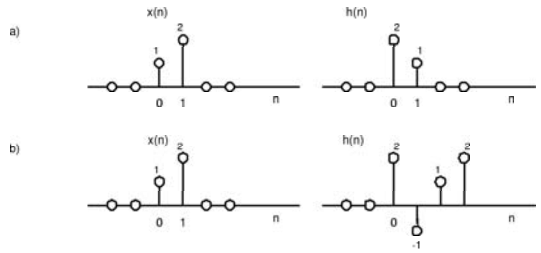
Problem set #1

Discrete Signals and Systems

1. Classify the following systems with respect to linearity and time-invariance

a)	$y(n) = 2x(n)$	*e)*	$y(n) = (-1)^n$
b)	$y(n) = x(n) + 1$	*f)*	$y(n) = (-1)^n\, x(n)$
c)	$y(n) = x(n+1)$	*g)*	$y(n) = (-1)^{x(n)}$
d)	$y(n) = \cos x(n+1)$		

2. Consider a linear time-invariant system with input $x(n)$ and impulse response $h(n)$. Determine the output of the system for each of the following cases:



3. Consider a linear time-invariant system with input $x(n)$ and impulse response $h(n) = \delta(n-k)$ (k is an integer number). Determine the output of the system $y(n)$. How would you call this system?

4. Consider a linear time-invariant system with input $x(n)$ and impulse response $h(n) = u(n)$. Determine the output of the system $y(n)$. How would you call this system?

5. Consider an amplitude modulator system with input $x(n)$ and output $y(n)$, defined by

$$y(n) = Ax(n)\cos(\omega n) \tag{1}$$

   Determine if this system is linear and time-invariant.

6. Determine the convolution of the following signals:

$$\begin{array}{lll} a) & h(n) = a^n u(n) & x(n) = b^n u(n) \\ b) & h(n) = a^n u(n) & x(n) = b^n[u(n) - u(n-N)] \end{array} \tag{2}$$

7. Consider two linear time-invariant systems with impulse responses $h_1(n), h_2(n)$. Show that if you connect the two systems in series or in parallel the overall system is still linear and time-invariant. Determine the impulse response of the system in both cases.

8. Consider two signals $x(n), y(n)$ with finite support $\{0, \ldots, M-1\}$ and $\{0, \ldots, N-1\}$. Determine the maximum length of their convolution $x(n) * y(n)$.

9. Show that the response of a linear time invariant system to a periodic input with period N is a periodic signal with period N.

10. Consider a finite difference equation

$$y(n) + \frac{1}{2}y(n-1) = x(n) \tag{3}$$

Show that this equation defines two systems. Compute the impulse responses in both cases.

11. Consider a system defined by the difference equation

$$y(n) + \frac{1}{2}y(n-1) = x(n) \tag{4}$$

Given the output of the system, $y(n) = \delta(n-1) + 2\delta(n-2) - \delta(n-3)$, determine the input.

12. Give an example of a system in which it is not possible to obtain the input, knowing the outout.

13. Determine the Fourier transform of the following signals

    $a)$ $x(n) = \delta(n-k),\ k \in \mathbb{Z}$ $\quad$ $c)$ $x(n) = a^n u(n)\ |a| < 1$

    $b)$ $x(n) = u(n) - u(n-N)$ $\quad$ $d)$ $x(n) = a^n u(-n)\ |a| > 1$

14. Consider a signal $x(n)$ with Fourier transform $X(e^{j\omega})$. Determine the Fourier transform of the following signals

    $a)$ $x(n-n_0),\ n_0 \in \mathbb{Z}$ $\quad$ $g)$ $e^{j\omega_0 n}x(n),\ |a| < 1$

    $b)$ $nx(n)$ $\quad$ $h)$ $x^*(n)$

    $c)$ $x(-n)$ $\quad$ $i)$ $x^*(-n)$

    $e)$ $Re\{x(n)\}$ $\quad$ $j)$ $Im\{x(n)\}$

    $f)$ $Odd\{x(n)\}$ $\quad$ $k)$ $Even\{x(n)\}$

15. Let $x(n), X(e^{j\omega})$ be a real signal and its Fourier transform. Prove the following properties (symmetry)

    $a)|X(e^{-j\omega})| = |X(e^{j\omega})|$ $\quad$ $c)$ $\arg X(e^{-j\omega}) = -\arg X(e^{j\omega})$

    $b)Real\{X(e^{-j\omega})\} = Real\{X(e^{j\omega})\}$ $\quad$ $d)$ $Im\{X(e^{-j\omega}) = -Im\{X(e^{j\omega})\}$

16. Consider a signal $x(n)$ with Fourier transform $H(e^{j\omega})$. Determine the Fourier transform of the expanded signal

    $$y(n) = \begin{cases} x(n/2) & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

17. Consider two linear and time invariant systems with frequency responses, $H_1(e^{j\omega}), H_2(e^{j\omega})$. Determine the frequency responses of the series and parallel connections of the two systems.

Bom trabalho!