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Multiple-Model Adaptive Control of an Air Heating Fan using Set-Valued Observers

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***Abstract*— This paper addresses the problem of controlling the temperature of an air heating fan with an unknown flow input rate. This time-varying uncertainty in the dynamics of the plant significantly reduces the performance of the closed-loop system, if a single (fixed) non-adaptive controller is used. Moreover, the average temperature of the air flowing through the system, that can be seen as an offset on the corresponding dynamics, is also (slowly) time-varying and highly dependent on the ambient temperature. Therefore, an alternative approach to this problem is proposed, by resorting to a novel multiple-model adaptive control methodology that relies on set-valued observers to identify the operating region of the plant. The suggested method is evaluated experimentally, demonstrating a loss of performance of about 2% when compared to the (unrealizable) perfect model identification scenario. As a shortcoming, the computational requirements due to the use of SVOs are considerably larger than the ones needed for an LTI non-adaptive controller.**

I. Introduction

When dealing with realistic applications, the model of a system is only known up to some level of precision, due to uncertain parameters and unmodeled dynamics. In these circumstances, a non-adaptive controller may not achieve the desired closed-loop performance, e.g., to guarantee a given level of attenuation from the exogenous disturbances to the performance outputs, for the whole range of uncertainty. To overcome this problem, several solutions have been proposed in the literature of adaptive control (cf. [1], [2], [3], [4]).

In this paper, we focus our attention on an adaptive control architecture, referred to as Multiple-Model Adaptive Control (MMAC). In particular, we take advantage of the recent advances in the Set-Valued Observers (SVOs) theory to invalidate models of the plant that are not compatible with the current input/output sequences, as described in the sequel – see [5], [6].

The plant considered is a Process Trainer PT326 developed by Feedback Instruments Ltd. [7] (Fig. 1). The atmospheric air is drawn by a blower (on the left), and passes through a heater (①) and a tube (②) before returning to the atmosphere. The goal is to regulate the temperature of the air, $T(\cdot)$ – measured by a thermocouple sensor with output variable $y(\cdot)$ – using the heater (①) in Fig. 1, with input signal $u(\cdot)$.

The air flow $q(\cdot)$ can be manually manipulated by changing the throttle opening ($\theta°$) from 10° to 165° (degrees), and, as shown in the sequel, has a significant impact on the dynamic behavior of the system. Minimum and maximum throttle openings, $\theta_{min} = 10°$ and $\theta_{max} = 165°$, correspond to minimum and maximum flows, respectively. Finally, the system is also affected by the ambient temperature T_a. However, this (uncontrollable) variable is only responsible for generating a (slowly) time-varying offset on the output, and hence it does not change the incremental dynamics of the plant.

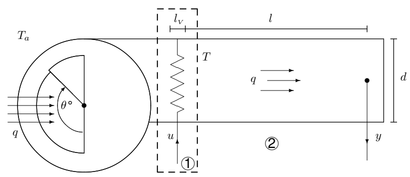

Fig. 1: Scheme of air heating system.

The nonlinear model of the system is approximated by scheduling between several local models, each of which corresponding to a pre-specified set of values of the throttle opening. Figure 2 depicts the responses of the system, for three different throttle openings (30°, 70°, and 130°), to a square-wave input signal, using a sampling frequency of $1000\,Hz$. The different offsets of the measured variable, $y(\cdot)$, are related to each operating point, and are (slowly) time-varying and highly dependent on the ambient temperature, T_a. Therefore, these offsets should not be used for the identification of dynamics of the plant.

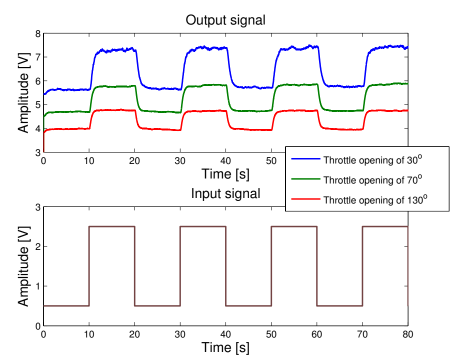

Fig. 2: System response to square wave input signal for the three throttle openings and sample period of $1\,ms$.

