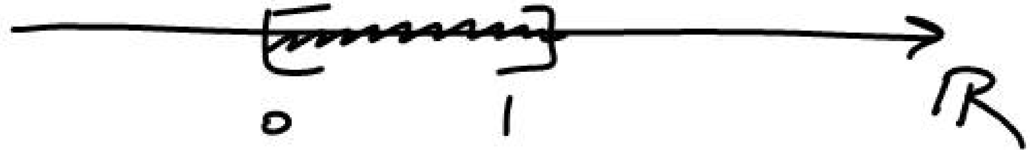
Goal: $(1-\alpha)x + \alpha y \in H_{s,n}$

2) $\langle s, (1-\alpha)x + \alpha y \rangle = \langle s, (1-\alpha)x \rangle + \langle s, \alpha y \rangle$

$$= (1-\alpha) \underbrace{\langle s, x \rangle}_{r} + \alpha \underbrace{\langle s, y \rangle}_{n}$$

$= r$

$S = [0,1]$



(closed)

$S = [0,1[$

(not closed)

$$B(c,R) = \{x \in V : \|x - c\| \le R\}$$

$\|\cdot\|$

$V = \mathbb{R}^n$

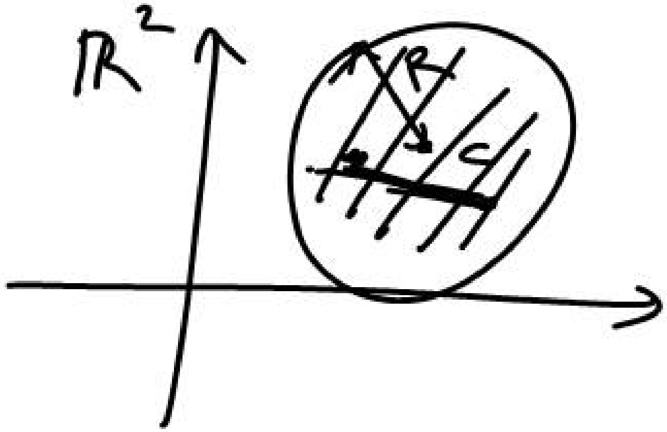
$\|x\| = \sqrt{x_1^2 + \cdots + x_n^2}$

$\|\cdot\| : V \to \mathbb{R}$

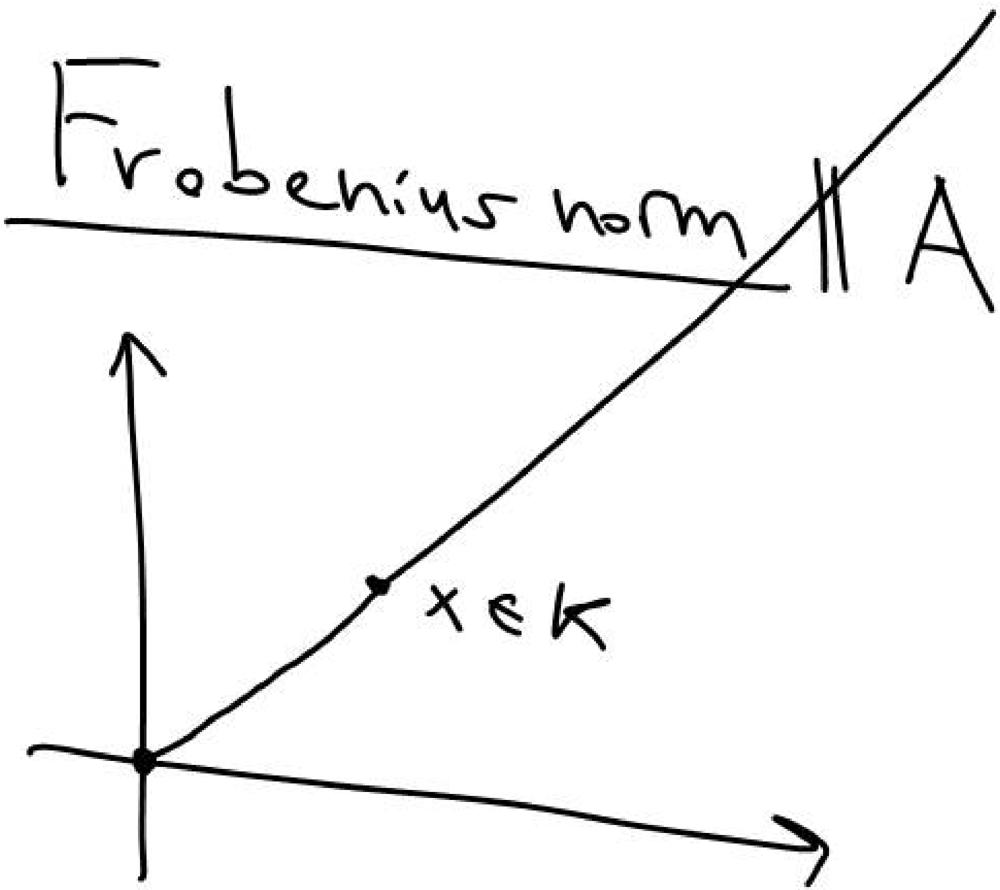
(1) $\|x\| \ge 0 \ \forall_{x \in V}$; $\|x\| = 0 \iff x = 0$

