

Image Segmentation

Motivation



- segmentation: splits the image domain into **regions**
- each regions should have a **semantic meaning** (object)
- in practice: we assume **homogeneity**

Problem:

Find [conected] groups of pixels with similar features (e.g., color, texture)

label Image

Segmentation can be represented by a **label image**

$$J_k \in \{0, 1, \dots, O-1\}$$

where O is the number of objects in the image.

0	0	0	0	2	0
0	1	1	0	2	2
0	1	1	0	2	2
0	0	0	0	0	2
0	3	3	3	0	0
0	0	3	3	3	0

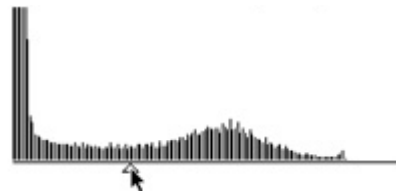
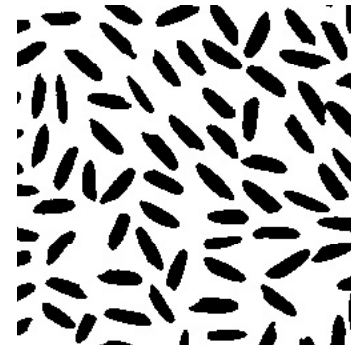
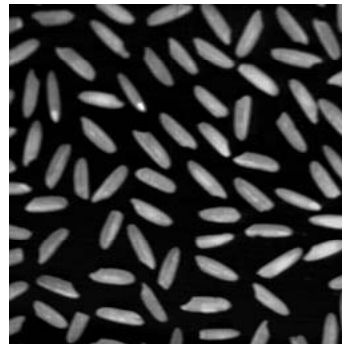
Binary segmentation is a special case with $O=2$.

binary segmentation: thresholding

binary segmentation

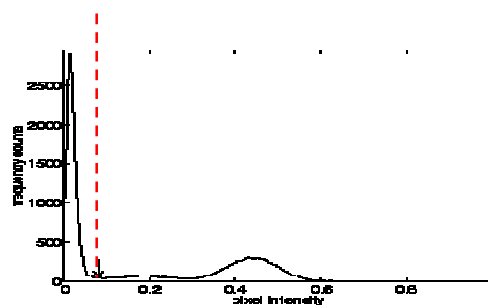
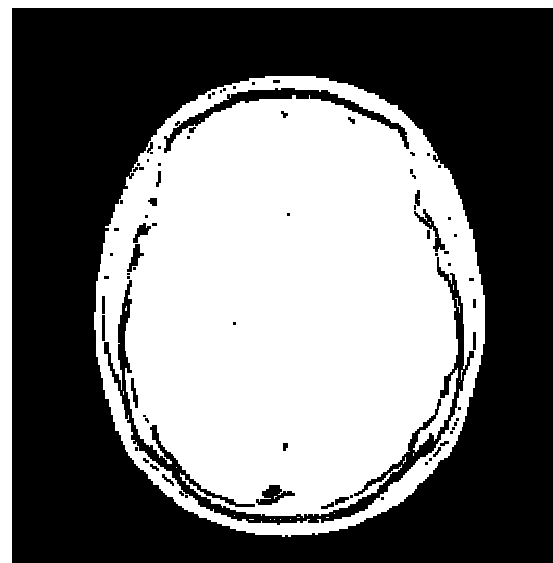
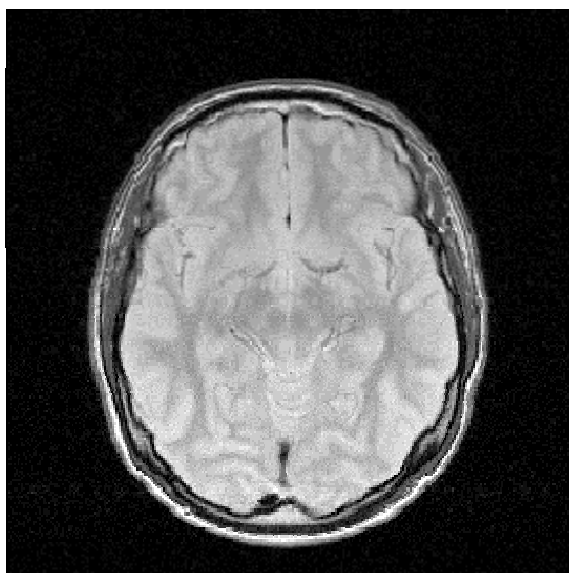
thresholding

$$J(m,n) = \begin{cases} 1 & I(m,n) > T \\ 0 & \text{caso contrário} \end{cases}$$



histogram

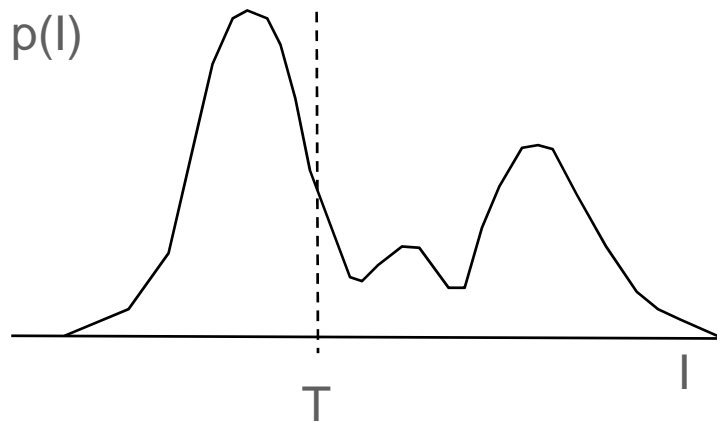
Example



automatic choice of the threshold

Can be done using na histogram of the data by **heuristic methods** or **statistical methods**

Otsu method (1979)



$$\sigma^2 = \sigma_{\text{int}}^2(T) + \sigma_{\text{ext}}^2(T)$$

$$\sigma_{\text{ext}}^2(T) = P_o(T)(\mu_o(T) - \mu)^2 + P_f(T)(\mu_f(T) - \mu)^2$$

