TRAJECTORY ANALYSIS IN NATURAL IMAGES USING MIXTURES OF VECTOR FIELDS

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ABSTRACT

This work introduces a new approach to modeling object trajectories in image sequences. Trajectories performed by natural objects (*e.g.*, people, animals) typically depend on the position of each object in the scene and can change in an unpredictable way. Despite this diversity, there is often a small number of typical motion patterns based on which it is possible to explain all the observed trajectories. To achieve this goal, we model each of these motion patterns using a motion field and allow objects to switch between fields in a space-varying, possible probabilistic, way. Our approach provides a space-dependent motion model which can be estimated using an expectation-maximization (EM) algorithm. Experiments with both synthetic and real data are presented to illustrate the ability of the proposed approach in modeling different motion patterns.

Keywords: trajectories, vector fields, hidden Markov models, EM algorithm.

1. INTRODUCTION AND PRIOR WORK

Trajectory analysis has been widely used to model typical motion patterns and is an important tool in several applications, such as video surveillance. Different trajectory analysis problems, such as classification, clustering, and anomaly detection [1], have been addressed in the past decade.

Many approaches to trajectory analysis problems are based on (dis)similarity measures between trajectories. The success of this class of methods depends on the ability to find meaningful and stable similarity measures between trajectories. Euclidean distances were proposed in [2] but are only applicable between trajectories of the same length. The Hausdorff distance, used in [4, 5], is unable to distinguish trajectories sharing the same path.

Another class of techniques which has been applied in the context of visual surveillance relies on the alignment of the trajectories, using techniques such as, e.g., *dynamic time warping* [6] or the *longest common subsequence* [7]. Although those techniques can handle trajectories with different lengths, they perform poorly in the presence of noise. To surpass this problem, other alignment-based techniques, such as circular statistics [8], have been adopted; in that work, a single trajectory is modeled by a mixtures of Von Mises distributions. Another promising technique models trajectories using hidden Markov models and compares them using the cross likelihood.

Some authors argue that spatial distance may be not appropriate to describe the statistical nature of trajectories, since a pair of trajectories may be spatially close but correspond to quite different motions [10]. Knowledge of the context and structure of the scene plays an important role in modeling and recognizing activities. This observation underlies a class of methods which uses regions of the scene having a particular semantic importance, such as path intersections [11] or entry and exit points [12].

This paper introduces a novel approach to modeling trajectories in natural image sequences. We characterize the scene by a set of underlying motion vector fields. Each trajectory is composed by a set of consecutive segments, each of which is driven by one of these motion fields. Model switching is performed based on a probabilistic mechanism whose parameters depend on the position of the object. For example, in a traffic scenario trajectory changes are more probable in intersection (of two roads) than along a street. The proposed model is flexible enough to represent a wide variety of trajectories and allows the representation of space-varying behaviors as required.

Since we do not know the label of the true motion field at each instant of time nor the transition instants, we derive the EM algorithm for model parameter estimation. We propose an efficient algorithm which is able to estimate the model parameters (motion fields, switching matrix). Furthermore, this algorithm allows the use of space-varying vector fields and switching probabilities.

The rest of the paper is organized as follows. Section 2 describes the statistical model herein proposed. The learning mechanism is described in Section 3. Experimental results are presented in Section 4. Section 5 concludes the paper.

2. GENERATIVE MOTION MODEL

Let $\mathcal{T} = \{\mathbf{T}_1, ..., \mathbf{T}_K\}$, with $\mathbf{T}_k : \mathbb{R}^2 \to \mathbb{R}^2$, for $k \in \{1, ..., K\}$, be a set of K vector fields. The velocity vector at point $\mathbf{x} \in \mathbb{R}^2$ of the k-th field is denoted as $\mathbf{T}_k(\mathbf{x})$. At each time instant, one of these velocity fields is *active*. Formally, each object trajectory is generated according to