(2) $\|\alpha x\| = |\alpha| \, \|x\|$, $\alpha \in \mathbb{R}$, $x \in V$

(3) $\|x + y\| \le \|x\| + \|y\|$, $\forall_{x, y \in V}$



Frobenius norm $\|A\|_{Fr} = \sqrt{\sum_{ij} a_{ij}^2}$



$$X \in S^n \longrightarrow \lambda_1, \ldots, \lambda_n : \text{eigenvalues (in } \mathbb{R})$$

$$X \succeq 0 \iff \lambda_1, \ldots, \lambda_n \geq 0$$

$$X = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \succeq 0$$

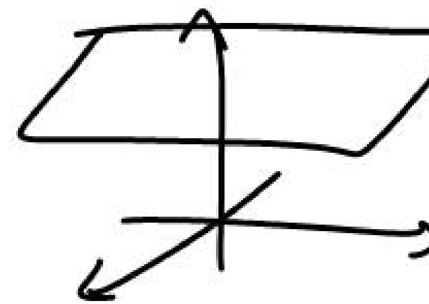
$\mathrm{eig}(X)$

$$X = \begin{bmatrix} x_1 & & & 0 \\ & x_2 & & \\ & & \ddots & \\ 0 & & & x_n \end{bmatrix}$$

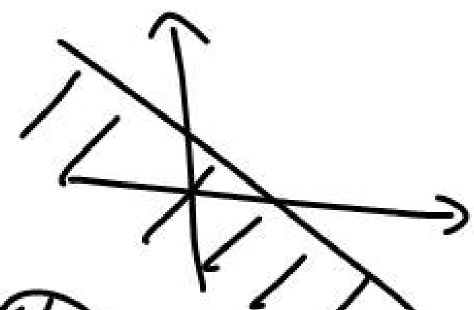
$$X \succeq 0 \iff \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \geq 0$$

Convex sets

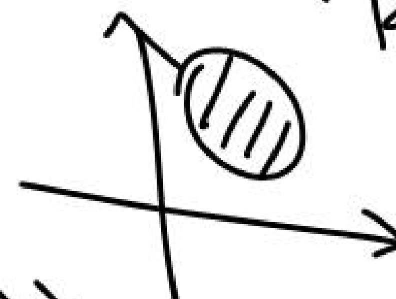
- Hyperplane



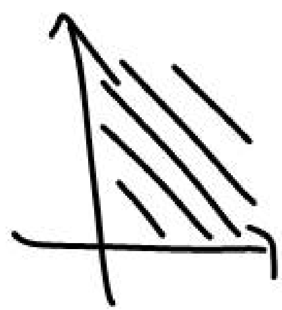
- Closed half-space



- Norm Balls



- Cones



$A : \mathbb{R}^n \to \mathbb{R}^m$: affine

"A = linear map + const."

