The ASIMOV High-Speed Link

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April 26, 2005
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Notation

For reference purposes, some of the most common symbols and acronyms adopted in this report are listed below.

Symbols

Matrices and vectors are denoted by upper-case and lower-case boldface characters, respectively.

\[ [A]_{i,j} \] \( ij \)-th element of matrix \( A \)
\[ [v]_i \] \( i \)-th element of column vector \( v \)
\( (\cdot)^* \) Complex conjugate
\( (\cdot)^T \) Transpose
\( (\cdot)^H \) Conjugate transpose (Hermitean)
\( \text{Re} \{ \cdot \} \) Real part
\( \text{Im} \{ \cdot \} \) Imaginary part
\( * \) Convolution
\( |z| \) Magnitude of a complex or real number
\( \|v\| \) Vector norm \( \|v\| = (v^*v)^{\frac{1}{2}} \)
\( u(t), U(\omega) \) Complex baseband signal in the time and frequency domains
\( \tilde{u}(t), \tilde{U}(\omega) \) Real passband signal
\( a(n) \) Symbol drawn from a complex constellation
\( \hat{a}(n), \tilde{a}(n) \) Symbol estimates at the input and output of a hard decision device (slicer)
\( \delta(\cdot) \) Dirac delta function (continuous argument) or Kronecker unit impulse (discrete argument)
\( E \{ \cdot \} \) Expected value
\( h(t) \) Time-invariant received PAM pulse shape
\( j \) \( \sqrt{-1} \)
\( L \) Oversampling factor in a discrete PAM sequence
\( n \) Discrete sequence index
\( T_b \) PAM signaling interval
\( \tau \) Continuous delay
\( \omega \) Continuous or discrete frequency
\( \omega_c \)  
Carrier frequency

\( x(t) \)  
Transmitted signal

\( y(t) \)  
Multipath-distorted received signal

### Acronyms

- **AGC**: Automatic Gain Control
- **ASC**: Autonomous Surface Craft
- **ASIMOV**: Advanced Systems Integration for Managing the coordinated operation of robotic Ocean Vehicles
- **AUV**: Autonomous Underwater Vehicle
- **BETR**: Band-Edge Timing Recovery
- **DDS**: Direct Digital Synthesis
- **DGPS**: Differential GPS
- **DPSK**: Differential PSK
- **DQAM**: Differential QAM
- **DSP**: Digital Signal Processor
- **DFE**: Decision-Feedback Equalizer
- **DMA**: Direct Memory Access
- **FIR**: Finite Impulse Response
- **FSE**: Fractionally-Spaced Equalizer
- **FSK**: Frequency-Shift Keying
- **GPS**: Global Positioning System
- **IF**: Intermediate Frequency
- **IIR**: Infinite Impulse Response
- **ISI**: Intersymbol Interference
- **LFSR**: Linear-Feedback Shift Register
- **LFM**: Linear Frequency Modulation
- **LMS**: Least-Mean-Square
- **LSB**: Least Significant Bit
- **MIPS**: Mega Instructions per Second
- **MISO**: Multiple-Input Single-Output
- **MSB**: Most Significant Bit
- **MSE**: Mean-Square Error
- **NCMA**: Normalized Constant-Modulus Algorithm
- **NLMS**: Normalized LMS
- **PAM**: Pulse Amplitude Modulation
- **PLL**: Phase-Locked Loop
- **PSK**: Phase-Shift Keying
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<td>Quadrature Amplitude Modulation</td>
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<td>RLS</td>
<td>Recursive Least-Squares</td>
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<td>SCS</td>
<td>Soft-Constraint Satisfaction</td>
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<td>SIMO</td>
<td>Single-Input Multiple-Output</td>
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<td>UART</td>
<td>Universal Asynchronous Receiver-Transmitter</td>
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<td>Ultra-Short Base Line</td>
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Chapter 1

General Overview

This report describes the ISR–IST contribution for the high-speed acoustic data link that was developed as part of the ASIMOV project. This contribution consists of several C software modules that implement most of the functionality of a modem, from physical-level synchronization and filtering to top-level data framing. This software was integrated with ORCA Instrumentation driver modules that interact with a custom-developed board based on a low-power Texas Instruments fixed-point TMS320C54x DSP.

As the modem software is relatively complex, documenting it from the strict perspective of software engineering is somewhat inadequate. Accordingly, this report addresses several issues, such as the transmitter and receiver signal processing structure, data format, software organization, and software parametrization. Section 1.3 provides an outline of this report.

1.1 Introduction

One of the main purposes of project ASIMOV is to demonstrate the potential applications of underwater autonomous vehicles (AUVs) for demanding scientific missions. Several factors that have hindered the widespread use of AUVs in practical applications can be traced back to the limited amount of data that can be exchanged in near real time between a (tetherless) vehicle and a mission control center using acoustic modems. Most notably, such lack of interactivity prevents end-users from assessing the unfolding of missions, and re-directing the vehicle when appropriate.

Although acoustic modems use sophisticated signal processing techniques to compensate for severe intersymbol interference (ISI) and other distortions that affect the transmitted waveforms as they propagate through dispersive underwater channels, fundamental limits restrict the data rates that can be reliably achieved. In general-purpose commercial acoustic modems, for example, the typical throughput is only 1 or 2 Kbps, which is clearly insufficient for ASIMOV, where a data rate of 30 Kbps has been specified for transmission of compressed images. To overcome this limitation, acoustic transmission always takes place vertically in the framework of project ASIMOV, thus providing a comparatively benign communication channel where high data rates are attainable. An Autonomous
2 General Overview

Autonomous Surface Craft (ASC)

Autonomous Underwater Vehicle (AUV)

Support Ship

Radio Link

High Rate Acoustic Link (Data)

Low Rate Acoustic Link (Commands/Data)

Low Rate Acoustic Link (Commands)

Figure 1.1: The ASIMOV acoustic communication system

Surface Craft (ASC) moving in tandem with the AUV is the key technical component that enables vertical transmission without restricting the AUV’s maneuverability. Besides relaying mission data from the AUV to a base station through a high-speed radio link, the ASC also provides navigational DGPS and USBL data to the AUV.

The main test site contemplated in the project is a shallow water volcanic plateau near the Azores islands, where numerous hydrothermal vents exist. In the missions of interest, the AUV is required to maneuver close to a rocky seabed at an approximate depth of 50m, and automatically detect the occurrence of bubble emissions from these vents. Sonar and video images are subsequently transmitted to the support ship through the ASC.

1.2 Systems Description

Two acoustic data links are used between the AUV and ASC (Figure 1.1). A high-speed unidirectional link using bandwidth-efficient M-PSK modulation operates at 30 Kbps with a carrier frequency of 61 KHz. It is mainly intended for transmission of compressed still images from the AUV to the ASC. A bidirectional, half-duplex, medium-speed link using binary FSK modulation with sliding carrier centered at 12 KHz transmits critical data between the two vehicles at rates up to 400 bps. It may also be used as a backward channel for ARQ protocols where the high-speed link provides the main forward channel. This kind of non-coherent modulation can tolerate the presence of significant multipath, which allows the medium-speed link to operate far from the vertical configuration at ranges up to about 3 Km. If this default modulation is ineffective in particular configurations that lead to difficult channel impulse responses, a fallback emergency mode using LFM (chirp) pulses at 20 bps may be selected. The latter mode successfully operates under virtually all conditions that may be encountered in practice. Communication with the AUV is therefore ensured at all times, even if the positioning systems malfunction and the
exact AUV location is lost.

1.2.1 Acoustic Compatibility and Directivity

Important design considerations involve the acoustic compatibility between the two systems, as it should be possible to transmit emergency commands to the AUV through the medium-speed link even when the high-speed link is active in the reverse direction. Although the operating frequencies for the two subsystems are non-overlapping, this requirement restricts the placement of transducers to avoid pre-amplifier cross-saturation while retaining a compact mechanical assembly.

The relatively low carrier frequency of 12 KHz used by the medium-speed link ensures that the acoustic signals suffer moderate absorption while propagating in the water, allowing direct communication between the AUV and a surface vessel over distances of several kilometers. Naturally, the high availability requirements in this communication link under significant uncertainty in transmitter/receiver locations can only be met if omnidirectional transducers are used at all endpoints (ASC, AUV, support ship). The medium-speed link is based on one of ORCA’s commercially available solutions, and its features will not be covered in detail here.

Transducer directivity in the high-speed link needs to be carefully considered, as there is a delicate balance between the accuracy of the positioning systems that ensure a favorable AUV/ASC configuration and the optimal directivity that minimizes intersymbol interference and fading in the communication channel. At the AUV, a top-mounted projector encased in a foam baffle creates an upward-looking directivity pattern whose main lobe has a width of about 60 degrees. It was experimentally verified that the residual acoustic energy that reaches the nearby omnidirectional transducer for the medium-speed link does not saturate the pre-amplifiers. The 61 KHz signal may therefore be rejected with bandpass filtering, allowing the medium-speed link to remain usable in the downward direction. Naturally, a transmission to the AUV at 12 KHz will likely corrupt any high-speed upward transmission, but simultaneous operation of the two links will only occur in emergency situations.

At the ASC, a foam baffle surrounding the single receiving hydrophone creates a directivity pattern similar to the one used at the transmitter. To reduce the effects of thruster and surface noise, both ASC acoustic transducers are mounted on a “torpedo” that is vertically lowered from the center of the craft to a depth of about 1.5 m (Figure 1.1).

1.2.2 Link Reliability and Complexity Requirements

Since mission-critical data will typically not be transmitted through the high-speed link, ensuring negligible error probability is not the primary design concern. Bit error rates of $10^{-4}$ to $10^{-3}$ may be sufficient for a human supervisor to verify that the vehicle operates as intended based on a sequence of images with a frame rate of about 0.5 Hz. If better performance is required, an outer layer should be used to code the modem input bit stream,
thus decreasing both the error probability and the effective data rate.

The AUV and ASC house a number of other subsystems for mission planning, guidance and control, positioning, sonar/video imaging, obstacle avoidance, etc. This is not intended as a dedicated testbed for acoustic transmission, and the resources that have been allocated to the communication system are somewhat limited. In particular, diversity techniques using multiple projectors/hydrophones have not been contemplated, and the available digital signal processing hardware was designed with rather stringent power consumption constraints. Therefore, the unusually high data rates that are required in the high-speed link are supported primarily by a favorable transmission geometry. Signal processing algorithms at the receiver are only meant to compensate for moderate distortions in the received waveforms.

1.2.3 Communication Hardware

The central component of both the transmitter and receiver is a single-processor board based on the Texas Instruments TMS320C54x 16-bit fixed-point DSP running at 120 MIPS (Figure 1.2). At the transmitter, the DSP receives bytes from a UART and calculates the required phases of the M-PSK signal, which are sent to a frequency synthesizer (DDS) for actual waveform generation. This signal is bandpass filtered to $61 \pm 15$ KHz before being power-amplified and applied to the projector. The filtering operation distorts the shape of the original square baseband signaling pulses, and introduces moderate intersymbol interference that must be compensated upon reception. Although suboptimal from the perspective of power efficiency, M-PSK modulation was chosen due to the simplicity with which bandpass waveforms can be generated using a DDS. Presumably, the availability of M-QAM constellations would have enabled enhanced performance when using coded modulation.

At the receiver, an analog front-end removes out-of-band noise components, provides automatic gain control and downconverts the signal to an intermediate frequency band between 0 and 30 KHz. After A/D conversion, all subsequent processing is performed by the DSP. The type of algorithms that may be used are conditioned by the native fixed-point precision and the internal memory size. Block-processing techniques are not well suited for this architecture, as no external memory is used for efficiency reasons.

Within the class of M-PSK constellations that are available at the transmitter, it
seems unreasonable to expect acceptable (raw) symbol error probabilities with more than 8 constellation points, since this would require excessively high transmit power. To attain the target rate of 30 Kbps, it was decided to transmit unencoded data at 15 Kbaud using 4-PSK. As discussed in Section 3.3.1, coded data are transmitted at the same baud rate using an 8-PSK constellation.

1.3 Outline of the Report

The present chapter described some acoustic link specifications and the hardware platform. The remainder of this report is organized as follows:

Chapter 2 addresses the signal processing blocks and algorithms that are used at the receiver (and, to a lower extent, at the transmitter) to map baseband waveforms into raw complex symbols and vice-versa.

The algorithms used for signal-space coding through Trellis-coded modulation are reviewed in Chapter 3. This low-level coding layer provides some improvement in output bit-error rate without sacrificing bandwidth by expanding the signal constellation.

Chapter 4 describes the structure and parameters of data frames. It discusses how data is formatted in hierarchical units and what mechanisms are available to resynchronize the decoding process even if some of the symbols in the frame are duplicated or lost at the receiver.

Finally, Chapter 5 provides an overview of the software modules that comprise the transmitter and receiver and discusses relevant issues such as the implementation of fixed- or floating-point arithmetic. It also describes how the programs can be parameterized by compiler options and provides example makefiles.
Chapter 2

Signal Processing Structure

This chapter describes the signal processing algorithms that are used at the transmitter and receiver. Naturally, the emphasis will be placed on the latter, which has much more stringent computational requirements.

Coherent modulation was chosen for the high-rate link, as it is the only signaling method whose spectral efficiency allows the target data rate of 30 Kbps to be attained within the available bandwidth. The choice of equalization and synchronization algorithms was constrained by the computational resources provided by a single Texas Instruments TMS320C54x 16-bit fixed-point DSP operating at 120 MIPS, which was used to implement the full receiver with no external memory. Due to the limitations of computational power and numerical precision, both conventional RLS algorithms and their fast (but numerically sensitive) versions [10] cannot realistically be implemented on this hardware platform, although they have been used as benchmarks when analyzing the experimental data gathered during field tests. Since there is an inevitable tradeoff between the complexity of equalization algorithms and the degree of distortion in the incoming signal that can be compensated, it is clear that this receiver will only operate reliably under favorable multipath conditions, such as those observed in vertical channels. The performance of this receiver is analyzed in [5, 8, 6, 7].

2.1 Signal Processing at the Receiver

Figure 2.1 shows the low-level signal processing blocks that are used to generate raw symbol estimates at the receiver. Often, these are collectively referred to as the “data pump”. Operating on the bandpass IF signal, the phase splitter and complex downconverter generate the in-phase and quadrature components of the equivalent baseband signal. These operations are performed by ORCA’s software for sample buffer management, and will not be covered in this report. The baseband signal is then symbol-synchronized, decimated and equalized. To avoid transient interaction between the equalization and timing recovery blocks that may lead to unstable behavior, these two systems are independently updated.

The general structure of Figure 2.1 is found in virtually any digital receiver. In the
context of underwater communications, the most widely used receiver architecture was proposed by Stojanovic [19], and comprises a decision-feedback equalizer (DFE) coupled with a phase-locked loop (PLL) for carrier recovery. This is often referred to as a sort of canonical receiver, against which any proposed system should be benchmarked [12, 1]. Computational constraints and the analysis of data collected at the test site have led to the adoption of algorithms for the ASIMOV receiver that differ from those originally proposed in [19].

In addition to the core signal processing functions that provide raw symbol estimates from the received waveform, the ASIMOV receiver is formed by several other blocks, including signal-space coding, scrambling, and frame/marker synchronization [9]. These are described in Chapters 3 and 4. Note that the slicer shown in Figure 2.1, and described in more detail in Section 2.1.3.1, is internal to the data pump. Its purpose is to provide low-delay preliminary decisions which are needed to adapt both the equalizer and the carrier recovery loop. Concurrently, the phase-corrected equalizer output is sent to a higher-level unmapping function which can take into account any underlying code structure to reduce the output bit error rate at the expense of increased decoding delay.

**Sampling Rates:** The baseband signal at the interpolator input is sampled at 60 kHz, i.e., oversampled by a factor of $L = 4$ relative to the symbol rate\(^1\) of 15 kbaud. The signal is decimated to $L = 2$ samples per symbol (30 kHz) at the equalizer input, which outputs estimates at the symbol rate of 15 kHz. All subsequent processing is done at this rate.