$$\mathbf{x}_t = \mathbf{x}_{t-1} + \mathbf{T}_{k_t}(\mathbf{x}_{t-1}) + \mathbf{w}_t, \quad t = 2, ..., L, \tag{1}$$

where $k_t \in \{1, ..., K\}$ is the label of the active field at time t, $\mathbf{w}_t \sim \mathcal{N}(0, \sigma^2_{k_t}\mathbf{I})$ is white Gaussian noise with zero mean and variance $\sigma^2_{k_t}$ (which may be different for each field), and L is the length in the trajectory. The initial position follows some distribution $p(\mathbf{x}_1)$. The conditional probability density of a trajectory $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_L)$, given the sequence of active models $\mathbf{k} = \{k_1, \ldots, k_L\}$ is thus

$$p(\mathbf{x}|\mathbf{k}, \mathcal{T}) = p(\mathbf{x}_1) \prod_{t=2}^{L} \underbrace{\mathcal{N}(\mathbf{x}_t|\mathbf{x}_{t-1} + \mathbf{T}_{k_t}(\mathbf{x}_{t-1}), \sigma^2_{k_t}\mathbf{I})}_{p(\mathbf{x}_t|\mathbf{x}_{t-1}, k_t)},$$

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where $\mathcal{N}(\mathbf{v}|\boldsymbol{\mu}, \mathbf{C})$ denotes a Gaussian density of mean $\boldsymbol{\mu}$ and covariance $\mathbf{C}$, computed at $\mathbf{v}$.

The sequence of active fields $\mathbf{k} = \{k_1, \ldots, k_L\}$ is modeled as a realization of a first order Markov process, with some initial distribution $P(k_1)$, and a space-varying transition matrix, *i.e.*, $\mathbf{B}_{ij}(\mathbf{x}_{t-1}) = P(k_t = j|k_{t-1} = i, \mathbf{x}_{t-1})$, where $\mathbf{B} : \mathbb{R}^2 \to \mathbb{R}^{K \times K}$ is a field of stochastic matrices. This model allows the switching probability to depend on the location of the object. The matrix-valued field $\mathbf{B}$ can also be seen as a set of K^2 fields with values in $[0, 1]$, under the constraint that $\sum_j B_{ij}(\mathbf{x}) = 1$, for any $\mathbf{x}$ and any i.

The joint distribution of a trajectory and the underlying sequence of active regions, is given by

$$p(\mathbf{x}, \mathbf{k}|\mathcal{T}, \mathbf{B}) = p(\mathbf{x}_1)P(k_1) \prod_{t=2}^{L} p(\mathbf{x}_t|\mathbf{x}_{t-1}, k_t)P(k_t|k_{t-1}, \mathbf{x}_{t-1}). \tag{2}$$

Although not explicitly denoted, $P(k_t|k_{t-1}, \mathbf{x}_{t-1})$ is a function of $\mathbf{B}$, $p(\mathbf{x}_t|\mathbf{x}_{t-1}, k_t)$ is a function of $\mathcal{T}$ and σ_{k_t}.

3. LEARNING THE VECTOR FIELDS VIA EM

In this section we address the estimation of the parameter set $\boldsymbol{\theta} = \{\mathcal{T}, \mathbf{B}, \boldsymbol{\sigma}\}$, where $\boldsymbol{\sigma} = (\sigma_1^2, \ldots, \sigma_K^2)$, from a set of observed trajectories $\mathcal{X} = \{\mathbf{x}^{(1)}, ..., \mathbf{x}^{(S)}\}$, where $\mathbf{x}^{(j)} = (\mathbf{x}_1^{(j)}, ..., \mathbf{x}_{L_j}^{(j)})$ is the j-th observed trajectory, with length L_j. The sequences of active fields, $\mathcal{K} = \{\mathbf{k}^{(1)}, ..., \mathbf{k}^{(S)}\}$, is not observed (*i.e.* it's hidden/missing).

