Nonlinear Optimization (18799 B, PP)
IST-CMU PhD course
Spring 2013
Instructor: jxavier@isr.ist.utl.pt
TAs: augustos@andrew.cmu.edu, ric.s.cabral@gmail.com

Important: The homework is due May, 6.

Homework 6

Instructions: read section 5.5 of [1].

Problem A. *(KKT conditions)* Consider the optimization problem

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}(x_1-a_1)^2+\frac{1}{2}(x_2-a_2)^2 \\ \text{subject to} & x_1+x_2=1 \\ & x_1^2+x_2^2\leq 4, \end{array} \tag{1}$$

with $x=(x_1,x_2)\in\mathbb{R}^2$ being the optimization variable. That is, we want to find the projection of a given point $a=(a_1,a_2)\in\mathbb{R}^2$ onto the set $C=\left\{x\in\mathbb{R}^2 \,:\, 1^\top x=1, \|x\|\leq 2\right\}$.

(a) Show that the first-order KKT conditions can be written as

$$\begin{cases} (1+\mu)x=a+\lambda 1 \\ 1^\top x=1, \|x\|\leq 2 \\ \mu\geq 0 \\ (\|x\|-2)\mu=0. \end{cases} \tag{2}$$

(b) Solve the KKT system for the case $a=(3,4)$, i.e., find $(x,\lambda,\mu)\in\mathbb{R}^2\times\mathbb{R}\times\mathbb{R}$ such that all conditions in (2) are satisfied.

Hint: it helps to make a sketch of the set C and of the point a.

(c) Solve the KKT system for the case $a=(5,0)$.

References

[1] S. Boyd and L. Vandenberghe. *Convex optimization*. Cambridge University Press, 2004.