To avoid the burden of handling synchronized threads associated with multiple processing blocks operating at different speeds, DSP systems commonly optimize throughput rates by processing data in “batch” mode, where each batch is a collection of consecutive signal samples that have been buffered into a single unit. By propagation these multisample vectors instead of the individual signal samples, DSP systems can reduce the overhead of function calls and task synchronization. This philosophy was adopted in the ASIMOV

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\(^1\)When rotationally-invariant TCM modulation is used in the ASIMOV high-speed link an actual symbol is four dimensional and consists of a pair of complex values, each belonging to a standard 8-PSK constellation. Strictly speaking, the symbol rate is therefore 7.5 kbaud. However, in the context of the core signal processing system of Figure 2.1 the term *symbol* will usually be used to denote a complex point belonging to a conventional (2D) constellation.
high-speed receiver, whose core signal processing blocks are triggered at symbol rate:

**Interpolator:** The Farrow interpolator receives vectors of 4 baseband input samples\(^2\) and generates an interpolated sample vector of the same dimension.

**Timing Recovery:** Band edge filters operate on 4-sample vectors of interpolated samples. Only one element of every filter output vector is retained, as all subsequent processing in the timing recovery loop is formally done at the symbol rate.

**Equalizer:** The equalizer operates on 2-sample vectors and generates scalar symbol estimates. Notice the slight inconsistency of Figure 2.1, which suggests that the same set of samples is input to the timing recovery block and the equalizer. In fact, the former processes the full 4-sample interpolated vector, whereas the latter is fed a vector decimated by a factor of 2.

**Carrier Recovery:** The carrier recovery block receives and outputs scalar samples.

As described in Section 3.3.1, for efficiency reasons the Trellis decoder for TCM uses a block sliding window, and can therefore be thought of as generating vectors of bits. However, it is triggered at the (TCM) input symbol rate of 7.5 kHz regardless of whether output bits are actually generated.

### 2.1.1 Timing Recovery

Very simple timing recovery algorithms may be derived by joint optimization of the optimal sampling instant and equalizer coefficients [19]. This approach is somewhat unreliable, as it requires the inclusion of a highly complex equalizer transfer function inside a feedback loop that may easily become unstable [13]. Therefore, an important practical requirement for the receiver of Figure 2.1 is that the timing recovery system be independent of the adaptive equalizer. Moreover, it must be relatively simple and remain effective for any M-PSK constellation even when the signal suffers from significant intersymbol interference, which rules out most of the techniques that are commonly used in other digital communication applications [16]. Based on these criteria, band-edge timing recovery (BETR) was selected [4, 11].

Let \( y_c(t) \) be a received PAM signal

\[
y_c(t) = \sum_{k=-\infty}^{\infty} a(k)h_c(t - kT_b) + \eta_c(t) ,
\]

(2.1)

where \( a(k) \) is the \( k \)-th transmitted symbol, \( h_c(t) \) is the received pulse shape, \( T_b \) is the signaling interval, and \( \eta_c(t) \) denotes additive noise. As detailed in Appendix A, BETR adjusts the optimum sampling offset \( \tau \) to iteratively maximize the power of a baud-rate-sampled sequence \( E\{|y_c(nT_b+\tau)|^2\} \). A block diagram of the system is shown in Figure 2.2. \( F_h \) and \( F_l \) are complex bandpass filters centered around the upper and lower band-edge

\(^2\)As discussed in Section 2.1.1.1, the number of samples that are actually read from the baseband sample buffer may change when wrap-around of the optimal sampling instant occurs. However, this should normally be an infrequent event that is of little relevance here.
frequencies $\pm r_b/2$, where $r_b$ is the signaling rate. The product of their outputs generates a signal containing a spectral line at $r_b$, whose phase is to be tracked. This signal is decimated to one sample per symbol, smoothed and normalized, and its imaginary part used to drive a loop filter that provides a timing correction. When used in a feedback configuration, the goal of the timing recovery loop is to drive to zero the error at the input of the loop filter of Figure 2.2. Further details may be found in Appendix A and in [4, 11]. This processing structure reflects the fact that, if the excess bandwidth in the spectrum of $h_c(t)$ is less than $r_b/2$, then the $T_b$-periodic BETR cost function has a single nonzero Fourier coefficient at frequency $r_b = 1/T_b$, in addition to the DC component [13].

In principle, the baud spectral line signal (BSL), i.e., the decimated product of band-edge filter outputs, could drive the loop filter directly, but then the feedback parameters would be dependent on the signal power, which is undesirable. As only the phase of the BSL is relevant, a more robust alternative proposed in [11] is to normalize it to unit magnitude prior to loop filtering. Straightforward generation of a unit-magnitude signal $U(n)$ from the BSL $r(n)$ as $U(n) = r(n)/|r(n)|$ requires square-root and division operations, which are rather computationally intensive. A more effective procedure proposed in [11] first passes $r(n)$ through a first-order lowpass filter to yield the smoothed BSL

$$R(n) = R(n-1) + k_2(r(n) - R(n-1)), \quad k_2 = 1/1024,$$

which is then approximately normalized by the iteration

$$U'(n) = \left(1 + k_3 \text{sgn} \text{Im}\{R(n)U^*(n-1)\}\right)U(n), \quad k_3 = 1/32,$$

$$U(n) = \left(1 - k_4(|U'(n)|^2 - 1)\right)U'(n), \quad k_4 = 1/16.$$

The normalized estimate of the optimal sampling instant updated by the loop filter, $\mu = \tau L/T_b$, is given by

$$\mu(n+1) = \mu(n) - K_p \text{Im}\{U(n)\}, \quad K_p = 5 \times 10^{-3},$$

where the proportional gain $K_p$ was chosen empirically. An integral term in (2.5) proved to be of no practical value, and was not included in the code to reduce the computational load.

### 2.1.1.1 Interpolator

Once the timing error is available, it may either be used to adjust the clock signal of the A/D converter, or generate synchronized samples using an interpolator. The latter
approach was chosen, as it allows a DMA controller to store the A/D samples in memory without DSP intervention. This leads to a purely discrete-time version of BETR, where the structure of figure 2.2 operates on a discrete sequence \( y(n) \) derived from the \( L \)-oversampled signal \( y_c(nT_b/L) \) by a piecewise-parabolic Farrow interpolator [2].

\[
y(n) = I(\tau, y_c(lT_b/L), l = n - 3, \ldots n).
\]

Specifically, define

\[
y_2(n) = y_c((n - 2)T_b/L) \\
y_{12}(n) = y_c((n - 1)T_b/L) - y_c((n - 2)T_b/L) \\
y_{103}(n) = (y_c((n - 1)T_b/L) - y_c(nT_b/L)) + (y_c((n - 1)T_b/L) - y_c((n - 3)T_b/L)) .
\]

Then

\[
y(n) = \left( (y_{12}(n) - y_{103}(n))\mu + y_{12}(n) + y_{103}(n) \right) \mu + 2y_2(n),
\]

where \( \mu = \tau L/T_b \). To minimize the number of products, this third-order FIR filter has a floating-point gain\(^3\) of 2, yielding \( y(n) = 2y_c((n - 2)T_b/L) \) for \( \mu = 0 \), \( y(n) = 2y_c((n - 1)T_b/L) \) for \( \mu = 1 \), and interpolating between these two samples for intermediate values of \( \mu \), such that

\[
y(n) \approx 2y_c((n - 2)T_b/L + \tau), \quad \tau \in [0, T_b/L].
\]

The time delay of about 2 samples between \( y_c(t) \) and \( y(n) \) is immaterial, as only the latter is processed at the receiver.

Whenever the value of \( \mu \) updated by (2.5) leaves the admissible interval \([0, 1]\), it must be aliased back to that range as follows:

- If \( \mu(n + 1) < 0 \), then \( \mu(n + 1) \leftarrow \mu(n + 1) + 1 \). Generate and buffer an extra sample of \( y \) by reinterpolating on the current Farrow filter internal samples.
- If \( \mu(n + 1) \geq 1 \), then \( \mu(n + 1) \leftarrow \mu(n + 1) - 1 \). The next time that a value of \( y \) is required, read two samples of \( y_c \) before interpolating (once).

### 2.1.1.2 Doppler Tracking Considerations

The wrap-around operations described above for \( \mu \) imply that the input sample buffer from the A/D converter is read slower (when \( \mu(n + 1) < 0 \)) or faster (when \( \mu(n + 1) \geq 1 \)) than nominally expected. Naturally, a buffer underflow or overflow will eventually occur for a sufficiently long data frame if the sampling clock is not adjusted. However, for typical design parameters this event can only happen when the sampling frequency offset is quite high, far exceeding the tracking ability of the timing recovery system.

In fact, BETR has limited tracking range and bandwidth due to the statistical nature of the underlying optimization criterion, and is most effective when used to compensate for

\(^3\)In the fixed-point implementation, however, the gain is unitary and \( y(n) \approx y_c((n - 2 + \mu)T_b/L) \). As mentioned in Section A.3 of Appendix A, this disparity is actually useful, as it compensates for a 6 dB difference between the fixed-point and floating-point gains of the band-edge filters.
small average misadjustments between the assumed and actual data rates due to Doppler or Tx/Rx clock offset. More sophisticated Doppler compensation systems were deemed unnecessary, as both the AUV and ASC will be stationary during acoustic transmissions. It seems unlikely that significant mean Doppler shifts would be observed even if the AUV and ASC where moving, since the tracking systems ensure that their relative speed is close to zero.

2.1.2 Equalization

Residual timing jitter and intersymbol interference are removed by a (linear) fractionally-spaced equalizer operating at 2 samples per symbol. The equalizer input vector, parameter vector and output are denoted by $y(n)$, $c(n)$ and $z(n) = c^T(n)y(n)$, respectively. In a variety of algorithms, the update recursion for $c(n)$ may be written as

$$c(n+1) = c(n) + \mu(n)y^*(n)e(n), \quad (2.11)$$

where $e(n)$ is a generalized error and $\mu(n)$ is a variable adaptation step. A fixed value for $\mu$ is difficult to select a priori because it depends on the channel characteristics, hence all the algorithms that were considered provide some means of dynamically adjusting the step. The recursion (2.11) is still valid for a decision-feedback equalizer (DFE) if $y(n)$ contains previous symbol decisions, in addition to the input samples $y(n)$. Although widely used in general-purpose underwater receivers to compensate for severe ISI, decision-feedback equalization proved to be of limited usefulness under the relatively benign acoustic channels linking the AUV and ASC. For that reason, this nonlinear structure was not built into the software.

Based on the analysis of experimental data gathered at the test site, three adaptation algorithms were considered to be sufficiently robust, yet simple, for inclusion in the real-time receiver:

**Soft-Constraint Satisfaction (SCS):** A blind algorithm closely related to the constant-modulus algorithm (CMA), but with enhanced convergence properties [20].

**Normalized Least Mean Squares (NLMS):** A reference-driven algorithm similar to LMS, where the step depends on the norm of the equalizer input vector [10].

**Adaptive step-size LMS (SLMS):** Another reference-driven algorithm where both the equalizer output MSE and the adaptation step are optimized at each iteration [10].

Some details of these algorithms will be given in the next section. Blind algorithms were examined mainly as a convenient way of recovering from short-term channel fades that may occur as waves induce rapid oscillations of the ASC. Although their ability to compensate for ISI in steady-state was usually satisfactory, the initial convergence was rather inconsistent, sometimes ranging from a few hundred symbols to several thousand in consecutive data frames. While the clarification of such behavior is an interesting research topic, blind equalization was considered too unreliable for the goals set forth in ASIMOV.
12 Signal Processing Structure

The SLMS algorithm is very popular in underwater equalization, as its convergence and tracking properties approach those of recursive least-squares algorithms (RLS) with much lower computational complexity [3]. In the ASIMOV high-speed acoustic link, however, ISI is sufficiently low for SLMS to outperform NLMS only marginally [8]. Having somewhat lower complexity, NLMS was therefore adopted as the equalization algorithm in the standard ASIMOV receiver.

2.1.2.1 Equalizer Order

The fractionally-space equalizer has 20 coefficients per subchannel, i.e., a total of 40 coefficients. In reference-driven algorithms there are 8 anticausal coefficients per subchannel, so that the coefficient and input vectors can be represented as

\[ \mathbf{c} = \begin{bmatrix} \mathbf{c}^{(0)} \\ \mathbf{c}^{(1)} \end{bmatrix}, \quad \mathbf{y}(n) = \begin{bmatrix} \mathbf{y}^{(0)}(n) \\ \mathbf{y}^{(1)}(n) \end{bmatrix}, \quad (2.12) \]

where, for any \( i = \{0, 1\} \),

\[ \mathbf{c}^{(i)} = \begin{bmatrix} c_{N_1}^{(i)} & \ldots & c_{-1}^{(i)} & c_{0}^{(i)} & \ldots & c_{N_2}^{(i)} \end{bmatrix}^T, \quad N_1 = -8, \ N_2 = 11, \quad (2.13) \]
\[ \mathbf{y}^{(i)}(n) = \begin{bmatrix} y^{(i)}(n - N_1) & \ldots & y^{(i)}(n + 1) & y^{(i)}(n) & \ldots & y^{(i)}(n - N_2) \end{bmatrix}^T. \quad (2.14) \]

The values of \( N_1 \) and \( N_2 \) were empirically chosen from analysis of data collected at the test site. The initial coefficient vector is arbitrarily chosen as an impulse in subchannel 0:

\[ c_k^{(i)} = \begin{cases} 1 & \text{if } i = 0, k = 0, \\ 0 & \text{otherwise}. \end{cases} \quad (2.15) \]

Generation of the stationary “polyphase components” \( y^{(i)}(n) \triangleq y(nL+i), i = \{0,1\}, L = 2 \) from the (decimated) cyclostationary interpolator output \( y(n) \) involves no computational overhead when operating in batch mode, as the latter outputs a 2-element vector with the desired format\(^4\) \[ [y(2n) \ y(2n+1)]^T = [y^{(0)}(n) \ y^{(1)}(n)]^T \] in every symbol interval.

As usual, the training sequence is delayed by a suitable number of symbols \( -N_1 \), so that the required anticausal input samples (with respect to the reference symbol) are available to the equalizer. As detailed in Section 4.1, initial coarse synchronization of data frames is achieved by transmitting a chirp preamble, whose presence can be detected by correlation with an uncertainty of about one symbol interval. Upon convergence, the equalizer will automatically synthesize any fine-scale delay correction that is necessary to precisely align the reference and received sequences.

In blind equalization the notion of causal and anticausal samples is meaningless, as no reference sequence exists. However, given coefficient and input vectors of the form (2.12) with

\[ \mathbf{c}^{(i)} = \begin{bmatrix} c_{0}^{(i)} & \ldots & c_{N_2-N_1}^{(i)} \end{bmatrix}^T, \quad \mathbf{y}^{(i)}(n) = \begin{bmatrix} y^{(i)}(n) & \ldots & y^{(i)}(n - (N_2 - N_1)) \end{bmatrix}^T, \quad (2.16) \]

\(^4\)Note that here \( y(n) \) denotes the interpolated sequence decimated to \( L = 2 \) samples per symbol.
the equalizer will hopefully remove the ISI and converge to an arbitrarily delayed and phase
rotated symbol \(a(n - d) \exp j\phi\). The delay \(d\) depends on the initial coefficient vector; As a
rule of thumb, if the ISI is not too severe and \(c\) has a single nonzero element \(c_d^{(i)}\), then \(c_d^{(i)}\)
will remain the strongest coefficient upon convergence and \(c^T y(n) \approx a(n - d) \exp j\phi\), as
long as \(d\) is not too close to the filter edges \(0, N_2 - N_1\). By using the initialization strategy
\[
\begin{cases}
  1 & \text{if } i = 0, k = -N_1, \\
  0 & \text{otherwise},
\end{cases}
\]
(2.17)
the equalizer output sequence in blind and reference-driven mode will have a similar delay
(usually it will differ by 2 or 3 symbols at most), which helps to accommodate both classes of
equalization algorithms more homogeneously. Note that (2.15) and (2.17) actually denote
identical coefficient vectors \(c\), as different time indices were adopted.

## 2.1.2.2 Blind Algorithms

The Constant Modulus Algorithm (CMA) is a popular blind equalization algorithm due to
its simplicity and robustness [10]. The parameter vector \(c\) is adjusted so that the output
magnitude approaches a desired value \(R\). As the CMA cost function does not depend
on the output phase, the algorithm can tolerate arbitrary carrier frequency offset and
significant phase jitter in the input signal.

The normalized CMA (NCMA) algorithm is derived using a deterministic criterion, by
minimizing \(\|c(n + 1) - c(n)\|\) such that the a posteriori output magnitude equals \(R\) [15].
The update recursion is very similar to conventional CMA, except that the squared norm
of the input vector is used as a normalization factor.