3.1. Marginal MAP for parameter estimation

As mentioned above, the fact that the active label $\mathcal{K}$ are treated as missing data suggests the use of the EM framework. The *marginal maximum a posteriori* (MMAP) estimate of $\boldsymbol{\theta}$ is

$$\widehat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \; p(\boldsymbol{\theta}) \prod_{j=1}^{S} \sum_{\mathcal{K}} p(\mathbf{x}^{(j)}, \mathbf{k}^{(j)}|\boldsymbol{\theta}), \tag{3}$$

where each factor $p(\mathbf{x}^{(j)}, \mathbf{k}^{(j)}|\boldsymbol{\theta})$ has the form given in (2), and the sum over $\mathcal{K}$ has $K^{(\sum_j L_j)}$ terms; S is the number of the trajectories used for training.

Next, we will derive the E and M steps of the EM algorithm for solving (3). For the sake of simplicity, we will assume that the initial distributions $p(\mathbf{x}_1)$ and $P(k_1)$ are known.

3.2. The E-Step

This step computes the conditional expected value of the complete log-likelihood given the current estimates of the parameters $\widehat{\boldsymbol{\theta}}$. The complete log-likelihood is

$$\log p(\mathcal{X}, \mathcal{K}|\boldsymbol{\theta}) = \sum_{j=1}^{S} \log p(\mathbf{x}^{(j)}, \mathbf{k}^{(j)}|\boldsymbol{\theta}) \tag{4}$$

where each $p(\mathbf{x}^{(j)}, \mathbf{k}^{(j)}|\boldsymbol{\theta})$ has the form given in (2).

As is common in mixture models, we write the missing labels using binary indicator variables: the active field at time t of the j-th trajectory $k_t^{(j)} \in \{1, ..., K\}$ is represented by a binary vector $\mathbf{y}_t^{(j)} = (y_{t,1}^{(j)}, ..., y_{t,K}^{(j)}) \in \{0, 1\}^K$, where $y_{t,l}^{(j)} = 1 \Leftrightarrow k_t^{(j)} = l$. With this notation, the expected value of the complete log-likelihood $Q(\boldsymbol{\theta}; \widehat{\boldsymbol{\theta}}) \equiv \mathbb{E}\left[\log p(\mathcal{X}, \mathcal{K}|\boldsymbol{\theta})|\mathcal{X}, \widehat{\boldsymbol{\theta}}\right]$ becomes

$$Q(\boldsymbol{\theta}; \widehat{\boldsymbol{\theta}}) = \sum_{j=1}^{S} \sum_{t=2}^{L_j} \sum_{l=1}^{K} \bar{y}_{t,l}^{(j)} \log \mathcal{N}(\mathbf{x}_t^{(j)}|\mathbf{x}_{t-1}^{(j)} + \mathbf{T}_l(\mathbf{x}_{t-1}^{(j)}), \sigma_l^2 \mathbf{I}) + \sum_{j=1}^{S} \sum_{t=2}^{L_j} \sum_{l=1}^{K} \sum_{g=1}^{K} \bar{s}_{t,g,l}^{(j)} \; \log B_{g,l}(\mathbf{x}_{t-1}^{(j)}).$$

where $\bar{y}_{t,l}^{(j)} = P\left[y_{t,l}^{(j)} = 1 \middle| \mathbf{x}^{(j)}, \widehat{\boldsymbol{\theta}}\right]$ and

$$\bar{s}_{t,g,l}^{(j)} = P\left[y_{t-1,g}^{(j)} \, y_{t,l}^{(j)} = 1 \middle| \mathbf{x}^{(j)}, \widehat{\boldsymbol{\theta}}\right].$$

These probabilities are obtained by a slightly modified forward-backward procedure [13], namely to take into account the fact that the transition matrix depends on the trajectories.