critérium: minimization of variance inside the groups

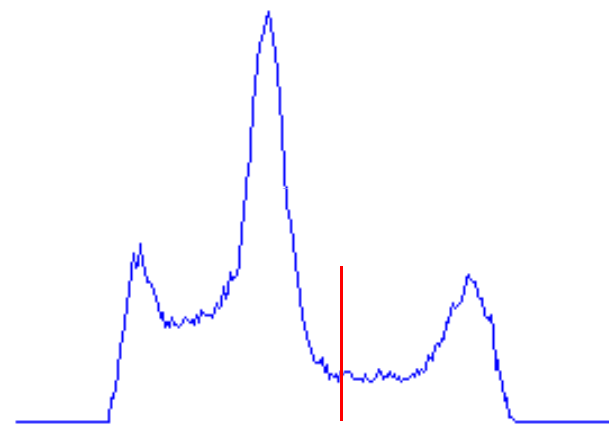
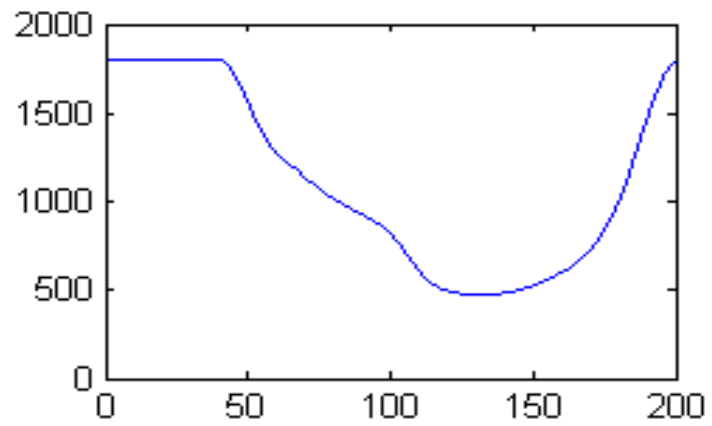
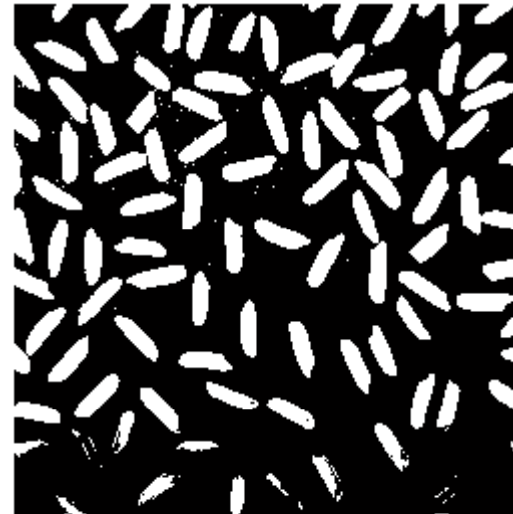
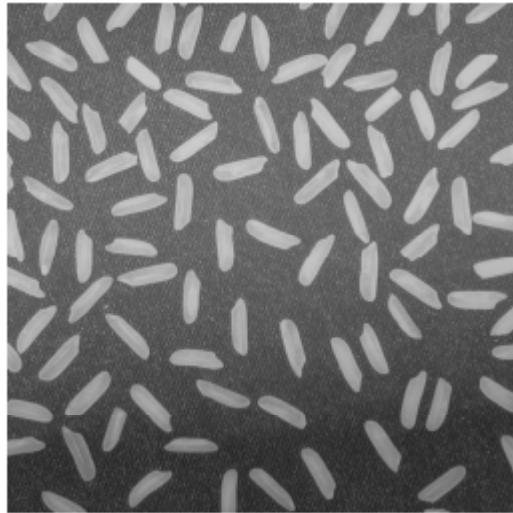
$$\sigma_{\text{int}}^2(T) = P_o(T)\sigma_o^2(T) + P_f(T)\sigma_f^2(T)$$

$P_o(T)P_f(T)$ class probabilities

$\sigma_o^2(T), \sigma_f^2(T)$ class variances

$$\sigma_{\text{int}}^2(T) = \sigma^2 - \sigma_{\text{ext}}^2(T)$$

Example

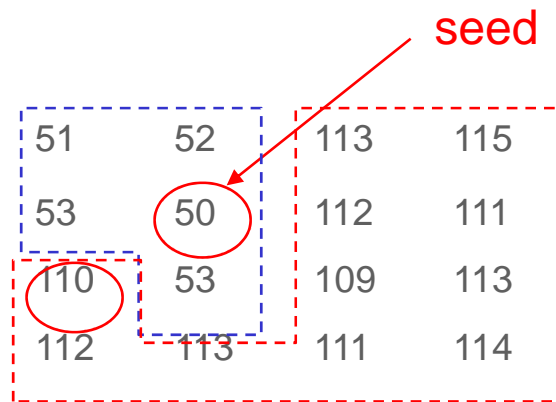


Dificulties



region growing

region growing



homogeneity criterion

$$|I(m,n) - \bar{I}| < T$$

Algorithm

- 1) **region initialization**: choose a seed
- 2) **test**: test if boundary points the homogeneity criterion $|I(x_1, x_2) - \bar{u}| < T$
- 3) **grow**: enlarge the region with boundary homogeneous points
- 4) **update**: update the boundary and average intensity
- 5) **cicle**: go to 2. if some boundary points are not tested .

classification of isolated pixels

Classification of isolated pixels

The simplest strategy consists of **independently** classifying each pixel

MAP classifier

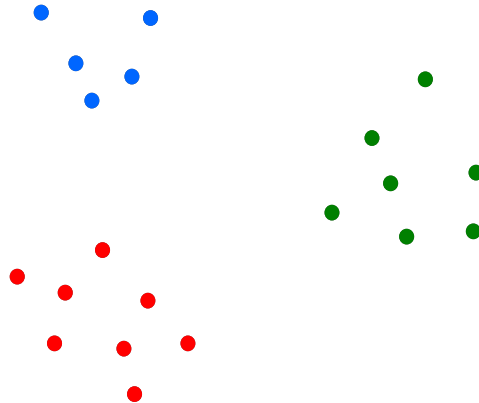
$$x_k = \arg \max_x P(x | I_k)$$

$$x_k = \arg \max_x P(I_k | x)P(x)$$

How can we estimate the probability distribution of each type of object?

supervised methods → the training set is not classified
unsupervised methods → classified training set

clustering methods



Goal: seek groups of pixels with similar features

O agrupamento é feito no espaço de características (cor, movimento, textura)