A. Main Contributions

The main contributions of this paper are as follows:

- The development of a multiple-model dynamic description for the Process Trainer PT326.
- The implementation of the MMAC/SVO controller for the plant, generalized to include an unknown plant offset, and that guarantees the stability of the closed-loop system.

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- The experimental evaluation of the proposed solution.

In addition to closed-loop stability, the MMAC/SVO controller also provides improved performance when compared to a single (fixed) non-adaptive controller. Moreover, it is shown that the deterioration, in terms of performance, when compared to the (non-realizable) perfect model identification scheme, is negligible, at least in the scenarios considered in this paper.

B. Organization of the Paper

This paper is organized as follows. Section II describes the dynamics of the air heating fan. The SVO-based approach to MMAC is introduced in Section III, and the experimental results are presented in IV. Finally, some conclusions regarding the proposed technique are discussed in Section V.

II. Dynamics of the Air Heating Fan

For each operating region, the dynamics of the Process Trainer PT326, illustrated in Fig. 1, was discretized with a sampling period of 200 ms, and modeled by the *ARMAX* structure described by

$$\begin{aligned} y(k) + a_1\, y(k-1) + \ldots + a_{n_a}\, y(k-n_a) = \\ = b_1\, u(k-n_k) + \ldots + b_{n_b}\, u(k-n_k-n_b+1) + \\ + e(k) + c_1\, e(k-1) + \ldots + c_{n_c}\, e(k-n_c), \end{aligned} \quad (1)$$

where $y(\cdot)$ is the measured temperature, $u(\cdot)$ is the control input, i.e., the power delivered to the heater, and $e(\cdot)$ is white Gaussian process noise. The coefficients a_i, b_j and c_m were experimentally assessed.

For all the (local) models considered, we used $n_a = n_c = 3$ and $n_b = 2$. Finally, the delay n_k is different for each of those models. These $ARMAX$ models can be described in state-space form by

$$S_i : \begin{cases} x_i(k+1) = A_i x(k) + B_i u(k) + L_i d_i(k) \\ y_i(k) = C_i x(k) + n_i(k) + \mathsf{b}_i(k) \end{cases}, \quad (2)$$

where i denotes the index of the local model, $\mathsf{b}_i(k) \in \mathbb{R}$ is the offset of the output variable, and $n_i(k)$ and $d_i(k)$ are the measurement noise and exogenous disturbances, respectively, at time k. For each value of the throttle opening, a different set of matrices A_i, B_i, C_i, and L_i, is obtained.

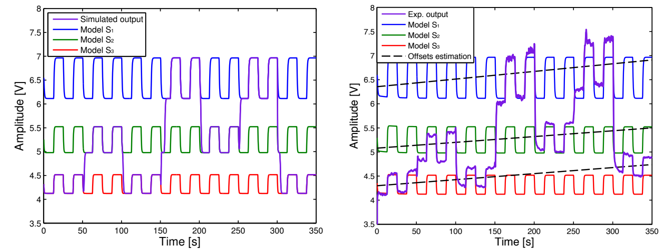
Figure 3a depicts the outputs of three different models (30°, 70°, and 130°) obtained in simulation, for the throttle opening sequence $130° \to 70° \to 130° \to 30° \to 70° \to 30° \to 130°$, with changes every $50\,s$. The measured output of the plant, obtained with the experimental setup, and using the same throttle opening sequence, is illustrated in Fig. 3b.

Remark 1: It should be noticed that the overheating of the tube impacts on the offsets of all models over time. Indeed, this overheating effect causes a variable offset in each model that is evident from Fig. 3b. $\diamond$

III. SVO-based MMAC

The large level of uncertainty of the model of the air heating fan significantly hinders the problem of designing a single *non-adaptive* controller for all the admissible models of the plant. To overcome this issue, several solutions are proposed in the literature of adaptive control based on a single plant model (cf. [1], [8], [9], [2], [3]).