3.3. The M-Step

In the M-step, the field and parameter estimation are updated as follows

$$\widehat{\boldsymbol{\theta}}_{\text{new}} = \arg\max_{\boldsymbol{\theta}} Q(\boldsymbol{\theta}; \widehat{\boldsymbol{\theta}}) + \log p(\boldsymbol{\theta}). \tag{5}$$

Next, we study this maximization, as well as the adopted priors, by looking separately at the maximization with respect to each component of $\boldsymbol{\theta} = (\mathcal{T}, \mathbf{B}, \boldsymbol{\sigma})$.

Updating $\widehat{\boldsymbol{\sigma}}$: adopting flat priors, *i.e.*, looking for usual maximum likelihood noise variance estimates, computing the partial derivative of $Q(\boldsymbol{\theta}; \widehat{\boldsymbol{\theta}})$ with respect to each component σ_k^2 of $\boldsymbol{\sigma}$, and equating to zero, yields

$$(\widehat{\sigma}_k^2)_{\text{new}} = \frac{\sum_{j=1}^{S} \sum_{t=2}^{L_j} \bar{y}_{t,k}^{(j)} \, \|\mathbf{x}_t^{(s)} - \mathbf{x}_{t-1}^{(s)} - \mathbf{T}_k(\mathbf{x}_{t-1}^{(s)})\|^2}{\sum_{j=1}^{S} \sum_{t=2}^{L_j} \bar{y}_{t,k}^{(j)}},$$

for $k = 1, ..., K$.

Updating $\widehat{\mathcal{T}}$: To estimate the motion fields we introduce some sort of regularization. Moreover, these fields live in infinite dimensional spaces, thus optimization with respect to them would lead to hard variational problems. To avoid this difficulty we adopt a finite dimensional parametrization, by representing each motion field as a linear combination over a common set of basis functions, *i.e.*,

$$\mathbf{T}_k(\mathbf{x}) = \sum_{n=1}^{N} \mathbf{t}_k^{(n)} \; \phi_n(\mathbf{x}), \tag{6}$$

where each $\mathbf{t}_k^{(n)} \in \mathbb{R}^2$ and $\phi_n(\mathbf{x}) : \mathbb{R}^2 \to \mathbb{R}$, for $n = 1, \ldots, N$, is a set of basis fields. Eq. (6) can be written in a compact way

$$\mathbf{T}_k(\mathbf{x}) = \Phi(\mathbf{x}) \, \boldsymbol{\tau}_k \tag{7}$$

where $\Phi(\mathbf{x}) = [\phi_1(\mathbf{x}), ..., \phi_N(\mathbf{x})]$ and $\boldsymbol{\tau}_k^T = [(\mathbf{t}_k^{(1)}), ..., (\mathbf{t}_k^{(N)})]$. Thus, estimating the field $\mathbf{T}_k$ becomes equivalent to estimating the coefficient vector $\boldsymbol{\tau}_k$.

To encourage smoothness of each velocity field $\mathbf{T}_k$, we adopt a zero mean Gaussian prior, with a covariance function chosen to assign low probability to large velocity differences between nearby locations

$$p(\boldsymbol{\tau}_k) \propto \exp\{-\frac{1}{2\,\alpha^2} \, \boldsymbol{\tau}_k^T \, \boldsymbol{\Gamma}^{-1} \boldsymbol{\tau}_k\}, \tag{8}$$

where α^2 is a global variance factor that allows controlling the "strength" of the prior (low variance corresponds to a strong prior).

The covariance $\mathbf{\Gamma}$ and the basis functions ϕ_i determine the covariance function of $\mathbf{T}_k$.