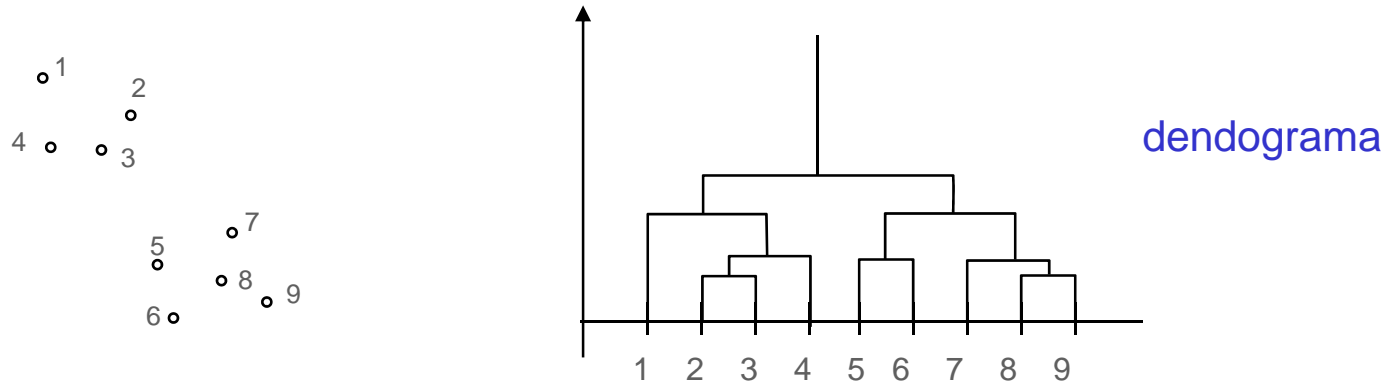
Métodos hierárquicos

Tem associada uma noção de **escala**.

métodos partitivos – dividem recursivamente o domínio da imagem em regiões cada vez mais pequenas.

métodos aglomerativos – associam regiões elementares para formarem regiões maiores com base num critério de homogeneidade.

Métodos aglomerativos



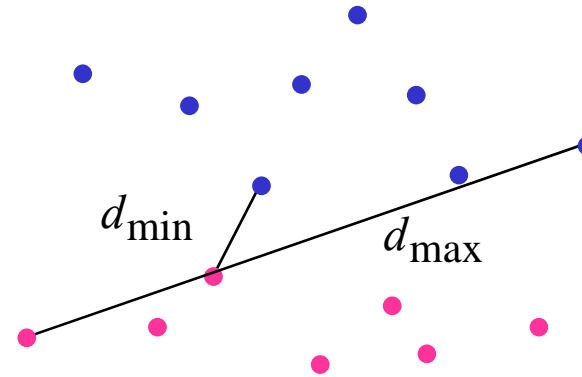
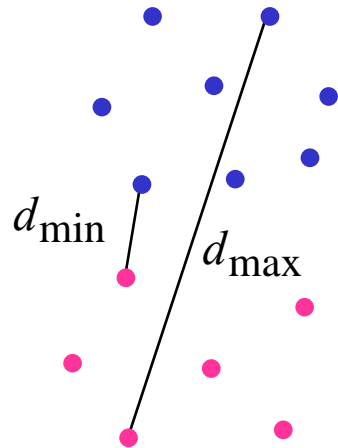
inicialização: definir uma coleção de regiões elementares (p.ex., pixels)

ciclo: até se obter um única região

- determinar o par de regiões mais próximas e fundi-los
- registar num dendograma as regiões associadas e a distância entre elas

segmentação: escolher uma distância máxima de fusão e obter os grupos de dados por análise do dendograma

Distância entre grupos



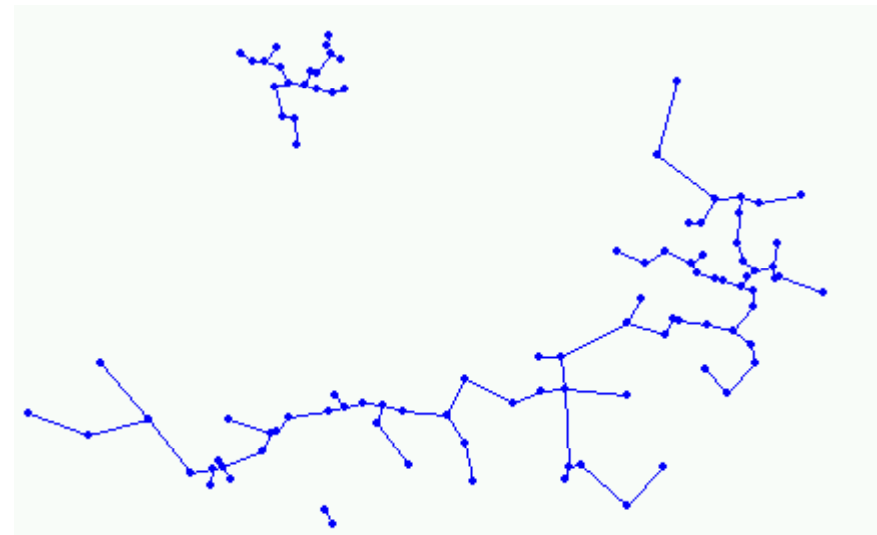
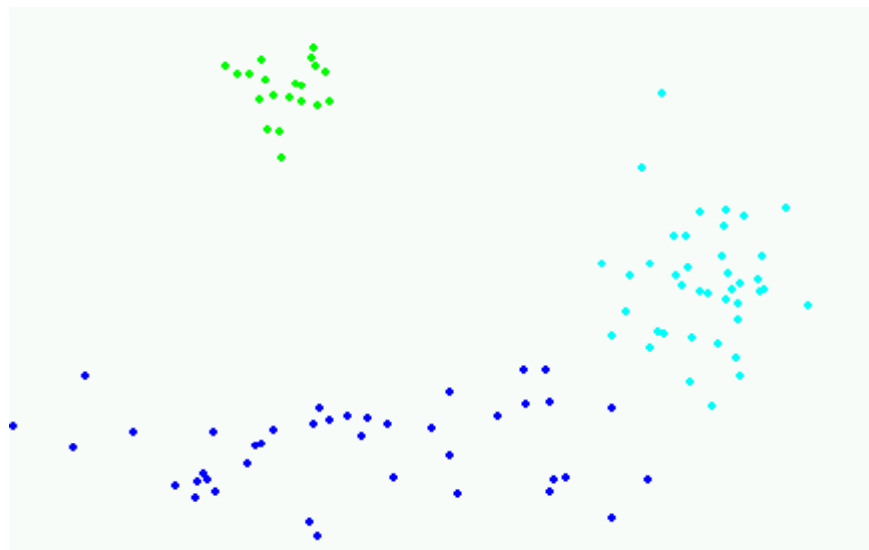
distância do mínimo $d_{\min} = \min_{x \in X, y \in Y} \|x - y\|$

método de ligação simples

distância do máximo $d_{\max} = \max_{x \in X, y \in Y} \|x - y\|$

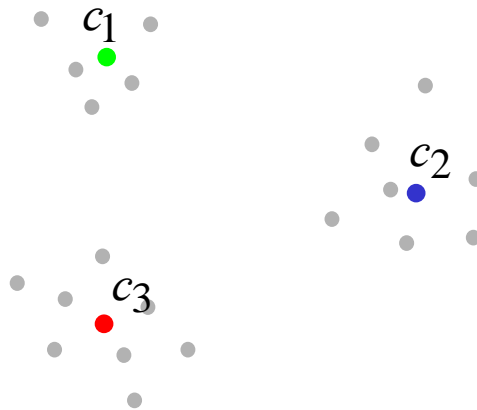
método de ligação completa

Exemplo



método de ligação simples

Método de k-Médias



Separar o conjunto de dados X em k subconjuntos disjuntos X_k
Aproximar os dados em cada subconjunto X_k por um centróide c_k