Soft-Constraint Satisfaction (SCS) is similar to NCMA, but a relaxation factor is
introduced when solving the deterministic constrained optimization problem [20]. This
modification reduces the number of undesirable fixed points of the adaptive algorithm for
which there is insufficient ISI cancellation. The parameters to be used in (2.11) are
\[
\begin{align*}
  \mu(n) &= \frac{1}{\|y(n)\|^2 + \delta_n} , \\
  e(n) &= \frac{z(n)}{1 - \bar{\mu} \left(1 - \frac{|z(n)|}{R}\right)} - z(n) ,
\end{align*}
\]
(2.18)
where \(\delta_n\) is a small bias term that prevents numerical overflow when \(\|y(n)\|^2\) is close
to zero and \(\bar{\mu}\) can, in principle, be chosen arbitrarily in the interval \(\bar{\mu} \in [0, 1]\) for stable
operation.

## 2.1.2.3 Reference-Driven Algorithms

In all reference-driven algorithms \(e(n)\) is the difference between the (phase-synchronized)
decision made by the slicer and the equalizer output. In NLMS the adaptation step is
given by
\[
\begin{align*}
  \mu(n) &= \frac{\bar{\mu}}{\|y(n)\|^2 + \delta_n} , \\
  \bar{\mu}_{TRN} &= 0.5 , \quad \bar{\mu}_{DD} = 0.1 ,
\end{align*}
\]
(2.20)
where $\delta_n$ has the same value of (2.18), and $\tilde{\mu} \in [0, 1]$ ensures stability when the channel is time invariant. The value $\tilde{\mu} = 0.5$ is used during training for fast convergence, whereas $\tilde{\mu} = 0.1$ is used in decision-directed mode to reduce constellation jitter and avoid equalizer divergence due to feedback of wrong decisions.

In addition to (2.11), SLMS uses the following two-step recursion to update the optimal step $g(n) = d^T(n) y(n)$ (2.21)

$$d(n + 1) = d(n) + y^*(n) [e(n) - \mu(n) g(n)]$$ (2.22)

$$\mu(n + 1) = [\mu(n) + \alpha \text{Re}[g(n) e^*(n)]]_{\mu_-}^{\mu_+} ,$$ (2.23)

where $\alpha$ is a small constant step that has little impact on performance and the bracket in (2.23) denotes clipping of $\mu(n + 1)$ to the interval $[\mu_- , \mu_+]$. The following numerical values were adopted

$$\mu(0) = 10^{-3} , \quad \mu_- = 10^{-5} , \quad \mu_+ = 10^{-2} , \quad \alpha = 2^{-15} \approx 3 \times 10^{-5} .$$ (2.24)

The auxiliary coefficient vector $d$ can be interpreted as the elementwise derivative of $c$ with respect to the current step $\mu$.

### 2.1.3 Carrier Recovery and Slicing

Carrier recovery is based on a simple and widely-used PLL algorithm [19, 13]. The instantaneous phase error is given by

$$e_\theta(n) = \text{Im}\{\hat{a}(n) \hat{a}^*(n)\} ,$$ (2.25)

where $\hat{a}(n) = z(n) \exp -j\theta(n)$ is the equalized symbol after phase synchronization. The corresponding slicer output $\tilde{a}(n) = D(\hat{a}(n))$ is based on a memoryless nearest-neighbor criterion denoted by the decision function $D(\cdot)$. The phase estimate is updated as

$$\theta(n + 1) = \theta(n) + G(z) e_\theta(n) ,$$ (2.26)

where $G(z) e_\theta(n)$ denotes filtering of the phase error sequence by a proportional-plus-integral (PI) loop filter

$$G(z) = G_p + \frac{G_i}{1 - \beta z^{-1}} , \quad G_p = 10^{-1} , \quad G_i = 1.7 \times 10^{-3} , \quad \beta = 1 - 1024^{-1} .$$ (2.27)

In reference-driven algorithms, the error term to be fed back to the equalizer is $e(n) = (\hat{a}(n) - \tilde{a}(n)) \exp j\theta(n)$, which prevents its coefficients from rotating to track common phase variations and thereby improves the overall stability.

### 2.1.3.1 Low-level Slicing for 4-PSK and 8-PSK

Computing the slicer output $\tilde{a}(n) = D(\hat{a}(n))$ for arbitrary constellations can be computationally intensive. To meet the real-time constraints of the receiver, specialized decision
functions were developed for the 4/8-PSK constellation shown in Figure 2.3. It should be stressed that the rules described in this section pertain to the low-level slicer shown in Figure 2.1. Higher-level mapping/unmapping functions, described in Chapter 3, may use Trellis-coded modulation or other coding strategies\(^5\) to reduce the output bit error rate.

The slicing process resembles the metric update procedure for Trellis-coded modulation described in Section 3.3.3. However, the numbering of constellation points in Figure 2.3 was chosen so that most of the code can be shared when slicing both 4-PSK and 8-PSK, and thus differs slightly from Figure 3.4, which follows [23]. The ordering is immaterial from the point of view of adapting data pump subsystems, and these specific indices were selected simply to streamline the code.

Given an arbitrary complex point \(x\), evaluating distances \(|x - y_i|^2\) with variable \(y_i = 4/8-\text{PSK}(i)\) is equivalent to computing the simpler inner product \(\text{Re}\{xy_i^*\}\) for decision purposes, since all points in the 4/8-PSK constellation have the same magnitude. The following algorithm is used:

1. Four reference points of the 8-PSK constellation are chosen,
   \[
y_0 = 1, \quad y_1 = j, \quad y_2 = \frac{1}{\sqrt{2}}(1 + j), \quad y_3 = \frac{1}{\sqrt{2}}(-1 + j).
   \]

2. The four inner products \(d(i) = \text{Re}\{xy_i^*\}, i = 0, \ldots, 3\) are calculated and stored. The index that maximizes the magnitude, \(i^* = \arg \max_i |d(i)|\), reveals that either \(y_{i^*}\) or \(-y_{i^*}\) is the constellation point closest to \(x\).

3. The phase error is \(\pm \text{Im}\{xy_{i^*}^*\}\), depending on whether \(y_{i^*}\) or \(-y_{i^*}\) is closest to \(x\). Since \(\text{Re}\{x(\pm y_{i^*})\}\) is necessarily positive for the nearest point, the appropriate phase error is simply obtained by computing \(\text{Im}\{xy_{i^*}^*\}\text{sgn} \{\text{Re}\{xy_{i^*}^*\}\}\).

4. Due to the symmetries of the constellation
   \[
d(i + 1) = \text{Re}\{ -jxy_i^*\} = \text{Im}\{xy_i^*\}, \quad i \in \{0, 2\},
   \]

\(^5\)As a special case, no coding or simple differential coding may be specified, as discussed in Section 3.3.2.
or, equivalently,

\[ d(i) = \text{Re}\{jx_{i+1}^a\} = -\text{Im}\{xy_{i+1}^*\}, \quad i \in \{0, 2\}. \]

Then, \( \text{Im}\{xy_{i+1}^*\} = (-1)^{i^*}d(i^* \oplus 1) \), where \( i^* \oplus 1 \) denotes an exclusive-or operation that complements the LSB of \( i^* \), swapping the indices 0 \( \leftrightarrow 1 \) or 2 \( \leftrightarrow 3 \). Hence, the table \( d(i) \) already contains all the required values to compute the phase error as

\[ e_{q}(n) = (-1)^{i^*}d(i^* \oplus 1)\text{sgn}(d(i^*)). \tag{2.28} \]

For 4-PSK constellations, only the first two points are considered in the previous algorithm. The choice between both constellations is made at compile time, as documented in Section 5.3.

**Interaction with high-level mapping:** The following points should be *carefully* observed when designing and selecting the mapping/unmapping functions to be used by the transmitter and receiver.

- Obviously, the low-level slicer should be compatible with high-level coding schemes, otherwise equalization and carrier recovery will fail, invalidating all subsequent processing as well. To ensure this, *any admissible coder must output a sequence of complex points that conform to the raw 2D constellation* of Figure 2.3. The same holds for the training sequence (and marker) mapping function.

- Ideally, no bits should ever come out of the low-level slicer, its complex output being used exclusively to adapt data-pump subsystems. However, this is strictly not true in the ASIMOV receiver because differential decoding is performed on these raw decisions to scan for start markers and the training sequence epilog, as discussed in Section 4.2. This *hard-coded* technique will only work if differential Gray-coded 4-PSK or 8-PSK is chosen as the training sequence mapping function at the transmitter. The correspondence between phase jumps and data bits is shown in Figure 3.5b,d. See Sections 4.3 and 5.3 for more details on transmitter and receiver parameterization.

### 2.2 Signal Processing at the Transmitter

As one would expect, the transmitter performs few signal processing functions. Most of its computational resources are devoted to frame formatting (Chapter 4) and coding (Chapter 6). The labelling of points is irrelevant, but the shape should be respected.

7 In data pump functions, the choice between differential 4-PSK and 8-PSK is made at compile time. See also Section 5.3.

8 For 8-PSK, the lookup table \( \{0, 3, 1, 2, 6, 5, 7, 4\} \) is used to map the internal indices of Figure 2.3 into bit triplets. The same table can be used for 4-PSK by discarding the LSB of each entry.
2.2 Signal Processing at the Transmitter

3) to generate a sequence of complex (2D) constellation points\(^9\) \(a(n)\) from the input bit stream \(b(n)\). Similarly to (2.1), the transmitted PAM signal may be represented as

\[
x_c(t) = \sum_{k=-\infty}^{\infty} a(k)p_c(t - kT_b) ,
\]

(2.29)

where \(p_c(t)\) denotes the pulse shape. Using specialized hardware, the continuous-time passband signal \(\hat{x}_c(t) = \text{Re}\{x_c(t) e^{j\omega_c t}\}\) may be generated from \(a(n)\) by direct digital synthesis (DDS), usually assuming rectangular \(p_c(t)\). Alternatively, an \(L\)-oversampled discrete-time version of (2.29) is first constructed,

\[
x(n) = \sum_{k=-\infty}^{\infty} a(k)p(n - kL) ,
\]

(2.30)

and \(x_c(t)\) is obtained at the output of a suitable reconstruction filter\(^{10}\). It is often advantageous to view (2.30) as the output of a single-input multiple-output (SIMO) system

\[
x(n) = p(n) \ast a(n) , \quad x(n) = \begin{bmatrix} x(nL) \\ \vdots \\ x(nL + L - 1) \end{bmatrix} , \quad p(n) = \begin{bmatrix} p(nL) \\ \vdots \\ p(nL + L - 1) \end{bmatrix} .
\]

(2.31)

As shown by (2.31), a SIMO filter built from the \(L\) baud-rate sampled “polyphase components” of \(p(n)\) efficiently computes a vector of samples in response to every input \(a(n)\). These \(L\) samples are simply queued to the D/A converter sequentially to generate the desired \(x_c(t)\). The ASIMOV transmitter may be configured (at compile time) to generate \(x_c(t)\) either by DDS, or according to (2.31) (Section 5.3). In the latter case, both rectangular and raised-cosine pulse shapes can be specified. Switching between the two types was not considered to be very relevant, so the call to create an appropriate SIMO filter was hard-coded into the modem startup function \texttt{ModemInit}. Currently, the following discrete-time raised-cosine pulse is used\(^{13}\)

\[
p(n) = \frac{\sin \pi n/L}{\pi n/L} \cdot \frac{\cos \beta \pi n/L}{1 - (2\beta n/L)^2} , \quad \beta = 1 ,
\]

(2.32)
Chapter 3

Coding

On-site measurements indicate that noise levels in the vicinity of hydrothermal vents can be two orders of magnitude higher than those commonly found in the open North Atlantic ocean up to hundreds of KHz [22]. Although experiments have shown that linear (feedforward) equalizers are able to converge under these conditions, an outer coding layer was included to reduce the error rate in the output bit stream.

The choice of a coding strategy must take into account the fact that bandwidth is very tight to meet the desired user bit rate of 30 kbit/s. Additionally, M-PSK constellations must be used due to hardware restrictions, which does not allow the available transmit power to be exploited in the most efficient way. Without coding, it seems unreasonable to expect acceptable error rates from constellations denser than 4-PSK, and the minimum required symbol rate would therefore be 15 kbaud. Considering the inefficiencies that are associated with the generation and filtering of waveforms, attaining this symbol rate would take up virtually all the available bandwidth.

Direct coding of the bit stream using convolutional coding or other forward error correcting techniques reduces the effective bandwidth available to the user. The constellation would then have to be expanded to 8-PSK at least to compensate for the added redundancy, as it is not possible to increase the symbol rate. This option was not pursued because it was not clear whether the inevitable increase in raw symbol errors would offset the coding gain. A better alternative is provided by trellis-coded modulation, which directly operates at the constellation level and introduces coding gain through temporal diversity without increasing neither the transmitted power nor the bandwidth [21]. This is achieved by expanding the signal set to compensate for the added redundancy but, unlike the coding scheme mentioned above, it is done in a way that guarantees a net performance gain relative to the uncoded case.

3.1 Coder and decoder structure

Using a rate $n/n + 1$ convolutional code, the TCM encoder maps blocks of $n$ bits into one of $2^{n+1}$ constellation points. In the ASIMOV high-speed link $n = 4$ is used, and each coded symbol is formed by two pairs of complex points in an $(8$-PSK$) \times (8$-PSK$)$
4D constellation. When interleaving is used, the equivalent channel at the receiver after equalization is assumed to be memoryless, and decoding of trajectories across constellation points can be done using the Viterbi algorithm with an Euclidean distance metric based on the known signal Trellis. Fig. 3.1 shows the general structure of the coder and decoder.

3.2 Interleaving

Convolutional interleaving/deinterleaving is used at the transmitter and receiver to simplify the software implementation [18]. In this kind of structure the complex values generated at the transmitter are sequentially shifted into a bank of $I$ parallel shift registers. Each successive register provides one more storage unit than the preceding one. The zeroth register provides no storage, i.e., it is a simple feedthrough connection. With each new value the commutator switches to a new register and the corresponding value is shifted in, while while the oldest value in that register is shifted out to the modulator. After the $(I-1)$-th register the commutator returns to the zeroth register and starts again. The deinterleaver performs the inverse operation, and the input-output commutators for both interleaving and deinterleaving must be synchronized.

Figure 3.2 illustrates an example of convolutional interleaving and deinterleaving for $I = 4$. Figure 3.2a shows values 1 to 4 being loaded; Each ‘×’ represents an unknown value that depends on the initial shift register internal states. Figure 3.2b shows some of the previous values shifted within the registers and the entry of values 5 to 8 to the interleaver input. Figure 3.2c shows values 9 to 12 entering the interleaver. At this time the deinterleaver is now filled, but nothing useful is being fed to the decoder yet. Finally, Figure 3.2d shows values 13 to 16 entering the interleaver, and at the output of the deinterleaver values 1 to 4 are being passed to the decoder. The process continues in this way until the entire sequence, in its original preinterleaved form, is presented to the decoder.

The performance of a convolutional interleaver is very similar to that of a block interleaver. An important advantage of convolutional interleaving is that the end-to-end delay is $I(I-1)$, and the memory required is $I(I-1)/2$ at both ends of the channel, which is half
the corresponding requirements for block interleaving. Moreover, the processing structure of Figure 3.2 lends itself to simple and elegant software designs. In keeping with the batch processing philosophy introduced in Section 2.1, synchronized commutators do not exist in the actual software implementation of the ASIMOV high-speed link. Instead, the interleaver receives and delivers $I$-element complex vectors with the original and interleaved values for all the shift registers, and the deinterleaver behaves analogously.

Similarly to the startup phase shown in Figure 3.2, dummy ('$\times$') values should continue to be fed to the interleaver after the end of a data block until the delay lines of the deinterleaver have been fully emptied. During both periods, a decision was made not to transmit these spurious values to avoid wasting bandwidth. This implies that the effective interleaving depth is not constant across a full data block, but rather increases from 1 to $I$ during the startup phase, remains at $I$ in steady state, and then decreases from $I$ to 1 in the block epilog. This results in weaker protection against error bursts during transient

---

Figure 3.2: Illustration of convolutional interleaving and deinterleaving with depth $I = 4$
3.3 Rotationally-Invariant TCM for M-PSK Constellations

Frequently, rapid phase variations in the received signal induce rotations of the input constellation that exceed the tracking rate of carrier recovery systems. This problem is particularly relevant in the present context, where clusters of points in the expanded constellation can have significant overlap due to noise. Depending on the phase symmetries of the constellation, arbitrary undetectable rotations may occur before carrier lock is regained. While a degradation in equalizer output is clearly visible during these events, the adaptation algorithms can usually recover spontaneously and numerical divergence seldom occurs. This problem is routinely addressed through differential coding of symbols, but that technique is ineffective in most TCM schemes because arbitrarily-rotated trajectories through the expanded constellation do not usually belong to the signal Trellis [21].