$A : \mathbb{R}^2 \to \mathbb{R}^3$

$$A(x) = \underbrace{\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{linear}} + \underbrace{\begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}}_{\text{constant}}$$

x

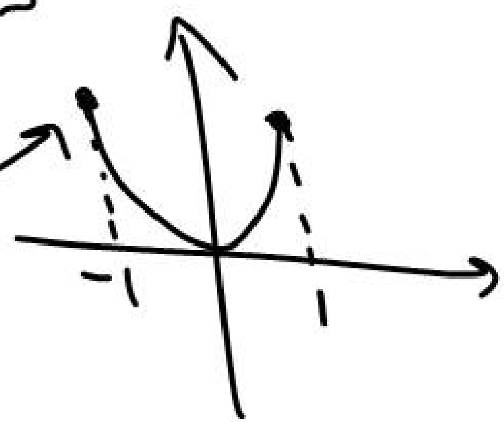
$S \subset \mathbb{R}^n$

- $A(S) = \{ A(x) : x \in S \}$

$A : \mathbb{R} \to \mathbb{R}^2$ $\quad A(x) = \begin{bmatrix} x \\ x^2 \end{bmatrix}$

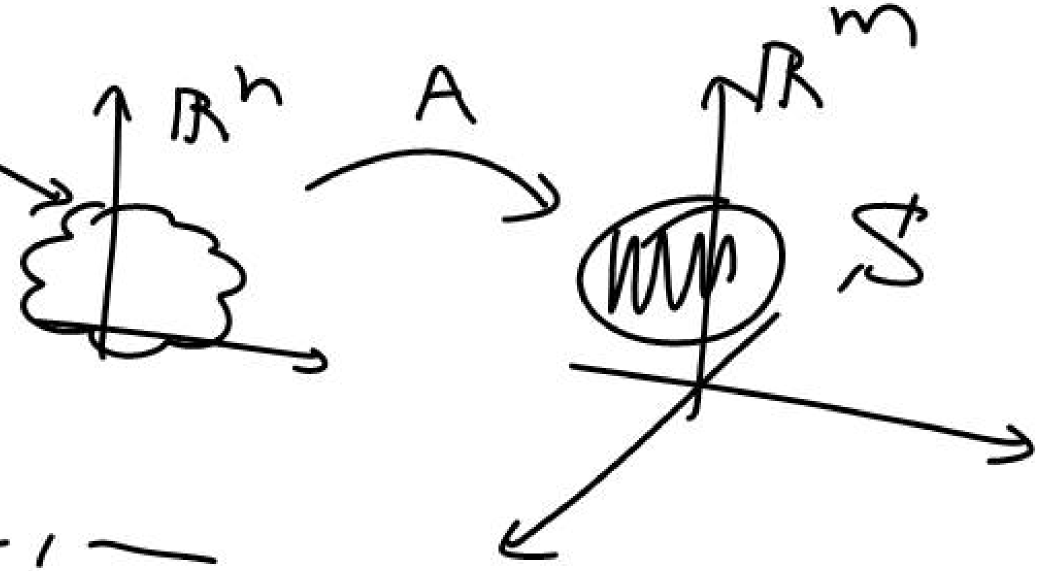
$S = [-1, 1]$

$A(S) \longrightarrow$



$$A^{-1}(S) = \{x \in \mathbb{R}^n : A(x) \in S\}$$

$A : \mathbb{R}^n \to \mathbb{R}^m$



$A : \mathbb{R}^2 \to \mathbb{R}$

$$A(x) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$