Critério

$$E = \sum_i \sum_{x \in X_i} \|x - c_i\|^2$$

← problema de otimização

Método de k-Médias (2)

Inicialização: escolher valores para os k centróides (p.ex., k observações)

Ciclo

classificação: classificar os padrões na classe com centróides mais próximos

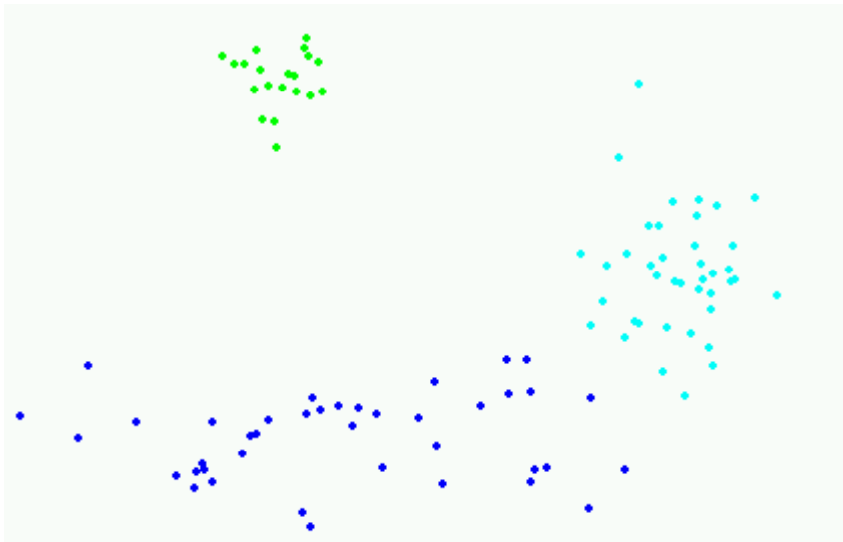
$$x \in X_i \quad \text{sse} \quad \|x - c_i\| < \|x - c_k\| \quad \forall k \neq i$$

atualização: recalculer os centróides

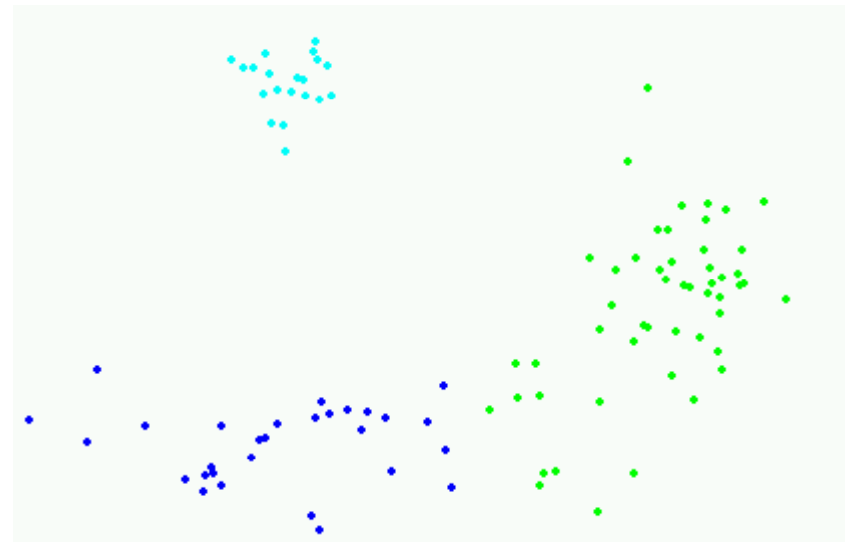
$$c_k = \frac{1}{\#X_k} \sum_{x \in X_k} x \quad (\text{se a distância for euclidiana})$$

Exemplo

dados



resultados do k-médias



Nota: mostra-se a classe correcta (cor) que não é usada pelo algoritmo de k-medias

Aplicação do K-médias



Segmentação com o método k-médias no espaço RGB, 9 classes

Aplicação do K-médias (2)



Aplicação do K-médias (3)



Segmentação com o método k-médias no espaço RGB, 9 classes

Aplicação do K-médias (4)



Classificação conjunta de pixels

Dependência espacial

Os métodos anteriores baseiam-se em dois princípios:

- estimação das propriedades de cada classe usando métodos de **aprendizagem não supervisionada** (métodos de agrupamento)
- **classificação independente** de cada pixel

Não exploram a dependência espacial: pixels vizinhos são provavelmente da mesma classe.

Como explorar a dependência espacial dos pixels ?

Minimização de Energia

Energia

$$E = \sum_k E(x_k) + \sum_{k,l \in V} E(x_k, x_l)$$

$E(x_k)$ custo de classificar o pixel k no objecto (classe) x_k

$E(x_k, x_l)$ custo de atribuir labels x_k, x_l a pixels vizinhos

V conjunto de pares de pixels vizinhos

Questões:

- como obter E ?
- como otimizar a energia E ?

(Energy minimization methods)

Métodos Baseados em Grafos

A minimização de

$$E = \sum_k E(x_k) + \sum_{k,l \in V} E(x_k, x_l)$$

quando x_k é uma variável binária tem solução óptima em tempo quase linear usando técnicas de **otimização baseada em grafos (Greig, 1989)**.

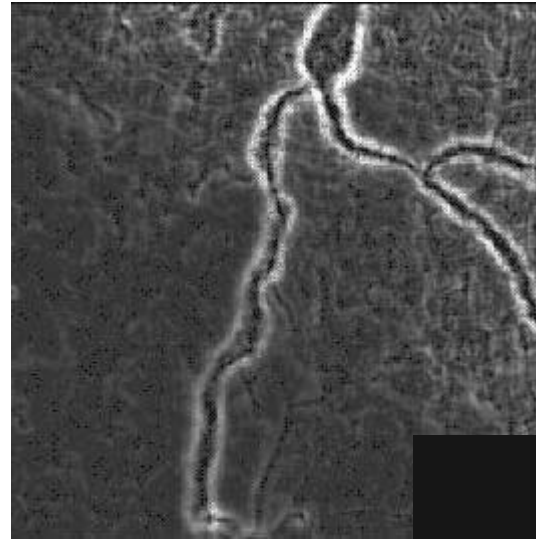
Essas técnicas podem ser aplicadas (de forma sub-ótima) para resolver problemas em que x_k não é binário.

Este é actualmente um **tema quente**

(procurar mais informação nas páginas de Zabih, Kolmogorov, Boykov)

deformable curves

challenge



how can we detect deformable curves?