In this paper, however, the focus is on a class of adaptive control architectures, referred to as Multiple-Model Adaptive



(a) (b)

Fig. 3: Simulated (a) and experimental (b) output of the plant. S_1, S_2, and S_3 were obtained for throttle openings of 30°, 70°, and 130°, respectively.

Control (MMAC)[1]. In terms of design, the idea behind the MMAC is to split the (large) set of parametric uncertainty, Ω, into N (small) subregions, Ω_i, $i \in \{1, \cdots, N\}$ – see Fig. 4 for an example where a single uncertain parameter is considered – and a non-adaptive controller for each of these subregions is synthesized. In terms of implementation, the goal is set to identify which region the uncertain parameters, ρ, belong to, and then connect to the loop the controller designed for that region.

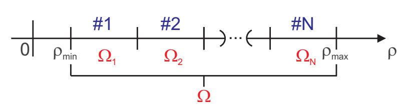


Fig. 4: Uncertainty region, Ω, for the parameter ρ, split into N subsets, Ω_i, $i = \{1, \cdots, N\}$.

Several MMAC architectures have been proposed that provide stability and/or performance guarantees, as long as a set of assumptions are met. For instance, [10] uses a parameter estimator to select a controller, guaranteeing stability of the closed-loop. Another MMAC approach, referred to as Robust Multiple-Model Adaptive Control (RMMAC), which was introduced in [11] and references therein, uses a bank of Kalman filters for the identification system and a hypothesis testing strategy to select the controllers. For this case, although simulation results – see, for instance, [11], [12] – indicate that high levels of performance are obtained, the only guarantees that can be provided are in terms of stability – see [13], [14]. In [15], calibrated forecasts are used to guarantee the stability of the closed-loop. This approach was later on extended in [16], to provide stability guarantees for several MMAC architectures. The theory of unfalsified control – see [17], [18], [19], [20], [21], among others [22], [23] – uses the controlled output error to decide whether the selected controller is delivering the desired performance or not. Other MMAC approaches increase the number of uncertainty regions in order to improve performance, whenever a given condition is satisfied (cf. [24]). The authors of [22] use a Lyapunov-based approach to select controllers, and hence require an in-depth knowledge of the plant. Some of the assumptions required by these methodologies are often unnatural or cannot be verified in practice.

The approach adopted in this article is somewhat different to the above MMAC architectures, although using a line-

[1]For a list of advantages of MMAC, over other adaptive control architectures, the reader is referred, for instance, to [4].

of-thought similar to that of the unfalsified control theory. Rather than trying to identify the *correct* region of uncertainty, by hypothesis testing or parameter estimation, the *wrong* regions are excluded. In other words, if the time-evolution of the inputs and outputs of the plant cannot be explained by a model with uncertain parameter ρ, such that $\rho \in \Omega_i$, then region Ω_i cannot be the one to which the uncertain parameter belongs. The invalidation of these uncertainty regions is addressed by using Set-Valued Observers (SVOs), taking advantage of the recent developments presented in [5], [6].

In summary, the approach provided in this paper is to use SVOs to decide which non-adaptive controllers should *not* be selected. Similarly to other MMAC architectures, we use a bank of observers – in our case, SVOs –, each of which *tuned* for a pre-specified region of uncertainty.

A. Preliminaries and Notation

The class of systems considered in this paper, typically referred to as uncertain Linear Parameter-Varying (LPV) systems, can be described by

$$\left\{\begin{array}{rcl} x(k+1) & = & A(k,\rho(k))x(k)+B(k,\rho(k))u(k) \\ & & +L(k,\rho(k))d(k) \\ y(k) & = & C(k,\rho(k))x(k)+N(k,\rho(k))n(k), \end{array}\right. \tag{3}$$