The terms of $Q(\boldsymbol{\theta};\widehat{\boldsymbol{\theta}})+\log p(\boldsymbol{\tau}_k)$ that depend on $\boldsymbol{\tau}_k$ are (dropping the $1/2$ factor) equal to

$$\sum_{j=1}^{S}\sum_{t=2}^{L_j}\bar{y}_{t,k}^{(j)}\|\mathbf{x}_t^{(j)}-\mathbf{x}_{t-1}^{(j)}-\Phi(\mathbf{x}_{t-1}^{(j)})\boldsymbol{\tau}_k\|^2+\boldsymbol{\tau}_k^T\,\mathbf{\Gamma}^{-1}\boldsymbol{\tau}_k. \quad (9)$$

Computing the gradient with respect to $\boldsymbol{\tau}_k$ and equating to zero, leads to a pair of linear system of equations,

$$\left(\mathbf{R}_k+\frac{\mathbf{\Gamma}^{-1}}{\alpha^2}\right)\,\boldsymbol{\tau}_k=\mathbf{r}_k \quad (10)$$

where

$$\mathbf{R}_k=\sum_{j=1}^{S}\sum_{t=2}^{L_j}\bar{y}_{t,k}^{(t)}\left(\Phi(\mathbf{x}_{t-1}^{(j)})\right)^T\Phi(\mathbf{x}_{t-1}^{(j)}) \quad (11)$$

and

$$\mathbf{r}_k=\sum_{j=1}^{S}\sum_{t=2}^{L_j}\bar{y}_{t,k}^{(j)}\left(\Phi(\mathbf{x}_{t-1}^{(j)})\right)^T(\mathbf{x}_t^{(j)}-\mathbf{x}_{t-1}^{(j)}). \quad (12)$$

Notice that since $\Phi(\mathbf{x}_{t-1}^{(j)})$ is $1\times N$, matrix $\mathbf{R}_k$ is $N\times N$ and $\mathbf{r}_k$ is $N\times 2$ (as is $\boldsymbol{\tau}_k$). Solving (10), yields $(\widehat{\boldsymbol{\tau}}_k)_{\text{new}}$, for $k=1,...,K$, which in turn define $\widehat{\mathcal{T}}_{\text{new}}=(\widehat{\mathbf{T}}_1,...,\widehat{\mathbf{T}}_K)_{\text{new}}$.

Updating $\widehat{\mathbf{B}}$: To address the estimation of the field of stochastic matrices $\mathbf{B}$, we follow the same strategy adopted for the fields $\mathbf{T}_k$, *i.e.*, we represent this field on a set of scalar basis functions, $\psi_i(\mathbf{x}):\mathbb{R}^2\rightarrow\mathbb{R}$, for $m=1,\ldots,M$,

$$\mathbf{B}(\mathbf{x})=\sum_{m=1}^{M}\mathbf{b}^{(m)}\,\psi_m(\mathbf{x}) \quad (13)$$

where each "coefficient" $\mathbf{b}^{(m)}\in\mathbb{R}^{K\times K}$ is a stochastic matrix, *i.e.*, for any $m=1,...,M$, and any $p=1,...,K$, $\sum_{k=1}^{K}b_{p,k}^{(m)}=1$.

We must guarantee that this representation preserves the stochastic nature of matrix $\mathbf{B}(\mathbf{x})$. A sufficient condition for all the entries of the expansion to be non-negative is that $\psi_m(\mathbf{x})\geq 0$, for all $\mathbf{x}$ and all $m=1,...,M$. Moreover, since each $\mathbf{b}^{(m)}$ is a stochastic matrix,

$$1=\sum_{k=1}^{K}\sum_{m=1}^{M}b_{p,k}^{(m)}\,\psi_m(\mathbf{x})=\sum_{m=1}^{M}\psi_m(\mathbf{x})\sum_{k=1}^{K}b_{p,k}^{(m)}=\sum_{m=1}^{M}\psi_m(\mathbf{x}).$$

In conclusion, $\mathbf{B}(\mathbf{x})$, as given by (13), is a stochastic matrix if the basis functions verify the following conditions: at any point $\mathbf{x}$, $\psi_i(\mathbf{x})\geq 0$, for all m; and $\sum_{m=1}^{M}\psi_m(\mathbf{x})=1$. These two conditions are known as *partition of unity* property and are satisfied by B-spline basis functions [14], of which bilinear interpolating functions are a particular simple case, which we adopt in this paper.