Snakes

Kass, Witkin, Terzopoulos, 1987

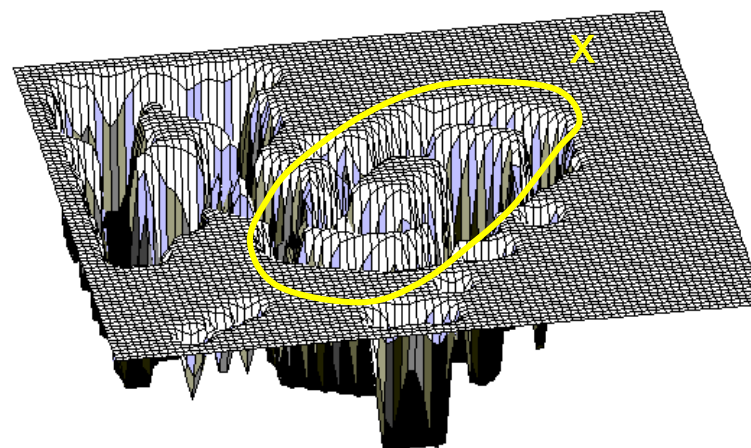


association problem



what point in the image domain
corresponds to each model point?

Snake idea



potencial
function

Energy

$$E = \int P(x(s)) ds + \alpha \int |\dot{x}(s)|^2 ds$$

The minimization of E is a variational calculus problem.

Potential functions

Potential functions should have valleys at the object boundary

gradient potential

$$P(x) = -\left\| \frac{du}{dx} \right\|$$

Cohen potencial

$$P(x) = - \sum_{c \in C} G_{\sigma}(x - c)$$

G_{σ} Gaussian kernel

C – set of boundary points

variational calculus

is used to minimize minimizar integrals

$$I(x) = \int_{s_0}^{s_1} F(s, x, \dot{x}) ds$$

where $x(s)$ na unknown function defined in the interval $[s_0, s_1]$

Example: Brachistochrone problem (1696)

Necessáry condition: **Euler equation**

$$F_x - \frac{d}{ds} F_{\dot{x}} = 0$$

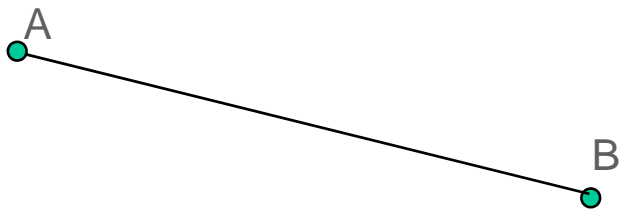
$$F_{\dot{x}\dot{x}} + F_{x\dot{x}}\dot{x} + F_{s\dot{x}} - F_x = 0$$

Brachistochrone problem

Johann Bernoulli presented this challenge in 1696 to the best mathematicians of that time

Problem:

Given two points A, B what is the trajectory of a point acted by the gravity force which starts from A and reaches B in the shortest time?

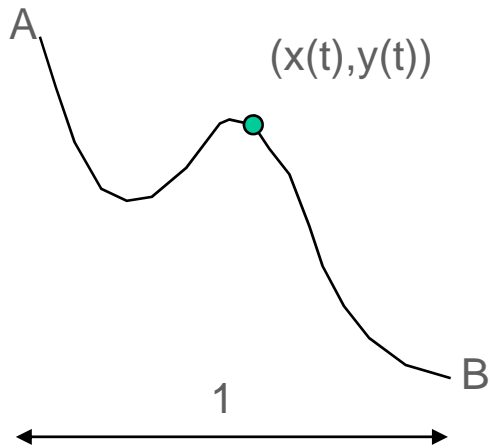


NÃO

<http://www-history.mcs.st-and.ac.uk/HistTopics/Brachistochrone.html>

Hector J. Sussmann and Jan C. Willems, 300 years of Optimal Control: from the brachistochrone to the maximum principle. IEEE Control Systems, pages 32-44, 1997.",

Brachistochrone problem

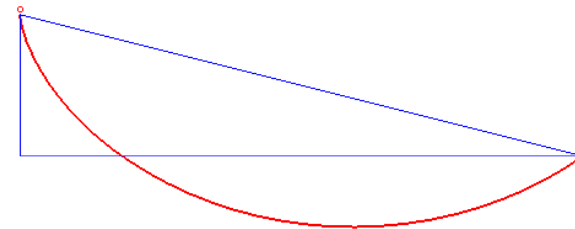


conservação da energia $v = \sqrt{2gy}$

comprimento de arco

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + y'^2} dx \quad y' = \frac{dy}{dx}$$

$$T = \int_A^B \frac{ds}{v} = \int_0^1 \frac{\sqrt{1+y'^2}}{2gy} dx$$



cicloide

Euler equation: how to get it?

Let $x(s)$ be a solution and let us consider an additive perturbation: $\tilde{x}(s) = x(s) + \varepsilon\eta(s)$

The integral

$$\eta(s_0) = \eta(s_1) = 0$$

$$I(\varepsilon) = \int_{s_0}^{s_1} F(s, x + \varepsilon\eta, \dot{x} + \varepsilon\dot{\eta}) ds$$

should be minimum for $\varepsilon=0$.

$$\frac{dI}{d\varepsilon}(0) = 0 \quad \frac{d}{d\varepsilon} \int_{s_0}^{s_1} F(s, x + \varepsilon\eta, \dot{x} + \varepsilon\dot{\eta}) ds = 0 \quad \int_{s_0}^{s_1} \eta F_x + \dot{\eta} F_{\dot{x}} ds = 0$$

Integrating by parts,

$$\int_{s_0}^{s_1} \eta F_x ds + [\eta F_{\dot{x}}]_{s_0}^{s_1} - \int_{s_0}^{s_1} \eta \frac{d}{ds} F_{\dot{x}} ds = 0 \quad \int_{s_0}^{s_1} \eta \left(F_x - \frac{d}{ds} F_{\dot{x}} \right) ds = 0$$

$$F_x - \frac{d}{ds} F_{\dot{x}} = 0$$

Energy minimization

Given the snake energy

$$E = \int P(x(s)) ds + \alpha \int |\dot{x}(s)|^2 ds$$

the Euler equation is

$$\alpha \ddot{x}(s) - \frac{dP}{dx}(x(s)) = 0 \qquad F_{\text{int}} = \alpha \ddot{x}(s) \qquad F_{\text{ext}} = -\frac{dP}{dx}$$