where $x(0)\in X(0)$, $x(k)\in\mathbb{R}^n$, $d(k)\in\mathbb{R}^{n_d}$, $n(k)\in\mathbb{R}^{n_n}$, $u(k)\in\mathbb{R}^{n_u}$, and $y(k)\in\mathbb{R}^{n_y}$, for $k\geq 0$. The (time-varying) vector of parameters, $\rho(\cdot)$, is such that $\rho(k)\in\mathbb{R}^{n_\rho}$. It is also assumed that $|d(k)| := \max_i |d_i(k)| \leq 1$, and $|n(k)| := \max_i |n_i(k)| \leq \bar{n}$. At each time, k, the vector of states is denoted by $x(k)$, and we define $X(0) := \text{Set}(M_0, m_0)$, where

$$\text{Set}(M,m) := \{q : Mq \leq m\} \tag{4}$$

represents a convex polytope. As an additional constraint, it is assumed that the matrices of the dynamics depend affinely on the vector of parameters.

Let $X(k+1)$ represent the set of possible states at time $k+1$, i.e., the state $x(k+1)$ satisfies (3) with $x(k)\in X(k)$ if and only if $x(k+1)\in X(k+1)$. The goal of an SVO is to find $X(k+1)$ based upon (3) and with the additional knowledge that $x(k)\in X(k), x(k-1)\in X(k-1),\cdots,x(k-N)\in X(k-N)$, for some finite horizon N. We further require that, for all $x\in X(k+1)$, there exists $x^\star\in X(k)$ such that, for $x(k)=x^\star$, the observations are compatible with (3). In other words, we want $X(k+1)$ to be the smallest set containing all the solutions to (3). A procedure for discrete time-varying linear systems was introduced in [25], and extensions to uncertain plants were presented in [5] and [6].

For plants with uncertainties, the set $X(k+1)$ is, in general, non-convex, even if $X(k)$ is convex. Thus, it cannot be represented by a linear inequality as in (4). The approach suggested in [5] is to overbound this set by a convex polytope, $\hat{X}(k+1)$, therefore adding some conservatism to the solution. A different method was presented in [6], that requires a smaller computational effort, while reducing the conservatism of the solution. Throughout the remainder of this article, we are going to use the former approach, in order to compute set-valued state estimates, $\hat{X}(k)$, of dynamic systems that can be modeled by (3).

Let $\mathcal{S}$ denote the set of *plausible* or *admissible* models of the plant to be controlled. We assume that $\mathcal{S}$ is a finite set, with cardinality N_S, and that each $S_i\in\mathcal{S}$ can be described by

$$S_i : \left\{\begin{array}{rcl} x_i(k+1) & = & A(k,\rho(k))x_i(k)+B(k,\rho(k))u(k) \\ & & +L(k,\rho(k))d_i(k), \\ y_i(k) & = & C(k,\rho(k))x_i(k)+N(k,\rho(k))n_i(k), \end{array}\right. \tag{5}$$

for each $i\in\{1,\cdots,N_S\}$, with $\rho(k)\in\Omega_i$ for all $k\geq 0$, and using a nomenclature similar to that of (3). Moreover, for any $i,j\in\{1,\cdots,N_S\}$, it is clear that

$$S_i = S_j \Leftrightarrow \Omega_i = \Omega_j.$$

Define W_d, W_n, and U, such that $d_i(j)\in W_d\subseteq\mathbb{R}^{n_d}$, $n_i(j)\in W_n\subseteq\mathbb{R}^{n_n}$, and $u(j)\in U\subseteq\mathbb{R}^{n_u}$, for all times j. The initial state of system S_i is represented by $x_o^i := x_i(0)\in X(0) := X_o\subseteq\mathbb{R}^n$. The sets W_d, W_n and U are assumed compact convex polytopes, and we define $W := W_d\times W_n$.

***Definition** 1:* [26] Let $d_i^j = d_j(i)$ and $n_i^j = n_j(i)$. Systems S_1 and S_2 are said to be *absolutely* (X_o,U,W)-*input distinguishable in* N *measurements* if, for any non-zero

$$\left(x_o^1, x_o^2, d_{0,N-1}^1, d_{0,N-1}^2, n_{0,N}^1, n_{0,N}^2, u_{0,N-1},\right)\in X_o^2\times W_d^{2N}\times W_n^{2(N+1)}\times U^N$$

there exists $k\in\{0,1,\cdots,N\}$ such that

$$y_1(k)\neq y_2(k).$$

Moreover, two systems are said to be *absolutely* (X_o,U,W)-*input distinguishable* if there exists $N\geq 0$ such that they are absolutely (X_o,U,W)-input distinguishable in N measurements.