Some of the most popular TCM codes for 2D QAM constellations are indeed invariant to 90° rotations, and systematic methodologies for constructing these codes have been outlined in the technical literature. However, their generalization to more than four phase ambiguities, as found in 8-PSK or 16-PSK, proved to be challenging. During the study of QAM schemes it was verified that rotational invariance was easier to obtain with multidimensional constellations, the reason being that some phase ambiguities may be removed by proper partitioning of the constellation. This property carries over to multidimensional M-PSK, and forms the basis of the rotationally-invariant TCM schemes developed in [23]. A multidimensional symbol can be sent over a two-dimensional channel (i.e., complex single-input single-output) by sending two of its components at a time.

Specific four- and eight-dimensional schemes using Trellis encoders with up to 64 states are described in [23] for both 8-PSK and 16-PSK. These provide coding gains ranging from 3.01 to 5.13 dB. Note that the equalizer cannot tolerate the large decoding delay of the Viterbi algorithm, and in decision-directed mode its adaptation is based on hard decisions provided by a memoryless slicer described in Section 2.1.3.1. This commonly-used approach essentially excludes 16-PSK TCM schemes, as the raw equalizer decisions, which do not benefit from any coding gain, would be too unreliable and lead to frequent divergence of the adaptation algorithms. Even if the equalizer does not diverge, occasional incorrect raw decisions will lead to a transient increase in its output MSE, thereby indi-

\footnote{It is easy to convince oneself that the insertion and deletion of dummy values during startup/epilog does not alter this.}
rectly affecting the performance of the Viterbi decoder. This prevents the full predicted TCM coding gain to be effectively realized in practical systems that incorporate adaptive subsystems such as channel equalizers.

### 3.3.1 TCM Encoder

Due to complexity constraints, one of the simplest schemes with underlying 8-PSK constellation was adopted for the ASIMOV high-speed link. Figure 3.3 shows the structure of the 4-state encoder and the Trellis diagram, with an associated coding gain of 3.01 dB. The state LSB is \( Y_0 \) and the MSB is the output of the left delay block in Figure 3.3a. Branches are labeled as input/output = \( Y_1 / Y_0 \). Note that \( Y_2, \ldots, Y_4 \) do not enter the state machine and have no influence on state transitions, therefore each branch in the Trellis diagram should actually be interpreted as a super-branch formed by 8 parallel branches for all possible values of these variables.

The encoder processes blocks of 4 consecutive bits \( b(4n), \ldots, b(4n+3) \), and outputs 4D complex symbols defined by pairs of points in an 8-PSK constellation. Prior to constellation mapping, bits \( b(4n), \ldots, b(4n+2) \) are differentially encoded according to

\[
\{Y_3 Y_2 Y_1\}(n) = (\{Y_3 Y_2 Y_1\}(n-1) + \{b(4n+2)b(4n+1)b(4n)\})_8, \quad (3.1)
\]

where \((::)_8\) indicates that the addition is performed modulo 8. Table 3.1 shows the pairs of 8-PSK points generated by the 4D constellation mapper of Figure 3.3. The 3-bit values given in the table should be interpreted as indices into the 8-PSK constellation shown in Figure 3.4.

The constellation may also be viewed as the superposition of the following 4D 2-PSK subconstellations \( S_0, \ldots, S_7 \)

\[
\begin{align*}
S_0 &: \{(0,4), (0,4)\} & S_4 &: \{(2,6), (2,6)\} \\
S_1 &: \{(1,5), (3,7)\} & S_5 &: \{(3,7), (1,5)\} \\
S_2 &: \{(2,6), (0,4)\} & S_6 &: \{(0,4), (2,6)\} \\
S_3 &: \{(1,5), (1,5)\} & S_7 &: \{(3,7), (3,7)\}
\end{align*}
\]
3.3 Rotationally-Invariant TCM for M-PSK Constellations

Table 3.1: 4D 8-PSK Constellation Mapping

<table>
<thead>
<tr>
<th>$Y_4Y_3Y_2Y_1Y_0$</th>
<th>Pt #1</th>
<th>Pt #2</th>
<th>$S_i$</th>
<th>$Y_4Y_3Y_2Y_1Y_0$</th>
<th>Pt #1</th>
<th>Pt #2</th>
<th>$S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$S_0(0)$</td>
<td>16</td>
<td>0</td>
<td>4</td>
<td>$S_0(2)$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>$S_1(0)$</td>
<td>17</td>
<td>1</td>
<td>7</td>
<td>$S_1(2)$</td>
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<td>3</td>
<td>$S_7(1)$</td>
</tr>
</tbody>
</table>

Figure 3.4: 8-PSK constellation for each half-symbol in rotationally-invariant TCM

where (Pt #1, Pt #2) indicates a pair of complex values that jointly define a 4D symbol, and {·,·} denotes the points that comprise a 2D 2-PSK subconstellation. Table 3.1 indicates the assignment of each 4D symbol to one of the $S_i$.

3.3.1.1 16-state TCM:

In addition to the 4-state TCM scheme described above, support was added at the transmitter for the 16-state coder described in Figure 6c of [23], which can provide a coding gain of 4.13 dB (see also Sections 4.3.1.1 and 5.3.1). Decoding requires a Viterbi algorithm that is significantly more complex than the one described in Section 3.3.3. The unmapping function was not developed, as it was considered to be too demanding for the available hardware, both in terms of memory and computational power.
3.3.2 Link operation with plain 4-PSK and 8-PSK

By proper choice of firmware, the ASIMOV high-speed link can also use plain 4-PSK, optionally with differential encoding. The encoder processes 2-bit blocks to produce a single complex point in the 4-PSK constellation depicted. As this simplified encoder is triggered at twice the rate of the TCM encoder of Figure 3.3, the overall rate at which input bits are read and output symbols are generated remains constant in both cases.

Figure 3.5a-b shows the signal constellations for plain and differentially-coded 4-PSK. In the latter case, each point should be interpreted as a phase jump relative to the previously transmitted complex symbol. As usual, Gray coding is used to minimize the bit error probability when symbol decoding errors occur between adjacent constellation points.

Fewer software changes are needed if the same constellation shape is used regardless of whether the input to the slicer is obtained directly from the equalizer output, after carrier synchronization, or by multiplying it with the conjugate of the previous decision in differential mode. This explains the 45-degree rotation of Figure 3.5a relative to standard 4-QAM/4-PSK, so that it coincides with Figure 3.5b up to a permutation of points.

Depending on the options used for compiling the transmitter and receiver, plain and differentially-encoded 8-PSK can also be specified, using the constellations shown in Figures 3.5c-d. Naturally, this will result in higher input-output bit rates if the triggering
rates given earlier for the various transmitter and receiver blocks are kept.

Similarly to 4-PSK, in terms of code compactness there are advantages in choosing identical uncoded and coded constellations, up to a permutation. In practice, differentially-Gray-encoded 8-PSK is generated by taking 3-bit blocks, using them as an index into the 3-bit permutation table \( \{0, 1, 3, 2, 7, 6, 4, 5\} \), and performing the mod-8 sum \((3.1)\) on the resulting value prior to constellation mapping according to Figures 3.4/3.5c. These two variants of 8-PSK were not used in ASIMOV because the noise levels at the test site would most likely lead to an unacceptably high error rate.

Finally, note that the description of PSK decoding in this Section pertains to high-level unmapping as an alternative to TCM. It should not be confused with the low-level slicer shown in Figure 2.1 and described in Section 2.1.3.1, which is hard-coded in the data pump for equalizer and carrier recovery loop adaptation.

### 3.3.3 TCM Decoder

Due to the presence of a convolutional encoder in Figure 3.3a, the generated sequence of 4D symbols traces a path through a predetermined Trellis (Figure 3.3b). Given the full set of noise-corrupted received symbols, the Viterbi algorithm can therefore be used to find the most likely sequence. Unlike the demodulation of convolutional codes based on Hamming distances, here the branch metrics are Euclidean distances that measure the disparity in the complex plane between an observed value and an arbitrary constellation point.

Given a pair of observed complex values \( x = (x_1, x_2) \), one could compute the Euclidean distance to any of the 32 4D points in Table 3.1 and thereby obtain the branch metrics. This, however, would be quite wasteful because the required computations are highly redundant. As the problem is separable, that is, Euclidean distances may be independently determined in each complex (2D) dimension and accumulated, a better alternative is to compute the distance between both complex values and a selected subset of the 8-PSK constellation of Figure 3.4, and from that regenerate all the required metrics.

Similarly to the memoryless slicing procedure described in Section 2.1.3.1 for 4/8-PSK, inner products \( \text{Re}\{y_i^HX_k\} \) are computed instead of distances \(^2-|x_k - y_i|^2 \) with constant norm \( y_i \). The computation of branch metrics proceeds hierarchically as follows:

1. Compute the inner product \( d(x_k, y_i) = \text{Re}\{y_i^HX_k\} \) for points 0, 1, 2, 3 of the 8-PSK constellation of Figure 3.4, \( y_i = 8-\text{PSK}(i) \).

2. Use these values to accumulate the (pseudo-)distance between \( x \) and the subconstel-

\(^2\)The minus sign is introduced for convenience, and the Viterbi algorithm should therefore select the path that maximizes the sum of these modified branch metrics.
lations $S_0, \ldots S_7$ of Table 3.1

$$d(x_1, y_0) \rightarrow S_0, S_6 \quad d(x_2, y_0) \rightarrow S_0, S_2$$

$$d(x_1, y_1) \rightarrow S_1, S_3 \quad d(x_2, y_1) \rightarrow S_3, S_5$$

$$d(x_1, y_2) \rightarrow S_2, S_4 \quad d(x_2, y_2) \rightarrow S_4, S_6$$

$$d(x_1, y_3) \rightarrow S_5, S_7 \quad d(x_2, y_3) \rightarrow S_1, S_1$$

As each 2D subconstellation of $S_i$ is 2-PSK, computing maximum pseudo-distances is trivial. If $d(x_k, y_i) > 0$ then $y_i$ is the closest point in the 2D constellation, otherwise the symmetrical $-y_i$ is closest and the correct metric is $d(x_k, -y_i) = -d(x_k, y_i)$.

3. Compute the pseudo-distance between $x$ and the 4 subconstellations

$$R_0 = S_0 \cup S_4, \quad R_1 = S_1 \cup S_5, \quad R_2 = S_2 \cup S_6, \quad R_3 = S_3 \cup S_7.$$  

For each $R_i$ this simply involves comparing the metrics for $S_i$ and $S_i+4$ and selecting the largest.

As pointed out in [23], the value of $Y_1Y_0$ at the encoder determines the subconstellation $R_i$ from which a 4D point will be transmitted, and this is the mechanism that introduces memory and provides coding gain. At this point, metrics have therefore been computed for the super-branches of Figure 3.3b. As usual, these are now added to the path metrics, a single survivor is selected for each state, and backtracking information is stored.

Pure maximum-likelihood sequence detection for long data packets is not really practical, as it involves a large decoding delay and requires significant storage for path backtracking. To circumvent these problems, the ASIMOV TCM decoder uses a simplified Viterbi algorithm where path metrics are recursively propagated across the whole packet, but backtracking information is only kept for a block-sliding window containing the most recent symbols. This technique reduces the memory footprint of the algorithm at the expense of a small penalty in output error rate. A block of (oldest) bits is extracted when the decoding window fills up, and the latter is then shifted in time, as detailed below:

1. The upper and lower time indices of the decoding window are denoted by $W_+$ and $W_-$, respectively. At the start of the decoding process they coincide with the initial time reference, which can arbitrarily be set to 0.

2. The upper index $W_+$ is incremented by one whenever the TCM decoder is triggered, and therefore coincides with the most recent time reference, $n$.

3. When the window length $W_+ - W_-$ reaches a prechosen maximum value $W$ (i.e., the decoding window is full) a block of $B$ symbols is decoded, and $W_-$ is advanced accordingly. These are the $B$ oldest symbols for which backtracking information is stored.

See also the discussion regarding mapper/unmapper reset in Section 4.2.1.
Chapter 4

Data Formatting

Data sent through any communication channel must be formatted so that it can be regen-
erated by a suitably synchronized receiver. While a layered protocol stack such as the ISO
model provides several synchronization mechanisms at different layers, this report only
covers those implemented at the physical level in the ASIMOV high-speed link. The next
sections describe in detail the format and design parameters of data packets.

4.1 Frame Structure

Data frames are sent from the transmitter to the receiver, formatted according to figure 4.1.

| Frame: Preamble >> Pre-silence >> Training sequence >> Data >> Post-silence |
| Training Sequence: Plain M-PSK symbols generated by LFSR |
| Data: (<repeat> Start Marker >> Interleaving matrix) >> Stop Marker |
| Interleaving matrix: <repeat> Interleaved vector (truncated during initial and fi-
  nal transients) |
| Interleaved vector: <repeat> Symbol (complex or higher-dimensional) |
| Symbol: Complex dimension 1 >> Complex dimension 2 >> . . . |

Figure 4.1: Frame format

Preamble: The preamble, typically a chirp or 2-PSK Barker code [17], is used for frame
detection and coarse synchronization of the receiver. Chirps were adopted in the ASIMOV
high-speed link, as their performance was found to be more robust than that of Barker
codes (with up to 13 chips). The baseband transmitted signal is given by

\[ x(t) = A \exp j\left(\omega_1 + \frac{\Delta \omega}{2T} t\right) t, \quad t \in [0, T], \quad (4.1) \]
where $\omega_0/2\pi = -7.5$ kHz, $\Delta\omega/2\pi = 15$ kHz, and $T \approx 10.7$ ms (or 640 sample intervals at a sampling frequency of 60 kHz). By differentiating the phase of (4.1), the instantaneous frequency of this signal is seen to increase linearly from $-7.5$ kHz to 7.5 kHz relative to the carrier frequency, thus covering the full signal bandwidth of 15 kHz. The amplitude $A$ coincides with that of the main data waveform in the frame. Detection of chirp preambles is based on a mixed energy and correlation criterion as follows.

- After baseband conversion and AGC, the squared magnitudes of the received samples are applied to an AR filter with a single pole at $z = 0.95$. Its (real) output provides a short-term estimate of the received signal power.

- Concurrently, the real and imaginary parts are quantized to 1 bit (0/1) and cross-correlated with a similarly quantized local replica of the desired chirp signal. This process is carried out efficiently by the same functions that are used for marker and training sequence scanning in the frame.

Suitable thresholds are defined for these variables, and a preamble is detected when both are exceeded.

**Pre- and Post-Silence:** The purpose of these signal gaps is to wait for echoes to die-out after signal transmission, thus improving the robustness of an ensuing detection process (preamble or training sequence scanning). At a sampling frequency of 60 kHz, the pre- and post-silence periods last for 300 and 100 sampling intervals, respectively.

**Training Sequence:** Similarly to the V.34 protocol, binary training data is obtained by applying a sequence of 1’s to a linear-feedback shift register (LFSR, or scrambler) whose initial state is initialized with 0’s. The constellation mapping function that converts these bits to complex symbols need not be the same one used for data. This allows, e.g., plain 4-QAM to be used during training and more complex signal mapping (differential, TCM) during actual data transmission.

The scrambler which is used for other purposes besides training, operates as follows. Given an input bit at time $n$, $u(n)$, the scrambled bit is computed as

$$s(n) = u(n) \oplus s(n-1) \oplus s(n-15),$$

(4.2)

where $\oplus$ denotes modulo-2 addition (xor). Whenever necessary, scrambling is inverted by

$$u(n) = s(n) \oplus s(n-1) \oplus s(n-15).$$

(4.3)

The feedback connection pattern ensures that a maximal-length sequence (or M-sequence) is generated, with period $2^{15} - 1$.