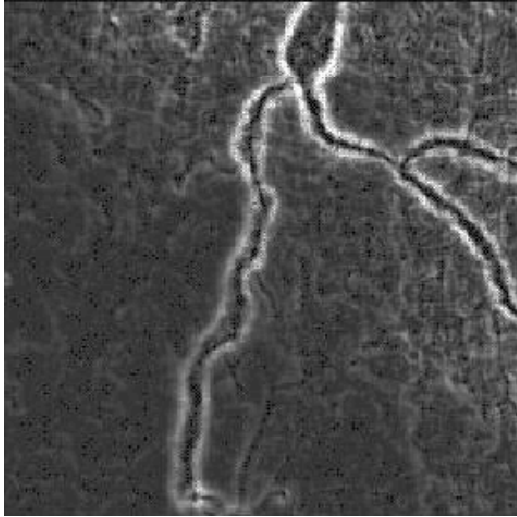
$$F_{\text{int}}(x) = F_{\text{ext}}(s) \qquad \text{equilibrium of forces}$$

Dynamic Euler equation

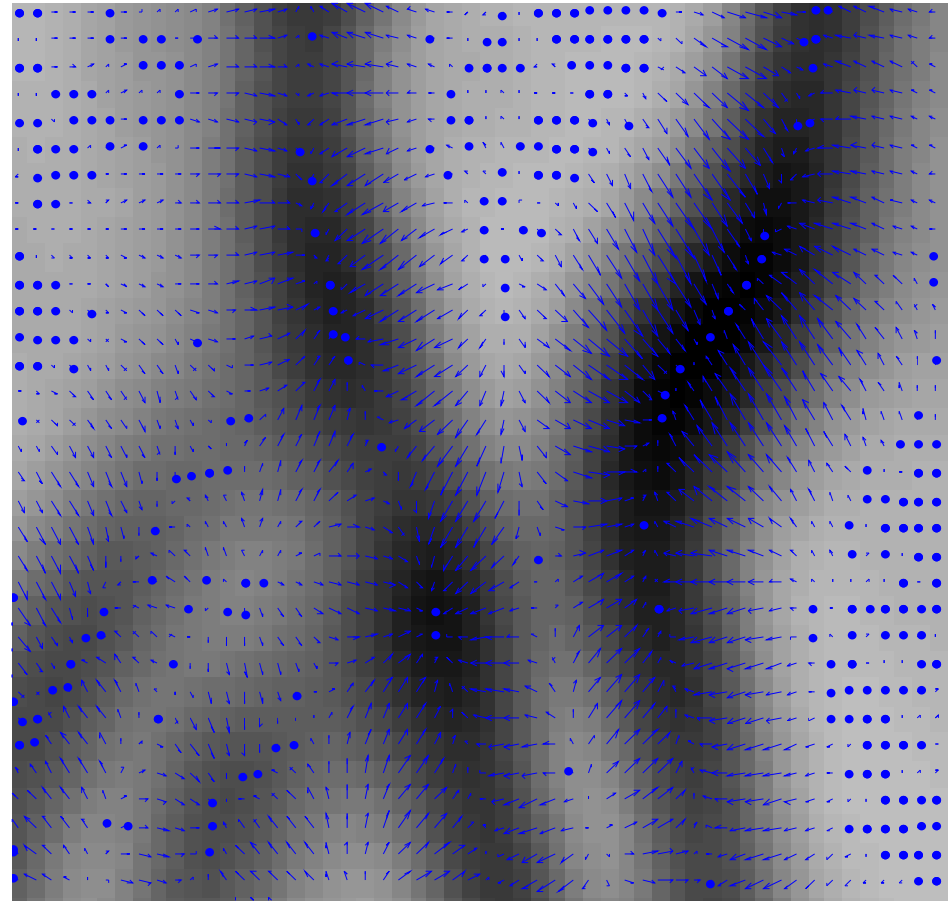
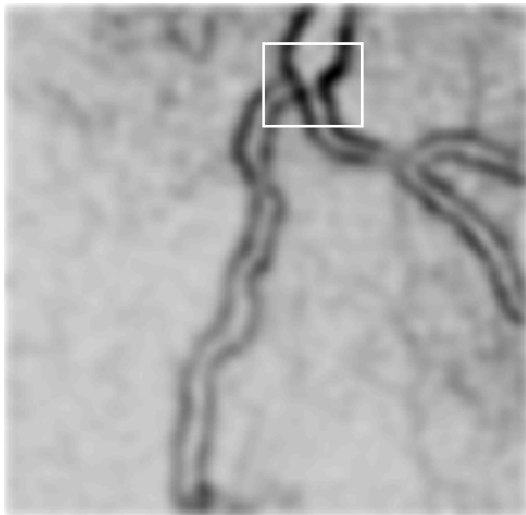
$$\frac{\partial x(s,t)}{\partial t} = F_{\text{int}}(s,t) + F_{\text{ext}}(s,t)$$

External force field

u



$P=-h*u$



Discrete Snakes

discrete model x_1, x_2, \dots, x_n

energy $E(x) = \sum_i P(x_i) + \alpha \sum_i \|x_i - x_{i-1}\|^2$

gradient

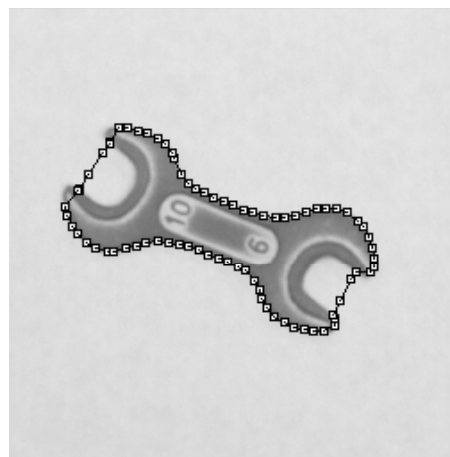
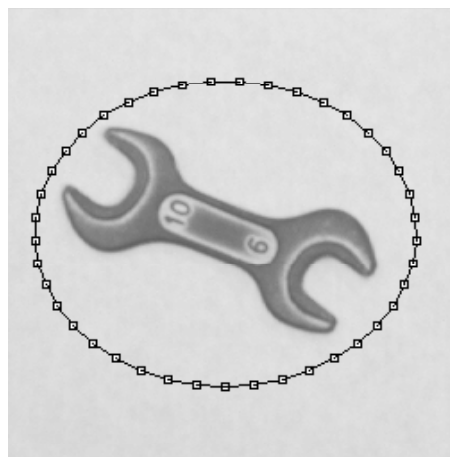
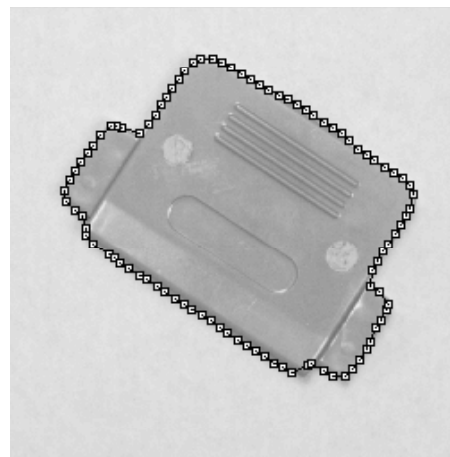
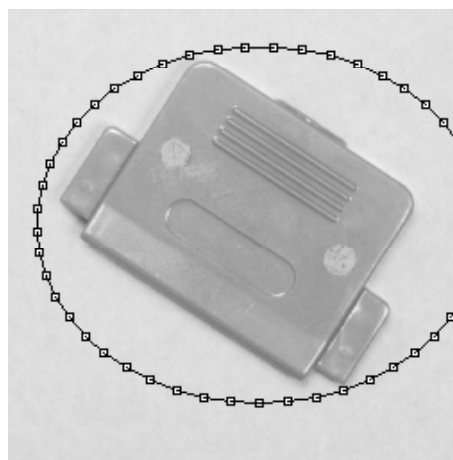
$$\frac{dE}{dx_k} = -F_{img}(x_k) + 6\alpha(x_k - \bar{x}_k)$$