Using the representation, we change the problem of estimating $\mathbf{B}$ into the problem of estimating a set of stochastic matrices $\boldsymbol{\beta}=\{\mathbf{b}^{(1)},...,\mathbf{b}^{(M)}\}$, by maximizing $Q(\boldsymbol{\theta};\widehat{\boldsymbol{\theta}})$, under the stochastic constraint. Inserting (13) into (5), and dropping all terms that do not depend on $\mathbf{B}$ (equivalently, on $\boldsymbol{\beta}$), the objective function can be written as

$$\mathcal{E}(\boldsymbol{\beta})=\sum_{j=1}^{S}\sum_{t=2}^{L_j}\sum_{l=1}^{K}\sum_{g=1}^{K}\bar{s}_{t,g,l}^{(j)}\,\log\sum_{m=1}^{M}b_{g,l}^{(m)}\,\psi_m(\mathbf{x}_{t-1}^{(j)}), \quad (14)$$

which should be maximized under the stochastic constraint. The Lagrangian for this constrained problem is

$$\mathcal{E}(\boldsymbol{\beta})+\sum_{m=1}^{M}\sum_{p=1}^{K}\lambda_{mp}\left(\sum_{k=1}^{K}b_{p,k}^{(m)}-1\right),$$

where the λ_{mp} are Lagrange multipliers. Since it is not possible to analytically solve the system of (non-linear) equations resulting from equating the gradient of this Lagrangian to zero, we surpass this problem, using the gradient projection (GP) algorithm [15].

Two main issues must be considered for the GP algorithm: (i) the computation of the gradient of the objective function; and (ii) the projection onto the constraint set. Concerning (i) it is simple to compute the partial derivatives of $\mathcal{E}(\boldsymbol{\beta})$ with respect the $b_{gl}^{(m)}$, which are given by

$$\frac{\partial\mathcal{E}(\boldsymbol{\beta})}{\partial b_{gl}^{(m)}}=\sum_{j=1}^{S}\sum_{t=2}^{L_j}\bar{s}_{t,g,l}^{(j)}\,\frac{\psi_m(\mathbf{x}_{t-1}^{(j)})}{\mathbf{B}(\mathbf{x}_{t-1}^{(j)})},$$

Concerning (ii), i.e., the projection of a matrix onto the set of stochastic matrices, this is equivalent to projecting each row of the matrix onto to the probability simplex; for this purpose, we use a fast $O(K)$ algorithm which was very recently proposed [16].

In summary, $\widehat{\mathbf{B}}_{\text{new}}$ is obtained by minimizing $\mathcal{E}(\boldsymbol{\beta})$ under the stochastic constraint, using the GP algorithm, with the projection step carried out by the algorithm described in [16].

4. EXPERIMENTAL RESULTS

Synthetic data:

Fig. 1 shows examples of trajectories synthesized by two fields corresponding to horizontal (west to east) and vertical (south to north) motion. An active field is randomly selected at the first instant of time, and the object is placed at one of two regions (which are obvious in Fig. 1). The object follows the initially selected field for a while and then may switch to other field. Switching between models can occur anywhere, but the switching probabilities are not constant: they are higher near the center of the image. This is achieved by letting the transition matrix be equal to identity everywhere except in a region near the center of the image, where the transition matrix has elements $B_{11}=B_{22}=0.8$, $B_{12}=B_{21}=0.2$.

Figs. 2 shows the vector fields estimates during the EM algorithm. It is clear that, in this experiment, the algorithm produced excellent estimates of the underlying motion fields, with convergence being attained in less than 10 iterations.

Fig. 3 shows the final estimates for the spatial-dependent transition matrix. Again, it is clear that our EM algorithm produced very good estimates of the true switching field $\mathbf{B}$.