4.1.1 Frame data

As depicted in Figure 4.1, the data portion of a frame is obtained by concatenating start markers and interleaving matrices, followed by a single stop marker.
4.1 Frame Structure

4.1.1.1 Markers and data scrambling

Start and stop markers are generated by the same scrambler used for training, by forcing suitable initial states defined in Section 4.3. These seeds are chosen such that the cross correlation between markers and the training sequence is weak for typical lengths. An equivalent scrambler (i.e., same order and tap coefficients) is applied to the input data to break long sequences of identical bits that might have a negative impact on symbol/carrier recovery and equalization. The initial state of this scrambler is reset to 0 at the start of each interleaving matrix.

4.1.1.2 Interleaving matrices

An interleaving matrix simply designates the concatenation of several interleaver output vectors, i.e., sets of complex values sequentially read from the delay lines of Figure 3.2 for each time instant.

To limit the propagation of errors when large interleaving depths are used, the delay line contents are discarded at the start of each matrix, making them self-contained. As discussed in Section 3.2, transient effects thus occur at the beginning and end of each matrix, as the delay lines are being filled and then emptied. Specifically, suppose that a depth-$I$ convolutional interleaver should generate a total of $VI$ complex values per matrix. Then the transmitter should process $V + I - 1$ vectors per matrix as follows:

- Let $n \geq 1$ denote the time index, and $v(n)$ the corresponding interleaver output vector. For $n = 1, \ldots, I - 1$ only the first $n$ elements of $v(n)$ contain symbols that should be transmitted. The remaining ones are invalid and should be discarded.
- For $n = I, \ldots, V$ the full vector $v(n)$ should be transmitted.
- For $n > V$ the interleaver should be fed with arbitrary (non-data) symbols. Only the last $V + I - n$ components of $v(n)$ contain valid symbols to be transmitted.

As expected, the total number of transmitted complex values is

$$2 \sum_{n=1}^{I-1} n + (V - I + 1)I = 2 \frac{(I - 1)I}{2} + (V - I + 1)I = VI.$$  \hfill (4.4)

The receiver processes the $V + I - 1$ vectors $v(n)$ according to:

- For each $n$, the transmitted components of $v(n)$ are demodulated and input to the convolutional deinterleaver. Arbitrary values are used for the discarded components.
- As depicted in Figure 3.2, all deinterleaver outputs are invalid while $n < I$. For $n = I, \ldots, V + I - 1$ full vectors are available at the deinterleaver output, yielding a total of $VI$ complex values.
4.1.1.3 Symbol mapping

As discussed in Section 3.3.1, symbols are 4D (complex pairs) when rotationally-invariant TCM is used, and 2D (complex scalars) in plain M-PSK. To simplify the software implementation, it is convenient for the interleaver/deinterleaver to always operate on plain complex numbers regardless of the underlying constellation dimensionality. Moreover, its input and output vectors should contain an integral number of symbols. For that reason, its depth is not directly specified, but rather set by the product of two configuration parameters: the number of constellation complex dimensions and the interleaving depth per constellation complex dimension.

Unmapping Delay When demodulating TCM symbols there is an initial latency while filling the Trellis window, as described in Section 3.3.3. Although bits are extracted from the Trellis on a block basis, the unmapping function stores these values in an internal bit buffer so that they can be conveyed to the caller one symbol interval at a time. The unmapping buffer’s smoothing action ensures that, after the initial latency, a suitable number of bits\(^1\) is available from the unmapping function as long as it is invoked once per symbol interval. This issue does not arise in plain M-PSK, where unmapping is memoryless.

Due to the unmapping delay, the trailing (deinterleaved) symbols of each matrix are only retrieved at the start of the next one. Because interleaving matrix lengths are chosen such that the Trellis window fills up at the start of all matrices other than the first one\(^2\), it would seem easy to design transmit/receive functions that could handle this delay with little code overhead. However, a design choice was taken to handle all matrices identically, and in a self-contained way. The transmitter thus generates dummy bits while feeding the mapper to generate the final \(W\) symbols in a matrix, where \(W\) denotes the unmapping delay, so that a full\(^3\) Trellis window supports the detection of any true data bit. These final bits are decoded but discarded at the start of the next frame, which introduces some inefficiency in the communication process. This was deemed acceptable for the operating parameters of interest.

As an alternative to having dummy trailer bits while keeping interleaving matrices strictly isolated, one could envision feeding the unmapping function with arbitrary symbols (ideally, as many as the unmapping delay) after processing all the interleaving vectors to extract the remaining data bits. This strategy was not adopted, as it would lead to unreliable decisions and corrupt the Trellis, which should be reused undisturbed when demodulating the next interleaving matrix\(^4\).

\(^1\)\(\log_2\) of the number of points in the constellation (cardinality).
\(^2\)See the discussion on mapper/unmapper reset in Section 4.2.1.
\(^3\)This is only approximately true when the block decoding strategy of Section 3.3.3 is used.
\(^4\)Arguably, this violates the principle of self-containment in interleaving matrices. See the discussion on mapper/unmapper reset in Section 4.2.1 for justification of this procedure.
4.1 Frame Structure

4.1.1.4 Character and bit stuffing

At the transmitter, input bytes are stored in a character buffer, typically filled by a serial port, and moved to a bit buffer as needed. The bit buffer provides the actual data for frame generation, and is filled on demand either by the character buffer or by internal bit stuffing functions. Stuffing amounts to insertion of one or more 8-bit default character codes (0xC1) into that buffer, to satisfy a read request from the frame generating function. The following algorithm is used:

1. If the buffer contains enough bits to satisfy the request, return the appropriate quantity.

2. If not, tentatively read one more characters from the character buffer. If none is available, insert a default code and return to step 1.

3. Otherwise, check if (a stuffed version of) that character still fits on the current interleaving matrix. If not, insert a default code and return to step 1.

4. Read the character, stuff it as appropriate, insert it into the bit buffer, and return to step 1.

Stuffing of an input character $c$ is required when it is similar to the default code. In that case, it is replaced by an expansion code and additional information for unstuffing, according to the following rules:

- The expansion code equals the default code with MSB set to 0.
- If $c(MSB-1)\ldots c(0)$ equals the corresponding bits in the default and expansion codes, then stuffing is required.
- In that case, $c$ is augmented by appending a 0 between $c(MSB)$ and $c(MSB-1)$. The resulting bit pattern equals the expansion code, concatenated by the original $c(MSB)$.

A similar two-buffer architecture is used at the receiver, where the character buffer is typically associated with an output serial port. Bits extracted from the incoming frame are stored in a bit buffer, and moved to a character buffer whenever enough data are available to regenerate one output byte, as described below:

1. Insert the bits from the frame scanning function into the bit buffer.

2. While the bit buffer contains at least a plain character’s worth of bits, examine the pattern. If it is a default code, remove it from the buffer and discard it.

3. If not, check whether it is an expansion code. If so, but not enough bits are available, return.

4. Read the required number of bits, unstuff if appropriate, write the result into the character buffer, and go back to step 2.
The bit buffers at the transmitter and receiver are emptied at the start of each interleaving matrix, making them self-contained. This effectively resynchronizes character boundaries following each start marker, which is a desirable feature in practice. Depending on the type of data being sent, it may be appropriate to use the contents of the current interleaving matrix, even if character boundary loss in the previous one led to extensive data corruption.

Packetization and Data/Command Modes: If specified at compile time, more complex bit and character management mechanisms may be used. In that operating mode, the bit stream is broken into a sequence of packets. Each packet starts with a synchronization flag, followed by the bits corresponding to an integer number of characters\(^5\). Bit stuffing is used to ensure that the start flag occurs only at the beginning of a packet. Should the receive modem lose synchronization of character boundaries, it may recover once the next start flag is detected. This may provide finer-grained synchronization than the one already available from start markers between interleaving matrices, but involves a loss in data throughput.

The maximum number of bits per packet, including the flag, is configurable. If the maximum packet size cannot contain an integer number of characters, including stuffing bits, then the packet will be (slightly) abbreviated. If not enough input data exist to fill the current packet, or the remaining number of data bits in the interleaving matrix cannot hold a full packet, then the packet will be truncated. When the input character buffer is empty (or the packet size is so small that it cannot hold one character plus the start flag), then packet start flags ('empty' packets) will be generated until the required number of symbols per matrix is attained.

Transmitter: All characters received through the serial port are stored in an input character buffer, which should be large enough to absorb all processing delays that are inherent in the operation of the modem. The transmitter may operate in data mode or command mode, and the character buffer is processed differently in both cases. In data mode, characters are removed from the character buffer and placed in a transmit bit buffer as they are needed to fill each packet. The bit buffer may also be filled by packet start flags and stuffing bits. Since it is filled 'on demand', the bit buffer need not be very large, but must be efficiently accessed.

In command mode the input characters are interpreted as configuration commands and parameters. Toggling between the two modes is achieved by transmitting a special sequence of characters known as a break flag (typically ++++) that may be redefined as appropriate. Input characters are processed sequentially, so that all commands must be interpreted before starting to transmit data frames and vice-versa. This means that data characters before and after a sequence of commands are completely isolated, and will never be included in the same frame. This may induce some inefficiency by allowing the last

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\(^5\)The number of bits per character is configurable, so ASCII or binary data may be handled simply and efficiently.
frame before a command sequence to be padded with empty packets instead of useful data. However, this limitation is deemed acceptable because (i) command sequences are not frequently used, and (ii) modem management procedures are thus greatly simplified.

**Receiver:** At this level, receiver operations mirror those of the transmitter. The incoming bit stream is scanned for packet start flags, destuffed, and sent to a bit buffer if the flag is not detected. The bit buffer size need only be slightly larger than the number of bits that make up a character. As soon as an appropriate number of bits is available from the bit buffer, a character is removed from it and stored in the output character buffer. To maintain synchronization of character boundaries, the bit buffer is reset when a packet start flag is detected. The output character buffer should be large enough to smooth out the latencies involved in sending data through the serial port.

Similarly to the transmitter, an input character buffer accepts commands and data, toggling between both modes when break flags are detected. When in data mode the receiver simply discards the contents of this buffer and resumes processing of incoming frames.

### 4.2 Layered Processing at the Receiver

The transmitter software generates frames according to the model of Figure 4.1 by a hierarchy of functions with no side effects, which hopefully makes the code relatively easy to understand and maintain. A similar software structure would be desirable for the receiver, but some compromise must be made between code modularity and the need to provide global resynchronization mechanisms upon detection of start markers in the incoming symbol stream. This section discusses some of the issues involved.

**Preamble Scanning:** The receiver continuously tries to detect and decode incoming frames. It may be convenient to interrupt it, e.g., when operating in command/data mode and new configuration parameters are received in the input character buffer. For that purpose, a pointer is passed to the preamble scanning function, so that it may be aborted by externally writing an appropriate (nonzero) value to this memory location. This feature would also be useful when implementing timeouts.

**Data Pump Reset:** Low-level signal processing functions (data pump) are reset before processing the training/data portion of a frame. Although the initial parameters for these systems are somewhat arbitrary, resetting them to a known default state limits the worst case behavior. This is relevant, e.g., in integrators and equalizers, which may take a long time to reconverge following an abrupt change in the properties of their input processes. Because frames may be arbitrarily spaced, there is no *a priori* reason to suppose that the data pump parameters should be preserved.
**Training Sequence:** Handling of training sequences is straightforward in reference-driven equalization. If the receiver is coarsely synchronized by the frame preamble, so that in each symbol interval the equalizer input vector contains most of the energy pertaining to the specified reference symbol, then the training process will automatically provide fine synchronization. In other words, the coefficient vector will evolve so that at time $n$ the equalizer output $z(n)$ approximately equals the reference symbol $a(n)$, and the overall discrete equivalent channel response approaches an impulse.

The situation is different when blind equalization is used because then, as discussed in Section 2.1.2.1, an unknown delay and phase rotation may exist upon convergence such that $z(n) \approx a(n - d) \exp j\phi$. To cancel the delay $d$ the receiver must scan for a known fragment of the training sequence and align itself accordingly. This is accomplished as follows:

1. The final 256 bits\(^6\) of the training sequence are generated and stored in the main data pump module.

2. Equalizer output symbols are differentially decoded by the low-level slicer of Figure 2.1 and stored in a bit buffer when approaching the end of the training sequence (specifically, when the nominal time left equals the temporal span of the local training replica plus 20 symbol intervals). Naturally, this requires the use of differential coding when mapping the training sequence. To attain the necessary efficiency, only 4-PSK and 8-PSK constellations can be specified (at compile time). For more details, see Sections 2.1.3.1 and 3.3.2.

3. When the differentially decoded bit buffer is full, pattern scanning begins. If the number of bit matches exceeds a threshold (90% of the buffer length), a flag is raised to signal training lock. Subsequent calls to the training function perform no function while this flag is set, effectively freezing the input waveform until the local time reference reaches the end of the training sequence. A sufficiently large buffer must exist to absorb the input samples that are acquired while the data pump is frozen.

4. If lock was not attained until the nominal end of the training sequence, pattern scanning is performed up to an additional 20 symbol intervals. No valid reference is available during this period, but that poses no problem because such scanning will only be required when blind equalization is used.

**Residual Phase Synchronization:** When training lock is attained the most recent input to the low-level slicer is saved by the data pump. Due to carrier synchronization the constellation should be aligned with Figure 2.3 at this point of the processing chain, up to a residual rotation ambiguity of $n\pi/4$ (8-PSK) or $n\pi/2$ (4-PSK) in blind equalization.

\(^6\)The stored bit pattern length depends on the width of `unsigned long` on the target processor, which is assumed here to be 32 bits.
4.2 Layered Processing at the Receiver

mode. Moreover, the last reference symbol is saved by the high-level management function invoked during training. These two complex values will subsequently be compared to eliminate the residual rotation before proceeding to the next frame decoding stage. This operation is essential if a non-differential mapping mode is used for interleaving matrix transmittal. With differential modes, it ensures that the differential decoder state is set appropriately, so that the first symbol is correctly decoded.

Marker Scanning: Marker sequences are handled similarly to training sequences, but no phase resynchronization is attempted upon successful detection. This could lead to receiver breakdown when non-differential mapping modes are used if carrier synchronization is lost in the middle of a frame. This potential problem was considered to be relatively unimportant in the context of the present work, which focuses on differential mappings. The following procedure is used for marker scanning:

1. When approaching the end of an interleaving matrix, low-level differential decoding and storage of received symbols is initiated (See Section 2.1.3.1).

2. When the interleaving matrix has been fully processed, the receiver enters a dedicated marker detection phase where differential decoding and storage continue. Events are timed so that the scanning buffer is filled with a full marker’s worth of bits 4 symbol (2D 4/8-PSK) intervals before its nominal endpoint. Marker scanning then begins.

3. Scanning continues until up to 4 symbol intervals after the marker nominal endpoint. This function promptly terminates upon marker detection, returning a suitable code (start/stop/none). A failure counter is decremented if no start marker is found in a time window of ±4 symbols, otherwise the counter is reset to a configurable value (currently 1) that defines the maximum allowed number of consecutive detection misses. If the failure counter reaches zero, then it is assumed that the receiver is hopelessly lost and frame decoding is aborted. If not the next interleaving matrix is processed.

4. As mentioned in Section 4.1.1.4, character boundaries are aligned at the start of each interleaving matrix by emptying the output bit buffer. To limit the propagation of errors, the data scrambler (Section 4.1.1.1) is reset to a known state.

4.2.1 Mapper/Unmapper Reset:

To ensure that interleaving matrices are self-contained, the unmapping function should be reset\(^7\), in addition to the scrambler and bit buffer. This is problematic when TCM is used because resetting the Trellis path metrics reduces the error protection at the start of a new (deinterleaved) matrix. Additionally, resetting the state of the built-in differential

\(^7\)Naturally, resetting of the bit buffer, scrambler, and unmapping function should be mirrored by equivalent operations at the transmitter.
encoder/decoder in rotationally-invariant TCM will cause a decoding error at the receiver if the received signal constellation is rotated relative to the one assumed at transmission.

To tackle this problem, two distinct reset levels were considered. A hard reset is performed at the start of the first interleaving matrix, setting the Trellis state, path metrics, differential encoder/decoder state and decoding window to default values. A soft reset is performed at the start of all remaining matrices, which performs no operation at the transmitter and forces the Trellis window to a default state at the receiver. This will also implicitly resynchronize the buffer of (differentially encoded) symbols.