$$F_{img} = -\frac{dP}{dx}$$
$$\bar{x}_k = \frac{x_{k+1} + x_{k-1}}{2}$$

recursion

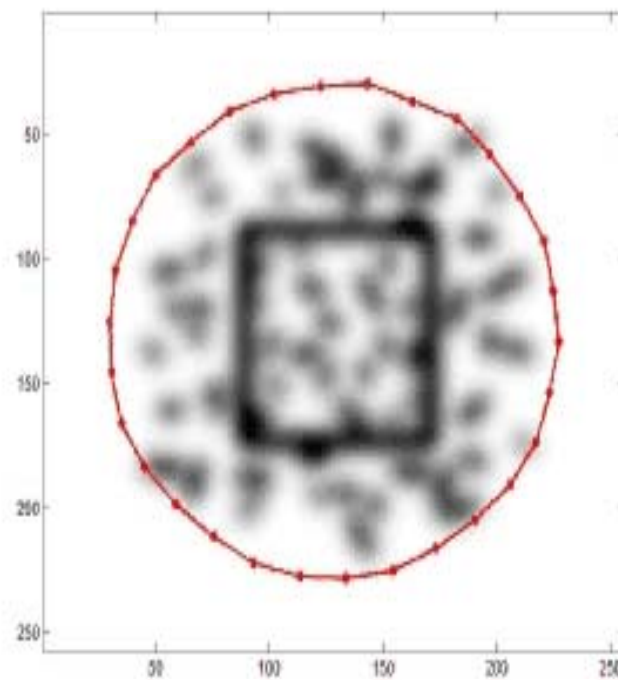
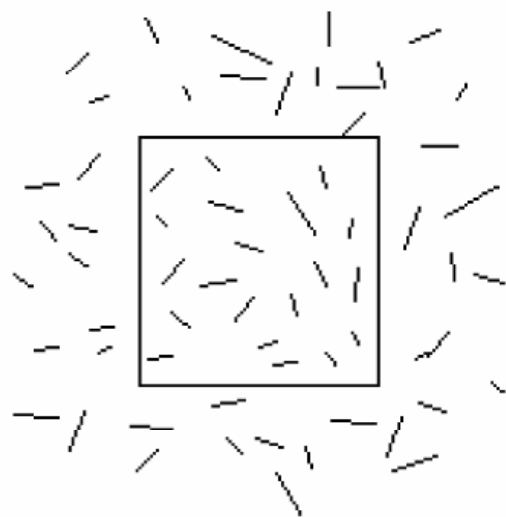
$$x_k(t+1) = x_k(t) + F_{img}(x_k(t)) - \beta(x_k(t) - \bar{x}_k(t))$$

Example

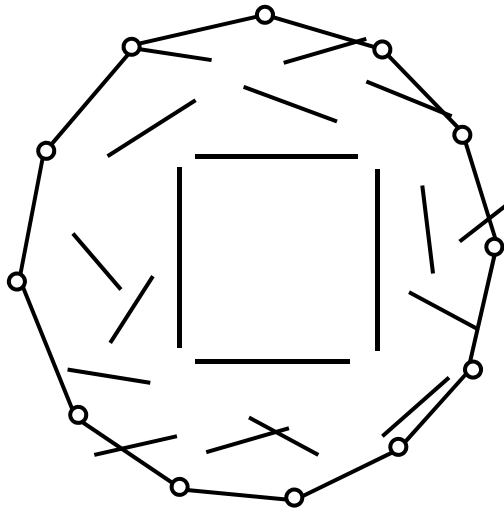


work with Jacinto Nascimento

Example



Robustness problem



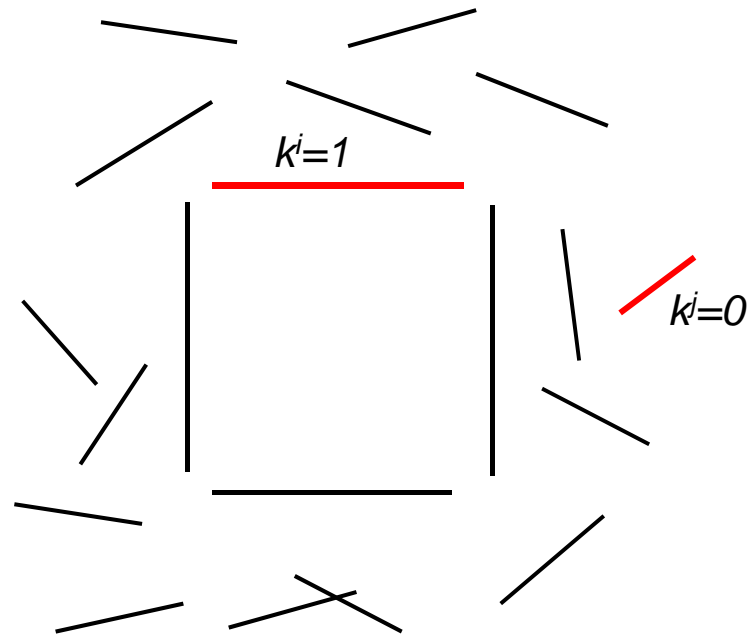
what data are valid ?

How to avoid the attraction toward wrong data?

solution

Nascimento & Marques, Image and Vision Computing, 2003

Assign a binary label to each visual feature (segment): k_j



Visual features are **segments**

Ideal case

(known labels)

Estimation

$$\hat{x} = \arg \min_x \sum_i P(x_i)$$

$$P(x) = \sum_j k^j P^j(x) + (1 - k^j)K$$

Difficulty: how to estimate the curve when the labels are unknown?

