Problem set #2

z Transform

1. Determine the z transform and its convergence region for each of the following signals

$a)$	$x(n) = \delta(n)$	$d)$	$x(n) = \delta(n-k),\ k > 0$
$b)$	$x(n) = \delta(n+k),\ k > 0$	$e)$	$x(n) = u(n)$
$c)$	$x(n) = u(n) - u(n-N)$	$f)$	$x(n) = nu(n)$

2. Determine the z transform and its convergence region for each of the following signals

$a)$	$x(n) = a^n u(n)$	$d)$	$x(n) = -a^n u(-n-1)$
$b)$	$x(n) = e^{j\omega_0 n} u(n)$	$e)$	$x(n) = e^{j\omega_0 n} u(-n-1)$
$c)$	$x(n) = a^{n+2} u(n-3) + 2\delta(n-1)$	$f)$	$x(n) = a^{n-1} u(-n-2) - 3\delta(n+1)$

3. Consider the signals

$$x(n) = 2^n u(n) + (-3)^n u(-n-1), \qquad y(n) = 3^n u(n) + (\frac{1}{2})^n u(-n-1)$$

Determine their z transform and region of convergence (if it exists).

4. Consider the two sided signal $x(n) = a^n u(n) + b^n u(-n-1),\ a, b \in \mathbb{C}$. When does the z transform of $x(n)$ exist?

5. Determine the inverse z transform for the following cases

$a)$	$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}}\ ,\ \lvert z\rvert > \frac{1}{3}$	$d)$	$X(z) = \frac{1}{1+z^{-1}}\ ,\ \lvert z\rvert < 1$
$b)$	$X(z) = \frac{1}{1+z^{-1}}\ ,\ \lvert z\rvert > 1$	$e)$	$X(z) = \frac{1}{1+z^{-1}}\ ,\ \lvert z\rvert > 1$
$c)$	$X(z) = \frac{1}{1-3z^{-1}} - \frac{2}{1+z^{-1}}\ ,\ 1 < \lvert z\rvert < 3$	$f)$	$X(z) = \frac{3}{1+2z^{-1}} - \frac{2}{1-\frac{1}{3}z^{-1}}\ ,\ \frac{1}{3} < \lvert z\rvert < 2$

6. Consider the LTI systems defined by the following transfer functions. Classify each of the with respect to causality and stability.

$a)$	$X(z) = \frac{1}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}\ ,\ \frac{1}{3} < \lvert z\rvert < 2$	$c)$	$X(z) = \frac{1}{1-3z^{-1}}\ ,\ \lvert z\rvert < 3$
$b)$	$X(z) = \frac{1}{1+\cdot 5e^{j\frac{2\pi}{5}}z^{-1}}\ ,\ \lvert z\rvert > 1/2$	$d)$	$X(z) = \frac{1}{(1-z^{-1})(1+5z^{-1})}\ ,\ \lvert z\rvert > 5$

7. Consider a LTI system with transfer function $H(z) = 1/(1-z^{-1}),\ |z| > 1$. Determine the z transform of the system output for the following input signals

$$x(n) = (\frac{1}{2})^n u(n) \qquad x(n) = (\frac{1}{3})^n u(-n-1)$$

8. Consider the z transform of a signal $x(n)$

$$X(z) = \frac{2 - \frac{3}{2}z^{-1}}{(1 - z^{-1})(1 - \frac{1}{2}z^{-1})}, \qquad |z| > 1$$

Determine $x(n)$. Perform a partial fraction expansion using as a ratio of polynomials of z^{-1}.

9. Consider a causal LTI system defined by the difference equation $y(n) = \frac{1}{3}y(n-1) + x(n)$.

   i) determine the transfer function $H(z)$. Plot the poles and zeros of $H(z)$ and its region of convergence.

   ii) determine the response of the system to the unit step $x(n) = u(n)$, using the z transform.

10. Consider the causal LTI system defined by $y(n) = y(n-4) + x(n)$.

    determine the impulse response.

    determine the transfer function of the system and plot its poles.

11. Consider a causal LTI system. We applied an input signal $x(n) = \delta(n) - 2\delta(n-1)$ to the system and observed an output $y(n) = 2\delta(n) - 3\delta(n-1)$.

    i) compute the system transfer function (and its ROC).

    ii) compute the system impulse response.

12. Consider the LTI system with transfer function

$$H(z) = \frac{z^{-1} - a^*}{1 - az^{-1}} \qquad a \in \mathbb{C}$$

   i) show that this is an all pass filter i.e., $|H(e^{j\omega})| = 1$, for all ω.

   ii) plot the pole and zero for $a = (1/2)e^{j\pi/8}$.

Note: the connection in series of several such systems is still an all pass system.

Bom trabalho! jsm