In the definition, we used the short-hand notation $v_{0,N}$ to denote a concatenation of a sequence of vectors

$$v_{0,N} := [v_o^{\mathrm{T}},\cdots,v_N^{\mathrm{T}}]^{\mathrm{T}}.$$

Unlike other definitions of distinguishability that can be found in the literature [27], [28], [29], Definition 1 is important when we want to guarantee that, regardless of the input signals, two systems can be distinguished in a given number of measurements.

B. MMAC/SVO Architecture

Figure 5 depicts the basic Multiple-Model Adaptive Control (MMAC) architecture adopted in this article, referred to as MMAC/SVO architecture for time-varying systems, where N_S possible dynamic models for the system were considered. The main idea in this architecture is to have an SVO, referred to as *Global SVO*, which is able to provide set-valued state estimates for all the admissible time-varying uncertainties of the plant. Therefore, unless none of the N_S families of models – which assume that the uncertain parameters are time-varying – is able to describe the dynamics of the actual plant, the Global SVO does never provide an empty set-valued estimate of the state. As described in the sequel, this estimate is used to reinitialize the remaining SVOs when all the models are invalidated.

Indeed, as stressed in [16], in the case of time-varying plants a model shall never be *disqualified* "forever". In fact, if the dynamics of the plant drift at a given time instant, then a previously discarded controller may be the appropriate one to be used from that moment on.

The block entitled *Logic* in Fig. 5 is responsible for selecting the controller connected to the loop, by taking

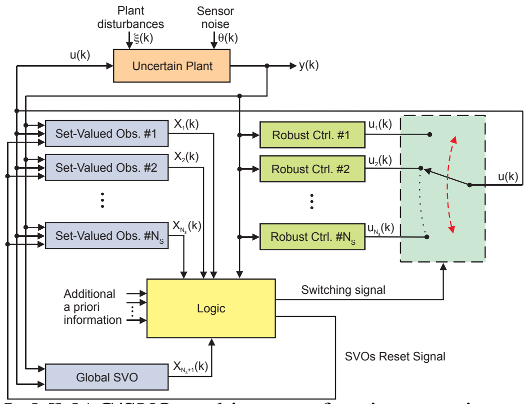


Fig. 5: MMAC/SVO architecture for time-varying systems. X_i is the set-valued state estimate provided by SVO $\#i$.

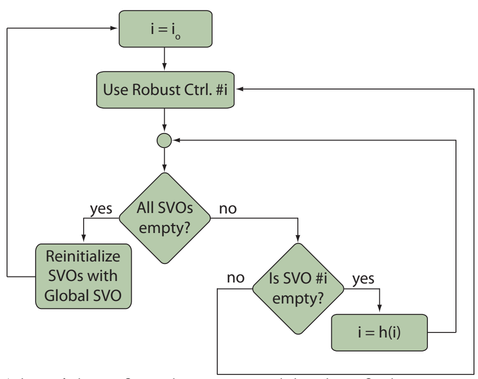


Fig. 6: Algorithm for the *Logic* block of the MMAC/SVO architecture for time-invariant systems.

into account the set-valued estimates of the state of the system provided by the SVOs. Several approaches can be used to tackle this decision problem. The one adopted in this article is depicted in Fig. 6, where $h : \{2, \cdots, N_S\} \to \{1, 2, \cdots, N_S - 1\}$ is a map satisfying $h(n) = n - 1$, and $i_o = N_S$ is the number of controllers. This strategy takes into account the fact that, if $\rho \in \Omega_i$, then SVO $\#i$ does never fail, i.e., the set-valued state estimate of the i-th SVO, $\hat{X}_i(k)$, is never empty. On the other hand, if $\rho \notin \Omega_i$, then it can happen that, for some t_o, we have $\hat{X}_i(k) = \emptyset$, for all $k \geq t_o$.