Natural Images: Flying Birds

Here we present results using natural image sequences used in the point correspondence literature [17]. The sequence $birds2$ (382 frames) contains two flocks of birds flying in the same area of the scene. In this experiment, the feature points are the centroids of the birds, obtained by simply thresholding the first channel of the images. The trajectories are obtained using the motion correspondence algorithm proposed in [18]. Fig. 4 shows the results of the motion estimation (magenta and blue arrows) superimposed with the trajectories (shown in yellow). First row shows the initialization, bottom row shows the final estimates. Here, the two main dynamics of the two flocks are correctly estimated. The number of iterations in the EM are the same as in the synthetic experiment.

5. CONCLUSIONS

We have presented an efficient and robust method for estimating mixtures of motion (velocity) vector fields from observed trajectories. In our approach, each trajectory is driven by one of a set

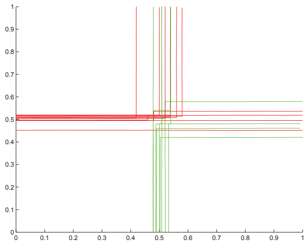

Fig. 1. Synthetic trajectories generated by the model described in the text.

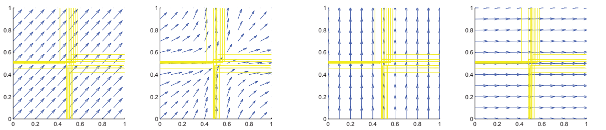

Fig. 2. Successive estimates of the two motion fields, from the trajectories shown in Fig. 1 (yellow lines), after 3 (left), and 9 (right) iterations.

of motion fields, with switching between fields controlled by a space-dependent probabilistic transition mechanism (modeled as a field of stochastic matrices). We have shown that this approach allows representing a wide variety of trajectories exhibiting space-dependent behaviors, without resorting to non-linear dynamical models (which are very hard to estimate).

We have proposed an EM algorithm to estimated the underlying fields along with the space-dependent switching model. The estimates are based on finite-dimensional parameterizations of all the fields, based on which we place a smoothness-inducing Gaussian prior on the motion fields. Almost all the update equations of the EM algorithm have simple closed form expressions, with the exception of the update of the field of stochastic matrices. To solve this update equation, we have proposed a gradient projection algorithm, based on a state-of-the-art fast algorithm to compute the projection onto the probability simplex.

Preliminary experiments, using both synthetic and real data, have shown that the proposed approach is able to model different motion patterns and that the proposed EM algorithm is able to estimate the motion and switching fields from observed trajectories.

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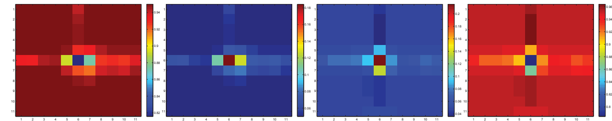

Fig. 3. Estimates of the transition matrix; from left to right B_{11}, B_{12}, B_{21}, B_{22}. We can observe that the model switching occurs at the center of the image.

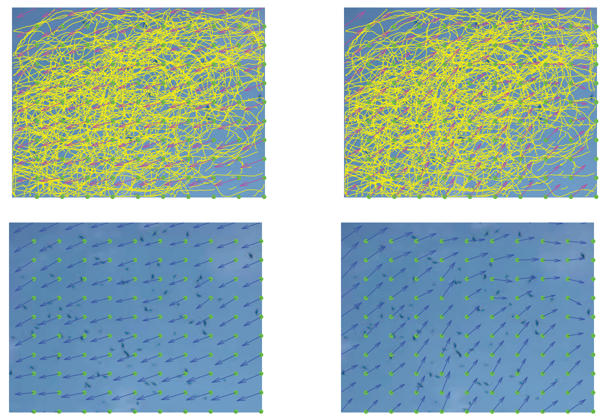

Fig. 4. Estimation of the vector fields in the birds sequence: (top row) tracks (yellow lines) superimposed with the estimated vector fields (magenta arrows); (bottom row) vector fields estimation (blue arrows); green dots are the points used in the greed.

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