The lengths of both the trellis window and interleaving matrices are selected so that state forcing is redundant if there is no loss in synchronization, that is, the forced state should equal the one that arises after normally processing a matrix. Specifically, the Trellis window should fill up when the first symbol of a (deinterleaved) matrix is sent to the unmapper.

Let $M$ be the total number of symbols per interleaving matrix, $W$ the maximum Trellis window size and $B$ the length of the block that is decoded from a full Trellis window (see Section 3.3.3). Note that $W$ exceeds the unmapping delay by 1.

First matrix: After an initial latency of $W - 1$ symbols to fill the Trellis window, blocks of data are decoded at intervals of $B$ symbols (the bits are stored in a buffer and gradually released over $B$ calls to the unmapping function). According to the “almost full” default Trellis condition assumed in (soft) resets, the next time for extracting a block would be one symbol after the end of the matrix (which lasts from time 0 to $M - 1$), hence

$$W - 1 + kB = M, \quad k \text{ integer.} \quad (4.5)$$

Remaining matrices: Blocks of data are decoded at intervals of $B$ symbols with no initial latency. As before, the next time for extracting a block would be one symbol after the end of the matrix,

$$lB = M, \quad l \text{ integer.} \quad (4.6)$$

Equating the two expressions it becomes apparent that $W - 1$ (unmapping delay) must be a multiple of $B$. Given $W$ and $B$, the interleaving depth and number of interleaving blocks are selected so that $M$ satisfies the first expression.

4.3 Transmitter and Receiver Configuration

This section describes the main variables that are used to parameterize the transmitter and receiver, and provides sample configurations. The text organization parallels the software implementation, where these variables are grouped into structs declared in

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*For 4D TCM symbols $M = VI/2$, where the right-hand term is derived from (4.4).*
several modules. More details on the software architecture and configuration options are given in Chapter 5.

### 4.3.1 Frame Parameters (modem.c)

The `FramePar` structure defines the overall frame parameters. It contains the following fields:

- **BaudRate**: Number of 2D constellation points transmitted per second in the data portion of a frame\(^9\).

- **Oversamp**: Number of samples generated or collected per 2D constellation point. The (baseband) sampling rate is `BaudRate*Oversamp`.

- **Constel**: Structure holding the signal constellation parameters for training, markers and data.

- **TrnBitsPerSymbol**: Number of bits per (2D) constellation point in training sequences and start/stop markers.

- **TrnMap**: Pointer to training sequence and marker mapping function. For compatibility with the low-level slicer of Section 2.1.3.1, this should be differentially-Gray-coded 4-PSK or 8-PSK.

- **BitsPerSymbol**: Number of bits per constellation point in data (inside interleaving matrices).

- **CDimPerSymbol**: Number of complex dimensions per data symbol, i.e., number of complex values that make up a symbol.

- **Map**: Pointer to data mapping function.

- **ResetMap**: Pointer to the resetting function of the data mapper.

- **UnMap**: Pointer to data unmapping function.

- **ResetUnMap**: Pointer to the resetting function of the data unmapper.

- **UnMapDelay**: Unmapper latency, i.e., number of symbols submitted to the unmapping function before the first decoded bit becomes available.
  
  - `UnMapDelay = 0` in memoryless and plain differential mappings.
  - `UnMapDelay` is 1 unit less than the Trellis window depth when TCM is used.

- **UnMapBlk**: Number of symbols to decode and remove when the Trellis window fills up. `UnMapDelay` should be a multiple of it.

- **PreWait**: Number of sampling periods between the end of the preamble and the start of the training sequence.

\(^9\)This parameter is something of a misnomer when 4D TCM is used, as symbols are formed by pairs of complex numbers and the real baud rate, i.e., the number of transmitted symbols per second, is actually half this value.
TrainSeqLen: Length of training sequence, i.e., number of 2D differential 4/8-PSK constellation points.

MarkLen: Length of start/stop markers, i.e., number of 2D differential 4/8-PSK constellation points.

StartMarkSeed, StopMarkSeed: Initial value to be loaded into the linear-feedback shift register that generates start/stop markers (Section 4.1.1.1). The same structure is used for training sequence generation, with 0 initial configuration.

IlCDimDepth: Interleaver depth per complex dimension in data symbols. The interleaving depth is thus IlCDimDepth\times CDimPerSymbol. This awkward format ensures that an integral number of (possibly multidimensional) data symbols fits in each interleaver input/output vector (Section 4.1.1.2).

IlBlk: (Equivalent) number of full vectors per interleaving matrix. As shown by (4.4), transient effects while filling and emptying a convolutional interleaver/deinterleaver are such that the total number of complex values in a matrix is a multiple of the interleaving depth.

MtxPerFrame: Number of interleaving matrices per frame.

PostWait: Minimum number of sampling periods after the frame epilog (stop marker) before a new frame may begin.

4.3.1.1 Mapping Functions

Pointers in the Constel structure can be set according to the following list of available constellation mapping functions. Inclusion of the actual source code for these functions is controlled by several compile-time definitions listed in Section 5.3.1. The practical procedure is demonstrated in the makefiles of Section 5.4. For each modulation type, mapping, unmapping and, if applicable, reset function names are indicated. As memoryless mappings have no internal state variables, their reset pointers should be set to NoReset, a dummy function that performs no operation.

Plain 2-PSK: PSK2Map, PSK2UnMap.


Plain 4-PSK: QAM4Map, QAM4UnMap.

4.3 Transmitter and Receiver Configuration

Double differentially-Gray-coded 4-PSK: DQAM4x2Map, DQAM4x2UnMap, DQAM4x2ResetMap, DQAM4x2ResetUnMap. This mode is the same as differential 4-PSK, but two complex constellation points are processed simultaneously. It was included in the project to test the handling of 4D symbols (used in rotationally-invariant TCM) by upper layers of the transmitter and receiver.

Plain 8-PSK: PSK8Map, PSK8UnMap.

Differentially-Gray-coded 8-PSK: DPSK8Map, DPSK8UnMap, DPSK8ResetMap, DPSK8ResetUnMap.

Rotationally-invariant 4-state TCM: TCM4Map, TCM4UnMap, TCM4ResetMap, TCM4ResetUnMap.

Rotationally-invariant 16-state TCM: TCM16Map, TCM16ResetMap. As described in Section 3.3.1.1, support for this TCM mode only exists in transmit mode due to limitations in the available computational power.

4.3.1.2 Differential Encoder/Decoder State:

All differentially-Gray-coded PSK modes share the same state variable to simplify saving and restoring it when a start marker is generated. Saving/restoring ensures that transmission of marker symbols is transparent as far as data symbols are concerned, even if the same differential PSK mapping function is used for both. This operation ensures consistency with TCM modes, which use an internal differential coder that is not affected by transmission of marker symbols using differential 4/8-PSK.

No similar save/restore is needed at the receiver because marker and training symbols are demodulated by a low-level slicer (Section 2.1.3.1), and never sent to the high-level unmapper that extracts data bits. Still, a single state variable is used for all differential PSK modes at the receiver as well.

4.3.2 Generic Data Pump Parameters (dpump.c)

Currently, the top level data pump module is configured simply by setting the behavior of the low-level slicer (Section 2.1.3.1) via two defines.

PSK8SYNC: If TRUE (1), the full constellation of Figure 2.3 is used when formulating symbol decisions to be fed back to the carrier recovery loop and equalizer. If FALSE (0) only the 4 points in the real and imaginary axes are considered. Note that 8-PSK mode should be selected when operating in TCM, even if marker and training symbols are 4-PSK. This may be considered somewhat suboptimal as it increases the probability of wrong decisions in 4-PSK symbols, thus leading to enhanced adaptation noise. On

\footnote{The same applies to training symbols, which are in fact generated by the same function used for markers. The fact that the saved differential encoder state before training is constant because all variables are reset at the start of a frame is not relevant.}
the other hand, recall that marker and training epilog detection mechanisms were introduced (Section 4.2) because there is some uncertainty in the temporal alignment of frames, so deciding \textit{a priori} which symbols should be decoded as 4-PSK or 8-PSK is problematic\footnote{Admittedly, there is virtually no ambiguity throughout most of the training sequence, but switching from 4-PSK to 8-PSK slicing near the epilog would complicate the code.}.

**DPSK8MARK**: If \texttt{TRUE} (1), differential decoding for training/marker scanning assumes the 8-PSK constellation of Figure 2.3. If \texttt{FALSE} (0), 4-PSK decoding is based on the restriction of Figure 2.3 to the real and imaginary axes.

Similarly to constellation mapping functions, it would be possible to declare a function pointer referencing any of the available equalization algorithms. This, however, was not done, and two specific function calls were built into the source code of \texttt{dpump.c}. Specifically, these are located in functions \texttt{DpTrain} and \texttt{DpDecode}, which manage all signal processing blocks during training and data/marker reception, respectively. On hindsight, and bearing in mind the likelihood that this code will be used as part of a testbed for evaluating equalization algorithms, the minor loss in performance associated with an indirect function call would be more than offset by the convenience of selecting the desired algorithm simply by filling in a variable.

### 4.3.3 Timing Recovery Parameters (\texttt{betr.c})

The \texttt{TrPar} structure defines several band-edge timing recovery parameters. The numerical values used in Chapter 2 are given between square brackets.

- \texttt{Fh}, \texttt{Fl}: Upper and lower band-edge filter parameters. For each one, the following variables are defined:
  - \texttt{NDelays} \[= 3\]: Filter order (number of elements in state vector).
  - \texttt{DelayLine}: Filter state vector
  - \texttt{FbCoeffs}, \texttt{FfCoeffs}: Feedback and feedforward coefficients. The values are given in Tables A.2 and A.3.
  - \texttt{ScaleFb}: Scaling factor for \texttt{FbCoeffs[0]}, to be used in fixed-point arithmetic (see Table A.3).

- \texttt{FLoop}: Loop filter parameters.
  - \texttt{Kp} \[= 5e-3\], \texttt{Ki}: Proportional and integral gains in (2.5). The integral gain is currently unused.

### 4.3.4 Carrier Recovery Parameters (\texttt{psksync.c})

The \texttt{CrPar} structure defines carrier recovery parameters.
4.3 Transmitter and Receiver Configuration

FLoop: Loop filter parameters.

\textbf{SPE:} Sum of phase errors at the output of the leaky integrator in (2.27).
\textbf{Kp} [\text{= 1e-1}], \textbf{Ki} [\text{= 1.7e-3}]: Proportional and integral gains in (2.27).

4.3.5 Equalization Parameters (equaliz.c)

The equalization module is configured by the following \texttt{#defines}. Typical values, some of which were mentioned in Section 2.1.2, are given between square brackets.

\textbf{MIDCOEF [1]}: Initial equalizer coefficient for delay 0 in channel\footnote{As discussed in Section 2.1.2, the number of channels, or polyphase components, in a fractionally-spaced equalizer equals the oversampling rate when a single physical sensor is used.} 0. All the remaining coefficients are set to 0. This parameter is mainly relevant in blind equalization, where a null coefficient vector is an undesirable equilibrium point of most cost functions.

\textbf{EQNORMEPS [10]}: Small positive constant that is added to the equalizer input vector norm to prevent overflow. Used in SCS and NLMS.

\textbf{SCSREF [1]}: Desired equalizer output modulus for the SCS algorithm.

\textbf{SCSSTEP [0.5]}: SCS step size.

\textbf{SLMSARLG [-15]}: $\log_2$ of SLMS step adaptation rate.

\textbf{SLMSSTHI [1e-2]}: SLMS Maximum step.

\textbf{SLMSSTLO [1e-5]}: SLMS Minimum step.

\textbf{SLMSSTINI [1e-3]}: SLMS Initial step.

\textbf{NLMSSTHI [0.5]}: NLMS step size during training.

\textbf{NLMSSTLO [0.1]}: NLMS step size in decision-directed mode.

\textbf{EQCHAN [2]}: Number of equalizer input channels. The decimation factor after timing recovery (see Section 2.1) is calculated so that the resulting oversampling rate matches this parameter. For proper operation, it should divide the input oversampling factor (FramePar.Oversamp).

\textbf{EQLENPERCHAN [20]}: Number of equalizer coefficients per input channel. The requirement of equal orders on all channels is not a serious restriction when the equalizer is linear and operates on a single sensor. It would be more problematic in a decision-feedback equalizer or when operating with multiple sensors.

\textbf{EQFILL [8]}: Number of anti-causal samples per channel. As usual, the equalizer reference is delayed to emulate the existence of noncausal samples. This is achieved in a transparent way by reading from the input sample buffer (via the timing recovery section).
loop) during startup of the data pump, i.e., whenever a new frame has been detected. The procedure is harmless, though pointless, when blind equalization is used.

Provision was made for a compile-time switch (EQDESIGN, see Section 5.3) which would enable the calculation of causal/anti-causal equalizer orders as appropriate for a given scenario. This option was not implemented, so the equalizer order is currently hard-coded based on EQCHAN, EQLENPERCHAN and EQFILL.

### 4.3.6 Sample Configuration: 4-PSK

The parameters provided below configure the transmitter and receiver for operation in basic differentially-Gray-coded 4-PSK mode with no interleaving.

```c
/* In modem.c */
FramePar = {15000, /* BaudRate */
    4, /* Oversamp */
    { /* Constel */
        2, /* TrnBitsPerSymbol */
        2, /* BitsPerSymbol */
        1, /* CDimPerSymbol */
        DQAM4Map, /* TrnMap */
        DQAM4Map, /* Map */
        DQAM4ResetMap, /* ResetMap */
        DQAM4UnMap, /* UnMap */
        DQAM4ResetUnMap, /* ResetUnMap */
        0, /* UnMapDelay */
        0 /* UnMapBlk */
    },
    300, /* PreWait */
    1000, /* TrainSeqLen */
    16, /* MarkLen */
    0xc07f, /* StartMarkSeed */
    0x2459, /* StopMarkSeed */
    1, /* IlCDimDepth */
    480, /* IlBlk */
    77, /* MtxPerFrame */
    400}; /* PostWait */

/* In dpump.c */
#define PSK8SYNC FALSE
#define DPSK8MARK FALSE
```
Notes:

1. The sampling rate is 60 kHz (BaudRate*Oversamp).

2. Differential 4-PSK is used to generate 1000 (TrainSeqLen) training symbols (TrnMap), for a total of 2000 bits (TrainSeqLen*TrnBitsPerSymbol).

3. By design, the same mapping function is used to generate marker symbols. Each marker is 16 symbols long (MarkLen), or 32 bits. According to (4.2), the bit pattern of start markers is obtained by initializing a 16-bit linear-feedback shift register (scrambler) with the lower 15 bits of StartMarkSeed

   \[ s(n-15)\ldots s(n-1) = 100000001111111b = 0xc07f & 0x7fff, \]

and then applying to it a sequence of 32 1’s. The stop marker bit pattern is generated similarly based on StopMarkSeed. Both seeds are chosen so that markers have low correlation with each other and the training sequence epilog. Note that the marker and training sequence lengths are far smaller than the repetition period of the maximal length sequence output by the scrambler (\(2^{15} - 1 = 32767\) bits).

4. Differential 4-PSK is also used for data (Map, UnMap). As discussed in Section 4.3.1.2, even in this case transmission of marker symbols is transparent as far as data symbols are concerned. Each symbol is formed by a single complex constellation point, therefore the number of complex dimensions per symbol (CDimPerSymbol) is 1.

5. It is important to ensure that the number of bits per symbol (TrnBitsPerSymbol, BitsPerSymbol) matches the cardinality of the associated constellations. In 4-PSK the number of bits should be \(\log_2 4 = 2\).

6. No window is needed for decoding differential 4-PSK, therefore UnMapDelay and UnMapBlk are set to 0 (see also Sections 3.3.3 and 4.2.1).

7. In the absence of coding, there is no advantage in interleaving symbols. The interleaving depth is thus set to 1 (IlCDimDepth*CDimPerSymbol).

8. Interleaving matrices are 480 vectors long (IlBlk). For depth 1 each vector is formed by a single complex value, yielding 480 4-PSK symbols per matrix.

9. There are 77 matrices per frame (MtxPerFrame), totaling 36960 data symbols or 73920 bits. The overall number of symbols, including training and start/stop markers is \(1000 + 77(16 + 480) + 16 = 39208\), or 78416 bits, lasting for about 5.2 s.