In summary, the main strategy in the algorithm is to start by using any controller in the initial set of plausible controllers, and then remove from the loop controllers whose corresponding models of the plant have been disqualified. For the sake of simplicity, the controllers are selected in a sequential fashion, in this case, i.e., if model $\#N_S$ is invalidated, we switch to controller $\#N_S - 1$, while if model $\#N_S - 1$ is invalidated, we switch to controller $\#N_S - 2$, and so on. However, other algorithms can be used, as long as the selected controller does never correspond to a previously *falsified* (or invalidated) model.

If all but the Global SVO provide empty set-valued state estimates for the plant, it means that none of the N_S models is able to describe the observed input/output data in the whole time-range. Thus, we conclude that the dynamics of the plant have drifted from one region of uncertainty to another, and hence all the other SVOs should be reinitialized.

In order to ensure closed-loop stability, we posit the following assumptions.

Assumption 1: Let $\mathcal{S}$ be the (finite) set of *admissible* models of the plant. If $S_i \in \mathcal{S}$ and $S_j \in \mathcal{S}$, with $S_i \neq S_j$, then S_i and S_j are absolutely (X_o, U, W)-input distinguishable in N sampling times.

Assumption 2: Let:

1) the initial state of the plant satisfy $x(0) \in X_o$;
2) the control input sequence satisfy $u(j) \in U$ for all $j \geq 0$;
3) the sequence of disturbances satisfy $(d(j), n(j)) \in W$ for all $j \geq 0$.

Assumption 3: There exists $T_{\min} > 0$ such that, if $\rho(k) \in \Omega_j$, then there exist time indexes k_1 and k_2 such that

1) $|k_2 - k_1| \geq T_{\min}$;
2) $k_1 \leq k \leq k_2$;
3) $\rho(\kappa) \in \Omega_j$ for all $\kappa \in [k_1, k_2]$.

In other words, Assumption 1 is used to guarantee that the models in $\mathcal{S}$ can be distinguished from each other (in the sense of Definition 1), while Assumption 2 ensures that the input signals sufficiently excite the system to allow distinguishability. Finally, Assumption 3 guarantees that the dynamics of the system to be controlled are *sufficiently slow*, so that the identification subsystem in Fig. 5 is able to select the appropriate model of the plant.

From these assumptions, we can conclude the following result.

Theorem 1: [30] Consider a dynamic system, S_r, described by (3), such that $\rho(k) \in \Omega = \Omega_1 \cup \cdots \cup \Omega_{N_S}$. Suppose Assumptions 1–3 are satisfied and that controller, C_i, designed for the region of uncertainty Ω_i, asymptotically stabilizes the system (3) with $\rho \in \Omega_i$. Then, the closed-loop system with the MMAC/SVO architecture for time-varying plants is input/output stable, for sufficiently large $T_{\min}$.

This result provides guarantees that the closed-loop system is stable, for *sufficiently slow* time-variations of the dynamics. The proof of the theorem (which is fully described in [30]) can be sketched as follows:

1) Given Assumptions 1 and 2, we can guarantee that, if the dynamics of the plant remain modeled by the same LPV description for a sufficiently large time interval, the SVOs will be able to invalidate all but the "correct" model of the plant.
2) Assumption 3 ensures that indeed the dynamics of the system remain in the same region of uncertainty for a sufficiently large time interval.
3) Since each *local* controller is guaranteed to asymptotically stabilize the system for the corresponding region of uncertainty, and using similar arguments to those in [16], it can be shown that the closed-loop system is stable.

C. MMAC/SVO for the Air Heating Fan

The dynamics in (3) can be readily transformed into the model described by (5), where the offset, $b_i(\cdot)$, is modeled as a low-frequency disturbance, added to the measured output of the system. The SVOs allow us to define (time-varying) upper and lower bounds on this disturbance that can aid the invalidation of the models. However, due to the large level of uncertainty of this offset, which is highly dependent on the room temperature, we assume that the aforementioned upper and lower bounds correspond to the upper and lower saturations of the temperature sensor, respectively.

