Digital Signal Processing - DSP
(PDS - Processamento Digital de Sinais)
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Random Signals

Most signals observed in nature, in society and in economy are random signals. This means that everytime we perform a new observation we obtain a different signal. Examples of random signals are: speech, audio, ECG, EEG, economic series.

We will consider two signals in this work: Gaussial signals and speech signals. Gaussian signals can be automatically generated in a computer using a random number generator. The random generator produces a sequence of independent realizations of a Gaussian variable with distribution $N(0,1)$. The autocorrelation of this sequence is $r(k) = \delta(k)$ since different samples are uncorrelated. We can colect several variables in a single vector $x$with normal distribution $N(0, I)$. Dependence among the variables can be created by performing a linear transformation $y = Ax$.

Real signals are more complex. For example, speech signals have a rich time structure. They exhibit short range dependence among samples separated by less than 2 ms and long range depedence (3-15 ms) due to the periodic vibration of the vocal folds.

Gaussian Signals

1. Generate 1000 realizations of a Gaussian random vector $x(n) \sim N(0, I)$ with dimension $d = 2$. Visualize the data. *Suggestion:* use Matlab command *randn*.

2. Second order statistics: determine the mean and covariance matrix of the data using the following estimators

$$\hat{\mu} = \frac{1}{N}\sum_{n=1}^{N} x(n) \qquad \hat{R} = \frac{1}{N}\sum_{n=1}^{N}(x(n) - \hat{\mu})^T(x(n) - \hat{\mu}) \tag{1}$$

   Compare with the true values. *Suggestion:* use the commands *mean e cov*.

3. Determine the 2D histogram of vector x splitting the plane $\mathbb{R}^2$ into cells of amplitude 0.5×0.5.

4. Transform the data using the affine transform $y = Ax+b$ with $A = \begin{bmatrix} 1 & .8 \\ .5 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Visualize the transformed data. What differences do you notice.

5. Determine the mean and covariance of the vector y. What are the theoretical values you expect?

Speech signal

6. Read the signal speech.wav and visualize it. Find quasi periodic structure due to the vibration of the vocal folds.

7. Forget the time dependence and compute the histogram of speech. Does it look Gaussian? compare with a Gaussian pdf with the same mean and variance.

8. Select an interval in which the speech signal is quasi periodic. Determine the autocorrelation function $r(k) = \sum_{n=0}^{N-1} x(n)x(n+k)$ and plot it. Try to identify the short range dependence and the long range dependence.

9. How could you use the autocorrelation function to determine the period?