10. The low-level slicer (Section 2.1.3.1) is configured to assume a 4-PSK constellation both when providing symbol decisions for carrier recovery/equalization (PSK8SYNC) and when decoding bits for training/marker scanning (DPSK8MARK).
4.3.7 Sample Configuration: 4-State TCM

A more elaborate configuration is now provided, using interleaved 4-state rotationally-invariant TCM in data matrices. The training sequence and start/stop markers are generated as in the previous section. Sample makefiles are given in Section 5.4 to actually build the executables.

/* In modem.c */
FramePar = {15000, /* BaudRate */
            4, /* Oversamp */
            { /* Constel */
              2, /* TrnBitsPerSymbol */
              4, /* BitsPerSymbol */
              2, /* CDimPerSymbol */
              DQAM4Map, /* TrnMap */
              TCM4Map, /* Map */
              TCM4ResetMap, /* ResetMap */
              TCM4UnMap, /* UnMap */
              TCM4ResetUnMap, /* ResetUnMap */
              24, /* UnMapDelay */
              6 /* UnMapBlk */
            },
            300, /* PreWait */
            1000, /* TrainSeqLen */
            16, /* MarkLen */
            0xc07f, /* StartMarkSeed */
            0x2459, /* StopMarkSeed */
            16, /* IlCDimDepth */
            15, /* IlBlk */
            118, /* MtxPerFrame */
            400}; /* PostWait */

/* In dpump.c */
#define PSK8SYNC TRUE
#define DPSK8MARK FALSE

Notes:

1. Training data and markers are generated as in Section 4.3.6.

2. 4D TCM symbols are formed by pairs of complex 8-PSK constellation points, hence CDimPerSymbol is 2. These are generated by taking blocks of 4 bits (BitsPerSymbol), using them to generate an additional redundant bit, and indexing a 32-point 4D constellation with the resulting 5 element vector (Figure 3.3).
3. The mapping function for TCM (Map) is independent from the one used for training and markers. This ensures transparency in marker generation (Section 4.3.1.2).

4. The Viterbi decoder uses a Trellis window spanning 25 TCM symbols (UnMapDelay+1). Filling it at the start of the first interleaving matrix causes a latency of 24 symbol periods. As shown below, a judicious choice of matrix lengths (Section 4.2.1), and insertion of dummy padding symbols (Section 4.1.1.3), ensures that all remaining matrices experience the same latency as well.

5. When the Trellis window fills up, the 6 oldest TCM symbols are decoded (UnMapBlk). This parameter divides UnMapDelay, as required by (4.5)–(4.6) with

\[
B = \text{UnMapBlk} \times \text{CDimPerSymbol} = 12,
\]
\[
W - 1 = \text{UnMapDelay} \times \text{CDimPerSymbol} = 48.
\]

6. The interleaving depth is 32 (IlCDimDepth*CDimPerSymbol). This is the length of complex vectors handled by the convolutional interleaver and deinterleaver.

7. Each matrix comprises enough (2D) constellation points to fill 15 (IlBlk) full interleaving vectors, i.e., 480 complex values. This satisfies (4.5) with \( M = 480 \), \( W = 49 \), \( B = 12 \), \( k = 36 \), and (4.6) with \( l = 40 \).

8. Satisfying (4.5)–(4.6) ensures constant initial latency in interleaving matrices, even in the absence of soft resets (Section 4.2.1). Out of the 960 bits in each frame \( (\text{IlBlk} \times \text{IlCDimDepth} \times \text{BitsPerSymbol}) \), the last 96 \( (\text{UnMapDelay} \times \text{BitsPerSymbol}) \) are not data, but dummy bits that are needed to support this operating mode.

9. There are 118 matrices per frame (MtxPerFrame), totaling 113280 bits (101952 data, 11328 dummy). The overall number of bits, including training (2000) and start/stop markers (32 each) is \( 2000 + 118(32 + 960) + 32 = 119088 \), or 59544 complex values. Each frame therefore lasts about 4 s.

10. The low-level slicer (Section 2.1.3.1) is configured to assume an 8-PSK constellation when providing symbol decisions for carrier recovery/equalization (PSK8SYNC), and 4-PSK when decoding bits for training/marker scanning (DPSK8MARK).
Chapter 5
Software Organization

The transmitter and receiver software architecture was originally designed with the goal of allowing most of the operating parameters described in Section 4.3 to be changed on-the-fly by configuration commands conveyed through an input character buffer\textsuperscript{1}. While this would be a desirable feature in a dedicated testbed for underwater communications, it requires some effort to ensure that both channel endpoints share the same set of parameters at all times. This could be achieved, e.g., by including in every transmitted frame all the operating parameters, or engaging in a negotiation process following any parameter changes (at the transmitter) to resynchronize the receiver\textsuperscript{2}. Any of these options would involve some reduction in net bandwidth and an increase in code complexity.

Bearing in mind the limitations in manpower and available hardware, in the course of this work reconfigurability was eventually dropped in favor of higher net data rates and a lower memory footprint. Still, the software retained most of its original flexible design, so that the truncated functionality can be easily added in the future.

The choice between fixed- or floating-point arithmetic is one of the most important design issues, which is usually dictated by the target processor. Although the primary target for the ASIMOV project was a 16 bit fixed-point DSP (Texas Instruments TMS320C54x), an effort was made to allow easy porting of the code to other processors. As described in Section 5.2, the current design enables the specification of fixed- or floating-point arithmetic at compile time with virtually no performance penalty. Moreover, for improved performance one can select native compiler support for complex numbers (GNU C compiler).

5.1 Software Modules

This section lists all the higher-level software modules (.c and .h files), grouped by function, that are needed for building PC executables. The DSP version includes an optimized assembly language implementation of some numeric functions, as well as specialized low-

\textsuperscript{1}Typically, this buffer would be fed by a serial port.
\textsuperscript{2}The availability of a bidirectional medium-speed link might enable parameter changes at the receiver to be conveyed to the transmitter.
level modules, not described here, that initialize the target board and interact with its peripherals.

5.1.1 Modem Management and Testing

debug: Replacement of preamble scanning and sample I/O functions for offline modem testing.

dptest: Simple control module for offline testing of data-pump functions (bare-bones replacement of modem and sample/bit/character buffer management).

iosamp: Interface with A/D and D/A converters and sample queue management. File I/O when in debug mode (incomplete).

modem: High-level modem management functions. Generation and scanning of frame preambles, training sequences, markers and interleaving matrices (see Figure 4.1). In debug mode, the frame parameters are read from framepar.

5.1.2 Bit/Character Buffer Management


buffer: General-purpose bit and character buffer management.

charmng: Processing of character streams for command/data mode switching (incomplete).

cmd: Processing of modem configuration commands (incomplete).

tokens: Token searching in text strings for command processing.

5.1.3 Frame Preambles

barker: Generation and scanning of Barker code frame preambles (incomplete).

chirp: Generation and scanning of chirp frame preambles.

5.1.4 Data Pump

betr: Band-edge timing recovery. In debug mode, its parameters are read from trpar.

dpump: Main data-pump module. Management of symbol/caller recovery, equalization, and marker/training sequence scanning. In debug mode, its parameters are read from dppar.

equaliz: Equalization. In debug mode, its parameters are read from eqpar.

farrow: Farrow interpolation used in timing recovery.
psksync: Carrier recovery and low-level slicing (see Section 2.1.3.1). In debug mode, its parameters are read from crpar.

5.1.4.1 Filtering

filter: General-purpose filtering and coefficient adaptation functions, and prototype pulse generation. Supported filter structures include single-input single-output IIR (direct form II with and without transposition), multiple-input single-output FIR, and single-input multiple-output FIR. SISO IIR are used as band-edge filters in timing recovery (Appendix A), MISO FIR as fractionally-spaced equalizers, and SIMO FIR as pulse shapers at the transmitter.

pulsemsg: Modem-specific management of pulse shaping filters.

5.1.5 Mapping/Unmapping


tcm8psk: Bit-to-symbol mapping using trellis-coded-modulation in an expanded 8-PSK constellation. Includes a 4-state coder and Viterbi decoder, as well as a 16-state coder.

5.1.6 Numeric Functions

costab: 256-element cosine lookup table in fixed-point format (16 bits, Q15). It is used to generate $\exp j\phi$ factors used in carrier recovery and low-level slicing, equalizer adaptation, and chirp generation.

fixptdef: Definition of fixed-point numeric precision throughout the receiver signal processing chain.

numeric: Real and complex arithmetic functions in fixed and floating point. For maximum efficiency, these may be replaced by assembly-language versions optimized for a desired processor.

nzeros: Lookup table containing the number of zeros in all 8-bit integers. Used for scrambling and bit pattern scanning (markers or training sequence epilog).

5.2 Numeric Implementation

Careful consideration must be given to the fixed-point representation of values across the signal processing chain, as the numerical ranges involved vary significantly. With only 16 bits available in the target processor, one could hardly expect to accommodate all values using a single fixed-point format. Instead, values are appropriately scaled as they
propagate through the receiver, so that at any given point the numerical representation ensures suitable accuracy. This is typically achieved by supplying a scaling argument to binary operators, so that conversion to the output format is done as part of the final shifting operation whenever products are involved. The computational overhead of fixed-point conversion is therefore minimal (in fact, almost negligible), while making optimal use of special DSP registers that can hold extended-precision results after multiplication.

**Choice of Fixed-Point Precision:** Naturally, the floating-point implementation supplies a reference for choosing fixed-point formats. As a first step, a software “probe” is inserted at any desired location in the code to log floating-point values to file. These values are subsequently imported into Matlab to determine a sufficiently tight fixed-point format, with suitable safety margins to prevent overflow. With 16 bits available, the $Q_i$ fixed-point format places the decimal point to the right of bit $D_i$, $i = 0, \ldots, 15$, and uses two’s complement notation to represent values. This approximately corresponds to $i$ bits for the fractional part of a real number, 1 for its sign, and the remaining $16 - i - 1$ bits for the magnitude of its integer part.

Once the appropriate format for any variable of interest is selected, it is defined as a constant in `fixptdef`. In the code, scaling factors for numerical operations are then written as a function of the precisions of its source and destination variables. As the resulting expression is fully evaluated during compiler preprocessing, this approach involves no performance penalty. As an example, suppose that the chosen precision for variables $x$, $y$, $z$ is $Q_i$, $Q_j$, $Q_k$, respectively, and $z = xy$ is to be calculated. Then, the 32-bit integer $xy$ should be (arithmetically) right shifted by $i + j - k$ bits to yield $z$ in the desired format.

**Rounding:** Rounding to the nearest integer requires adding 1 to the bit located immediately to the right of the target LSB in the extended-precision result prior to shifting. This is important, e.g., in the leaky integrators used inside carrier and timing recovery loops, to avoid excessive buildup of rounding errors.

**Verification:** The fixed-point accuracy at any point can be ascertained by logging numeric values to disk in fixed- and floating-point implementations, importing both time series into Matlab, and comparing their evolution. Specifically, if a floating-point value $x$ is represented in fixed-point $Q_i$ by $x_i$, then $x \approx 2^{-i}x_i$. Significant deviations between the two values might be caused by coarse quantization ($i$ should be larger), or a buildup of numerical errors in previous processing stages.

In retrospect, being able to compare fixed- and floating-point implementations of essentially the same code simply by recompiling the program turned out to be a powerful means of accelerating the development cycle. It should be kept in mind that algorithmic, numerical and coding aspects become somewhat coupled when designing a fixed-point implementation; tuning the fixed-point format frequently suffices to obtain suitable accuracy at a given point of the signal processing chain, but sometimes major changes to the al-
algorithms are needed to get more stable numerical behavior. This was the case, e.g., with the relatively sharp band-edge filters described in Section A.3. The final direct form II transposed structure was chosen to avoid having internal variables with large numerical ranges, which led to unacceptable rounding errors in other filter architectures.

5.3 Compile-time Definitions

To avoid building unnecessarily large executables, extensive use is made of conditional compilation using #ifdef directives. This section lists several variables which should be defined at compile time to include various parts of the code. The procedure is demonstrated in Section 5.4. Configuration parameters for specific modules are described in Section 4.3.

TXMODEM: Include code for transmission.

RXMODEM: Include code for reception.

FIXPT: Generate fixed-point code only (integer arithmetic).

GNUCPLX: Use native complex data types of GNU GCC compiler (fixed- or floating point).

SCANDESIGN: Compute required values for marker scanning from frame parameters, otherwise use hard-coded values.

BREAKTST: Include code to scan for command/data escape sequences in the characters that are locally sent to the modem.

PACKETTST: Generate and scan for packet start flags in the decoded bit stream.

PULSE: Generate samples of the baseband waveform by filtering the symbol sequence with a PAM pulse (implemented as a raised-cosine SIMO filter, as described in Section 2.2. The oversampling factor is set by Oversamp, defined in Section 4.3). Otherwise, directly output the real and imaginary parts of symbols, effectively setting the transmit oversampling factor to 1.

DDS: Write (& read for debugging purposes) complex symbols as indices in an M-PSK constellation, according to the format of ORCA’s modulator hardware (see Section 2.2). This option should only be used when PULSE is not defined.

DSP: Include specific code for DSP implementation.

DEBUG: Include debugging code, mainly sample read/write to/from files.

MSS: Monitor dynamic memory allocation and use.

Other secondary definitions, listed below, control code generation for various subsystems. As these are less likely to be changed, they are defined in header files to limit the number of required command-line options.
5.3 Compile-time Definitions

BETRDESIGN (betr.h): Compute upper and lower band-edge filters for timing recovery (not implemented), otherwise use hard-coded values.

EQDESIGN (equaliz.h): Compute equalizer order (not implemented), otherwise use fixed-size coefficient and input vectors.

EQNORM (equaliz.h): Recursively compute the equalizer input vector norm by exploiting its temporal shift structure (Mandatory. Nonrecursive computation not implemented).

NLMS (equaliz.h): Include code for normalized LMS (nonblind) equalization.

SCS (equaliz.h): Include code for soft-constraint satisfaction blind equalization.

SLMS (equaliz.h): Include code for adaptive step-size LMS (nonblind) equalization.

MISOFIR (filter.h): Include code for MISO FIR filtering.

SIMOFIR (filter.h): Include code for SIMO FIR filtering.

SIS0II (filter.h): Include code for SISO type II filtering.

SISOTII (filter.h): Include code for SISO type II transposed filtering.

SISOFIR (filter.h): Include code for SISO FIR filtering.

5.3.1 Mapping Functions

Below is a set of compile-time variables that control the inclusion of source code for the various mapping functions listed in Section 4.3.1.1.

DPSK2: Include differential 2-PSK mapping functions (plain 2-PSK always present).

DQAM4: Include differential 4-QAM mapping functions (plain 4-QAM always present).

DQAM4x2: [Used for debugging purposes] Include 4D pseudo-differential 4-QAM mapping functions. Automatically incorporates DQAM4.

PSK8: Include plain 8-PSK mapping functions.

DPSK8: Include differential 8-PSK mapping functions. Automatically incorporates PSK8.

TCM4: Include rotationally-invariant 4-state 4D 8-PSK Trellis Coded Modulation (TCM) mapping functions.

TCM16: Include rotationally-invariant 16-state 4D 8-PSK TCM mapping functions\(^3\).

\(^3\)Only available at the transmitter. See also Sections 3.3.1.1 and 4.3.1.1.
5.4 Sample Makefiles

These makefiles (for GNU make) demonstrate how to build PC executables for the transmitter and receiver whose parameters are specified in Section 4.3.7.