As described in [31], the local non-adaptive controllers for each region of the dynamics of the system were designed so as to minimize the $\mathcal{H}_2$-norm of the tracking error – see [32] – and are typically referred to as Linear Quadratic Gaussian (LQG) controllers.

In order to avoid chattering due to the switching of the controllers, an integrator is connected to the loop, in between the controller and the plant – see [33]. In terms of controller design, it can be assumed that the integrator is part of the process. Figure 7 depicts the LQG controller (which can be seen as the combination of a Linear Quadratic Regulator (LQR), and a Kalman Filter (KF), based on the separation principle – see [34]), together with the augmented process (plant and integrator).

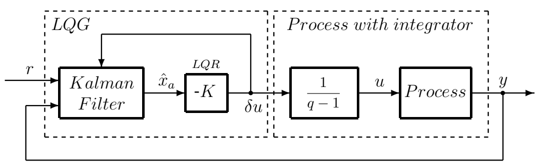


Fig. 7: Interconnection between the plant and the LQG controller.

The augmented process is, therefore, described by

$$\underbrace{\begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix}}_{x_a(k+1)} = \underbrace{\begin{bmatrix} A_i & B_i \\ 0 & 1 \end{bmatrix}}_{\bar{A}_i} \underbrace{\begin{bmatrix} x(k) \\ u(k) \end{bmatrix}}_{x_a(k)} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\bar{B}_i} \delta u(k) \quad , \qquad (6)$$

$$y(k) = \underbrace{\begin{bmatrix} C_i & 0 \end{bmatrix}}_{\bar{C}_i} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}$$

where x_a is the augmented state, $\bar{A}_i$, $\bar{B}_i$, and $\bar{C}_i$ are the augmented matrices, and δu is the incremental command action from the controller. For the sake of simplicity, the disturbances are omitted in this description. The design of the LQG controllers is performed using the quadratic cost function

$$J(\delta u) = \lim_{N\to\infty} \sum_{k=0}^{N-1} \left(e^2(k) + R\left[\delta u(k)\right]^2 \right), \qquad (7)$$

where $e(k) = r - y(k)$ is the tracking error and $R > 0$ is a weighting matrix. For $N \to \infty$, the discrete-time LQG controller for model $\#i$ is, finally, described by

$$\begin{cases} \hat{x}_a(k+1) = \left(\bar{A}_i - \bar{B}_i\mathfrak{K}_i - \mathfrak{L}_i\bar{C}_i\right)\hat{x}_a(k) - \mathfrak{L}_i e(k) \\ \delta u(k) \quad = -\mathfrak{K}_i\hat{x}_a(k) \end{cases}, \qquad (8)$$

where $\mathfrak{L}_i$ and $\mathfrak{K}_i$ are the observer and regulator gains, respectively. An anti-windup block was also designed to avoid transients caused by saturation of the command input.

Remark 2: In designing these controllers, the offset b_i is regarded as a (low-frequency) disturbance that is naturally rejected by the inclusion of the integrator at the plant input, and the plant is considered time-invariant. ⋄

IV. Experimental Results

Using the Process Trainer PT326, a series of tests have been performed in order to experimentally evaluate the behavior of the proposed control methodology. We start by analyzing the behavior of the local controllers, synthesized as described in Section III-C, followed by the performance evaluation of the MMAC/SVOs.

Throughout this section, we consider three local models for the plant, resulting from the throttle openings $\theta_1 = 30°$, $\theta_2 = 70°$, and $\theta_3 = 130°$. We denote by S_i the model of the system for $\theta = \theta_i$, and by C_i the associated LQG controller. Each of these models is also considered uncertain, in order to account for the whole range of acceptable throttle openings. For further details, the reader is referred to [31].

A. Experimental Evaluation of the Local Controllers

In the following experiments, we consider that the throttle opening is changed manually every 50 secs, according to the sequence 130°, 70° and 30°. Moreover, only a single LQG controller is used in each experiment. The results are summarized in Fig. 8.