EM method

Real case

(unknown labels)

Adaptive potencial

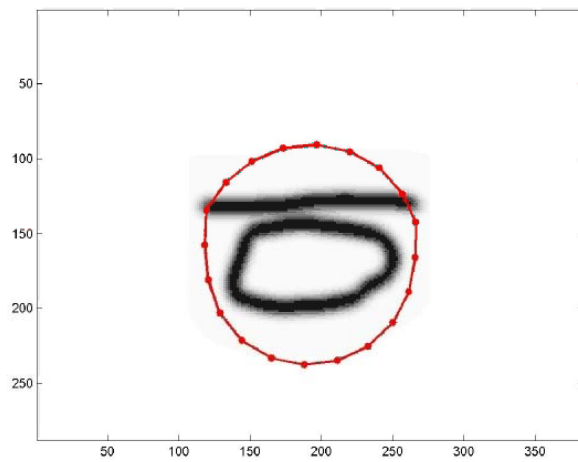
$$P_a(x) = \sum_j w^j P^j(x)$$

$$w^j = P(k^j = 1 / y^j, \hat{x})$$

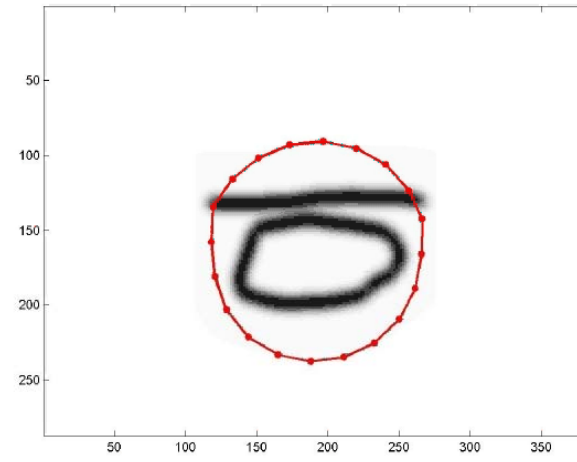
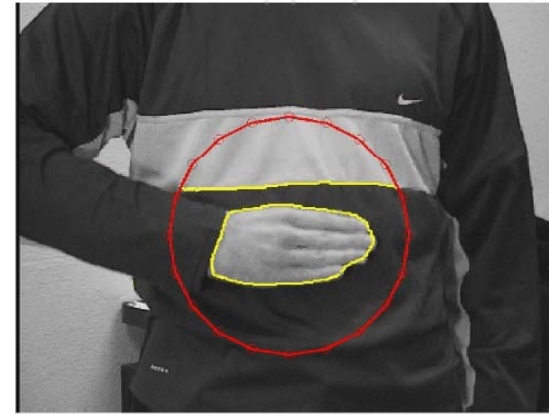
The adaptive potencial is obtained using the EM método for the estimation of parameters with incomplete data.

Example

Snakes



adaptative potential

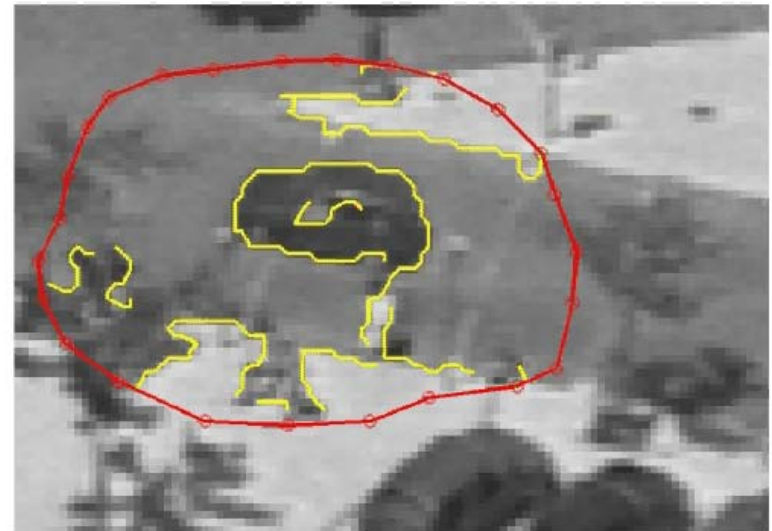


Exemplo

Snakes

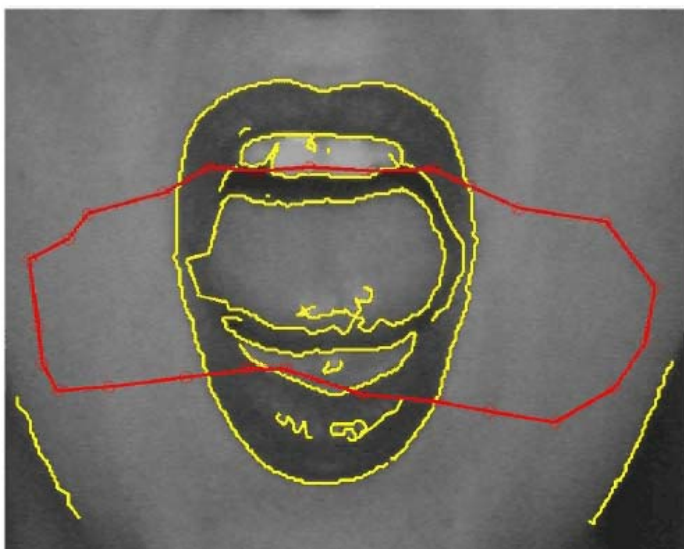


adaptative potential

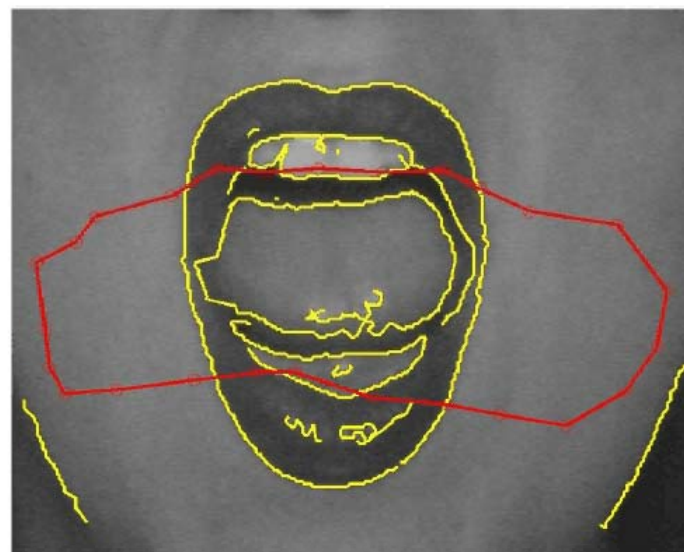


Exemplo

Snakes



adaptative potential



Exemplo