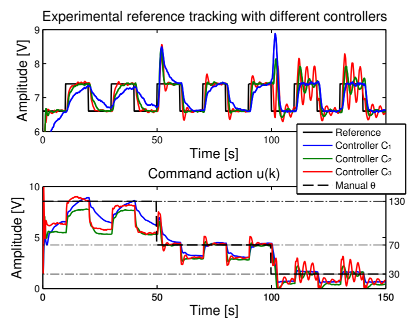


Fig. 8: Experimental closed-loop results for a time-varying throttle opening, using the three local controllers.

It should be noticed that none of the controllers exhibits reasonable performance for the whole range of the throttle opening, which indicates that non-adaptive control strategies may not be suitable for the problem at hand.

B. Experimental Evaluation of the MMAC/SVOs

Figure 9 illustrates a typical time-sequence of the results obtained using the MMAC/SVOs method described in this paper. The throttle opening changes every 70 secs. In this case, the SVOs take typically less than 10 secs to invalidate all but the "correct" model of the system. As a consequence, the tracking error is small, except during the transients between the switching of the non-adaptive controllers.

The results were also compared, through a series of experiments, with the so-called *Perfect Model IDentification* (PMID) method. In this unrealizable scheme, the appropriate controller is connected to the loop, by taking advantage of the information regarding the throttle opening. This method, of course, cannot be implemented in practice, since such information is assumed not to be available for the controller, and hence is used here just for evaluating the results obtained.

As depicted in Fig. 10, the results obtained with the PMID are similar to those of the MMAC/SVO. In fact, the tracking error and the control input are comparable, although the MMAC/SVO shows slightly larger transients during the changes of the throttle opening. In terms of RMS tracking error, the results obtained for 7 repetitions of the same experimental test are: $\mathrm{RMS}_{\mathrm{PMID}} = 1.05$, and $\mathrm{RMS}_{\mathrm{MMAC/SVO}} = 1.05$. The deterioration, in terms of RMS performance, that comes from the use of the SVO-based decision subsystem, is nearly 2%, for the scenario considered.

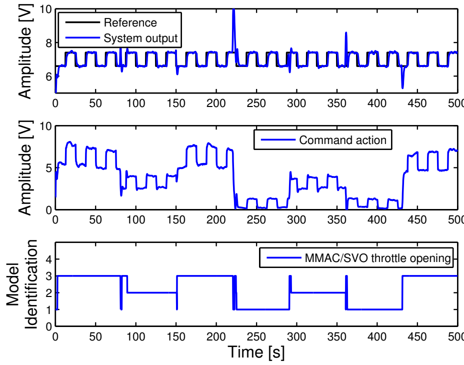


Fig. 9: Experimental results for the closed-loop system with the MMAC/SVOs.

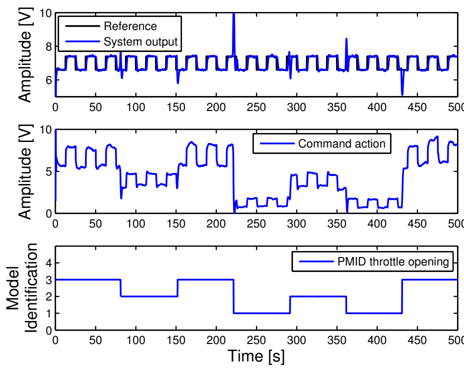


Fig. 10: Experimental results for the closed-loop system with the (unrealizable) perfect model identification.

V. Conclusions

This paper described the application of the Multiple-Model Adaptive Control methodology using Set-Valued Observers (MMAC/SVOs) to an air heating fan. The behavior of the proposed methodology was experimentally evaluated and it was shown that, at least for the scenarios considered, the deterioration in terms of RMS performance due to the SVO-based model selection is around 2%. As a shortcoming, the computational requirements of the SVOs are typically large when compared to the ones of a (single) non-adaptive controller. Nevertheless, for the present case, this did not jeopardize the practical implementability of the technique.

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