5.4.1 Transmitter

CFLAGS = -DTXMODEM -DDEBUG -DDQAM4 -DTCM4 -DPULSE -DSIMOFIR -DM_PI=3.14159265358979323846

TXOBJ = bitmng.o buffer.o charmng.o chirp.o
        costab.o debug.o filter.o iosamp.o map.o
        modem.o numeric.o nzeros.o pulsemng.o
        tcm8psk.o

txmodem: TXOBJ

modem.o: framepar.c

5.4.2 Receiver

CFLAGS = -DRXMODEM -DDEBUG -DDQAM4 -DTCM4 -DSCANDESIGN -DFIXPT -DM_PI=3.14159265358979323846

RXOBJ = betr.o bitmng.o buffer.o charmng
        chirp.o costab.o debug.o dpump.o
        equaliz.o farrow.o filter.o iosamp.o
        map.o modem.o numeric.o nzeros.o
        psksync.o tcm8psk.o

rxmodem: RXOBJ

modem.o: framepar.c
dpump.o: dppar.c
betr.o: trpar.c
equaliz.o: eqpar.c
psksync.o: crpar.c
Appendix A

Band-Edge Timing Recovery

Band-edge timing recovery (BETR) was originally proposed by Godard in [4], and further developed by Jablon [11]. This appendix briefly describes the basics of that method. The continuous-time received baseband PAM signal in the absence of noise is given by

\[ y_c(t) = \sum_k a(k) h_c(t - kT_b), \]

as in (2.1) for \( \eta_c(t) = 0 \). Assuming that the symbol sequence \( a(k) \) is uncorrelated, with zero mean and variance \( \sigma^2 \), the instantaneous power

\[ E\{|y_c(t)|^2\} = \sigma^2 \sum_{k=-\infty}^{\infty} |h_c(t - kT_b)|^2 \] (A.1)

is periodic, with period \( T_b \) (i.e., \( y_c(t) \) is a cyclostationary signal). A Fourier series expansion of the form

\[ E\{|y_c(t)|^2\} = \frac{\sigma^2}{T_b} \sum_{k=-\infty}^{\infty} c_k e^{j \frac{2\pi k}{T_b} t} \] (A.2)

yields

\[ c_k = \int_0^{T_b} \sum_{k=-\infty}^{\infty} |h_c(t - kT_b)|^2 e^{-j \frac{2\pi k}{T_b} t} dt = \int_{-\infty}^{\infty} |h_c(t - kT_b)|^2 e^{-j \frac{2\pi k}{T_b} t} dt \]

\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_c(\omega) H^*_c \left( \omega - \frac{2k\pi}{T_b} \right) d\omega. \] (A.3)

Baseband PAM pulse spectra are typically bandlimited to \( |\omega| < \pi/T_b \), and the same holds for the multipath-distorted replica \( H_c(\omega) \) (Figure A.1). Usually, the excess bandwidth\(^1\) of these pulses is smaller than 100\%, so that all \( c_k \) are zero when \( k \notin \{-1, 0, 1\} \). As \( c_{-1} = c_1^* \), (A.2) is seen to represent a nonzero-mean sine wave; The goal of BETR is to determine the instant \( t \in [0, T_b) \) where its maximum is reached, while reducing the jitter that arises from the random nature of the symbol sequence \( a(k) \). As seen from (A.3), the coefficients \( c_{-1} \) and \( c_1 \) only depend on the upper and lower portions of \( H_c(\omega) \), labeled \( H_h(\omega) \) and \( H_l(\omega) \)

\(^1\)The excess bandwidth of \( H_c(\omega) \) is defined as the frequency range above \( \omega = \pi/T_b \) where \( |H_c(\omega)| \neq 0 \), divided by the Nyquist bandwidth \( \pi/T_b \).
in figure A.1. As the mean value \( \langle E\{|y_t(t)|^2\rangle = \sigma^2 c_0 / T_b \) does not affect the location of the maximum, (A.2) can be replaced by the alternative cost function

\[
E\{|y_t(t)|^2\} - \frac{\sigma^2}{T_b} c_0 = \text{Re}\left\{ \frac{2\sigma^2}{T_b} c_1 e^{j\frac{2\pi t}{T_b}} \right\}. \quad (A.4)
\]

Noting that \( c_1 e^{j\frac{2\pi t}{T_b}} \) has constant magnitude, the maximum of (A.4) is reached when \( \text{Im}\{c_1 e^{j\frac{2\pi t}{T_b}}\} = 0 \). That is precisely the criterion used in BETR to adapt a PLL that tracks the optimal sampling instant (Figure A.2). The imaginary part of \( c_1 e^{j\frac{2\pi t}{T_b}} \) also vanishes at the minimum of the cost function, but that is an unstable point of the adaptation algorithm. Any jitter due to \( a(k) \) will move the operating point away from it, ensuring convergence to the desired timing estimate.

Consider the complex PAM signals \( y_h(t) \) and \( y_l(t) \), obtained by ideal bandpass filtering of \( y_c(t) \) so that their pulse shapes become \( H_h(\omega) \) and \( H_l(\omega) \), respectively (see Figure A.1). These filters are denoted by \( B_h(\omega) \) and \( B_l(\omega) \) in Figure 2.2. Then \( E\{y_h(t)y_l^*(t)\} \) is \( T_b \)-periodic, and can be expanded in a Fourier series as

\[
E\{y_h(t)y_l^*(t)\} = \sigma_n^2 \sum_{k=-\infty}^{\infty} h_k(t - kT_b)h_k^*(t - kT_b) = \frac{\sigma_n^2}{T_b} \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k}{T_b}} t. \quad (A.5)
\]
A.1 Band-Edge Filter Design

Table A.1: Lowpass prototype for band-edge filters

<table>
<thead>
<tr>
<th>a(0)</th>
<th>1</th>
<th>b(0)</th>
<th>0.00170386383818</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(1)</td>
<td>-2.65757729365461</td>
<td>b(1)</td>
<td>0.00511159151455</td>
</tr>
<tr>
<td>a(2)</td>
<td>2.46253341289720</td>
<td>b(2)</td>
<td>0.00511159151455</td>
</tr>
<tr>
<td>a(3)</td>
<td>-0.79132520853712</td>
<td>b(3)</td>
<td>0.00170386383818</td>
</tr>
</tbody>
</table>

Table A.1: Lowpass prototype for band-edge filters

where

\[ c_k = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_h(\omega) H_t^*\left(\omega - \frac{2k\pi}{T_b}\right) d\omega. \]  

(A.6)

It is clear from (A.5)-(A.6) that only \( c_1 \) may be nonzero, coinciding with the respective Fourier coefficient of \( E\{|y_c(t)|^2\} \). By replacing \( E\{\cdot\} \) with a time average of \( T_b \)-spaced samples and choosing \( t \in [0, T_b) \) to cancel the imaginary part of (A.5), the original cost function (A.2) is maximized. As the bandwidth of \( y_h(t) \) and \( y_l(t) \) is much smaller than that of \( y_c(t) \), this approach ensures better immunity against noise and jitter than would be possible if timing estimates were directly calculated from \( |y_c(t)|^2 \).

A.1 Band-Edge Filter Design

In practice, \( y_h(n) \) and \( y_l(n) \) are generated by rational bandpass filters \( F_h(\omega) = A_h(\omega)/B_h(\omega) \) and \( F_l(\omega) = A_l(\omega)/B_l(\omega) \), obtained from a common lowpass prototype

\[ F(\omega) = \left. \frac{A(z)}{B(z)} \right|_{z = e^{j\omega}} , \quad \frac{A(z)}{B(z)} = \frac{b_0 + b_1 z^{-1} + \ldots + b_p z^{-p}}{1 + a_1 z^{-1} + \ldots + a_q z^{-q}}. \]  

(A.7)

The transformation is given by

\[ \frac{B_h(\omega)}{A_h(\omega)} = \frac{B(\omega - \omega_h)}{A(\omega - \omega_h)}, \quad \frac{B_h(z)}{A_h(z)} = \frac{b_0 + (b_1 e^{j\omega_h}) z^{-1} + \ldots + (b_p e^{j\omega_h}) z^{-p}}{1 + (a_1 e^{j\omega_h}) z^{-1} + \ldots + (a_q e^{j\omega_h}) z^{-q}}, \]  

(A.8)

\[ \frac{B_l(\omega)}{A_l(\omega)} = \frac{B(\omega - \omega_l)}{A(\omega - \omega_l)}, \quad \frac{B_l(z)}{A_l(z)} = \frac{b_0 + (b_1 e^{j\omega_l}) z^{-1} + \ldots + (b_p e^{j\omega_l}) z^{-p}}{1 + (a_1 e^{j\omega_l}) z^{-1} + \ldots + (a_q e^{j\omega_l}) z^{-q}}, \]  

(A.9)

where \( \omega_h \) and \( \omega_l \) are the upper and lower band-edge frequencies, respectively. In the baseband case of figure A.1 \( \omega_h = 2\pi/(2T_b), \omega_l = -2\pi/(2T_b) \), but note that BETR can also be performed on passband waveforms. The filter order should be kept small when designing the lowpass prototype, so that real-time filtering does not become computationally complex. This rules out Butterworth filters. However, the roots of \( A(z) \) and \( B(z) \) should not be too close to the unit circle, as this leads to internal variables in the filtering structure that have large dynamic ranges and cannot be accurately represented in fixed-point arithmetic. This rules out elliptic filters. Finally, Chebyshev type I filters (equiripple in the passband, monotonic in the stopband) were chosen as a good compromise. Section A.2 provides Matlab code for computing the band-edge filters. For the given specifications, a prototype of order \( p = q = 3 \) was obtained with coefficients given in table A.1. Figure A.3 shows its magnitude and phase response.
Figure A.3: Lowpass prototype for band-edge filters (a) Magnitude/phase frequency response (b) Detail of passband

A.2 Matlab Code for Generating Band-Edge Filters

```matlab
% Compute band-edge filter responses for band-edge timing recovery (BETR)

fc = 0; % Carrier frequency
Tb = 1/15000; % Symbol interval
b = 1/Tb/2; % Excess bandwidth (rolloff/(2*Tb))
L = 4; % Samples/symbol (oversampling factor)

% Parameters for band-edge filters

ws = 2*pi/(Tb/L); % Sampling frequency
wh = 2*pi*(fc+1/(2*Tb)); % Continuous-time upper and lower band-edge center frequencies
wl = 2*pi*(fc-1/(2*Tb));

wh_be = 2*pi/ws*(wh - fix(wh/ws)); % Convert to discrete frequency 0..2pi
wl_be = 2*pi/ws*(wl - fix(wl/ws)); % Ideal (continuous-time) bandwidth of band-edge filters around upper and lower band-edge frequencies
```
A.3 Fixed-Point Representation

\[ Bw = 2\pi * (2*b); \] % Convert to discrete frequency 0..2pi
\[ Bw_{be} = 2\pi / ws * (Bw - \text{fix}(Bw/ws)); \]

\% Compute numerator and denominator coefficients of lowpass prototype:
\% Af,Bf -> H(w) = Bf(w)/Af(w) -> Hh(w) = H(w-wh), Hl(w) = H(w-wl)
\% For proper baud spectral line generation, the attenuation of the upper
\% band-edge filter should be sufficiently low within the lower band-edge
\% filter bandwidth, and vice-versa:
\% Hh(wl+Bw/2) << Hl(wl+Bw/2), Hl(wh-Bw/2) << Hh(wh-Bw/2)
\% If the prototype is real, so that H(w) = H*(-w), this is equivalent to the
\% single condition H(wh-wl-Bw/2) << H(Bw/2). For a rolloff of 100% (desirable)
\% this leads to a transition band of width 0 (undesirable). The following less
\% stringent condition is therefore adopted:
\% Hh(wl) << Hl(wl), Hl(wh) << Hh(wh)
\% Equivalently, for H(w) = H*(-w): H(wh-wl) << H(0). To reduce the overlap
\% between filters when the rolloff is close to 100%, it is considered that the
\% (single-sided) band-edge filter bandwidth is k*Bw/2, with 0<k<1.

\[ [Nf_{be},Wf_{be}] = \text{cheb1ord}((0.5*Bw_{be}/2)/\pi,(wh_{be}-wl_{be})/\pi,3,40); \]
\[ [Bf_{be},Af_{be}] = \text{cheby1}(Nf_{be},3,Wf_{be}); \]
\% Upper band-edge filter
\[ Bh_{be} = (Bf_{be}.*\exp(j*wh_{be}*(0:length(Bf_{be})-1))).'; \]
\[ Ah_{be} = (Af_{be}.*\exp(j*wh_{be}*(0:length(Af_{be})-1))).'; \]
\% Lower band-edge filter
\[ Bl_{be} = (Bf_{be}.*\exp(j*wl_{be}*(0:length(Bf_{be})-1))).'; \]
\[ Al_{be} = (Af_{be}.*\exp(j*wl_{be}*(0:length(Af_{be})-1))).'; \]

A.3 Fixed-Point Representation

Even with the chosen Chebyshev filter the poles and zeros are relatively close to the unit
circle, and the dynamic range for the internal variables of a direct form II IIR filtering
structure [14] cannot be accomodated with sufficient accuracy in the available 16-bit fixed-
point format. Fortunately, a direct form II transposed structure\(^2\) [14] proved to be suitable.

To ensure that filtering routines have good computational efficiency in fixed-point
implementations, filter coefficients should be represented in Q15 format. Yet, \(|a_i|, |b_i| < 1\)
cannot be guaranteed for arbitrary filter responses, and therefore a modification of the

\(^2\)This is used, e.g., in the \texttt{filter} function in Matlab.
filter structures, shown in Figure A.4, was introduced to account for this possibility. With the additional arbitrary coefficient $G$ the filter transfer function becomes

$$F(z) = \frac{G \left[ b_0 + b_1 z^{-1} + \ldots + b_p z^{-p} \right]}{1 + G \left[ a_1 z^{-1} + \ldots + a_q z^{-q} \right]}.$$  \hspace{1cm} (A.10)

By choosing $G$ with sufficiently large magnitude, all other coefficients in the numerator and denominator of (A.10) can be represented in Q15 format. Section A.4 provides Matlab code for generating both the fixed and floating-point filter parameters from the transfer function polynomials. They are listed in Tables A.2 and A.3 for the parameters assumed in Section A.2. Note that the coefficients of Table A.3 induce a DC gain of 2 relative to the floating-point frequency response. As explained in Section A.4, this stems from the fact that the numerator coefficients of Table A.2 were scaled by 2 prior to conversion to Q15 format in order to improve the numerical accuracy. As it turns out, this is convenient in practice because the floating-point implementation of the Farrow interpolator has gain 2, whereas the corresponding gain in fixed point is 1. The cascade of this inter-
A.4 Matlab Code for Computing Filter Parameters

Upper Band-Edge Filter

<table>
<thead>
<tr>
<th>a(0)</th>
<th>2.65757729365461</th>
<th>b(0)</th>
<th>0.00064113425497</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(1)</td>
<td>0.70710678118655 \cdot (-1 - j)</td>
<td>b(1)</td>
<td>0.00136005113801 \cdot (1 + j)</td>
</tr>
<tr>
<td>a(2)</td>
<td>0.92660838831551 \cdot j</td>
<td>b(2)</td>
<td>0.00192340276490 \cdot j</td>
</tr>
<tr>
<td>a(3)</td>
<td>0.21054944381730 \cdot (1 - j)</td>
<td>b(3)</td>
<td>0.00045335037934 \cdot (-1 + j)</td>
</tr>
</tbody>
</table>

Lower Band-Edge Filter

<table>
<thead>
<tr>
<th>a(0)</th>
<th>2.65757729365461</th>
<th>b(0)</th>
<th>0.00064113425497</th>
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</tr>
<tr>
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<td>b(2)</td>
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</tr>
<tr>
<td>a(3)</td>
<td>0.21054944381730 \cdot (1 + j)</td>
<td>b(3)</td>
<td>0.00045335037934 \cdot (-1 - j)</td>
</tr>
</tbody>
</table>

Table A.2: Floating-point band-edge filter coefficients (DC gain 1)

Upper Band-Edge Filter

<table>
<thead>
<tr>
<th>a(0)</th>
<th>21771 (Q13)</th>
<th>b(0)</th>
<th>42</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(1)</td>
<td>23170 \cdot (-1 - j)</td>
<td>b(1)</td>
<td>89 \cdot (1 + j)</td>
</tr>
<tr>
<td>a(2)</td>
<td>30363 \cdot j</td>
<td>b(2)</td>
<td>126 \cdot j</td>
</tr>
<tr>
<td>a(3)</td>
<td>6899 \cdot (1 - j)</td>
<td>b(3)</td>
<td>30 \cdot (-1 + j)</td>
</tr>
</tbody>
</table>

Lower Band-Edge Filter

<table>
<thead>
<tr>
<th>a(0)</th>
<th>21771 (Q13)</th>
<th>b(0)</th>
<th>42</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(1)</td>
<td>23170 \cdot (-1 + j)</td>
<td>b(1)</td>
<td>89 \cdot (1 - j)</td>
</tr>
<tr>
<td>a(2)</td>
<td>30363 \cdot -j</td>
<td>b(2)</td>
<td>126 \cdot -j</td>
</tr>
<tr>
<td>a(3)</td>
<td>6899 \cdot (1 + j)</td>
<td>b(3)</td>
<td>30 \cdot (-1 - j)</td>
</tr>
</tbody>
</table>

Table A.3: Fixed-point band-edge filter coefficients (DC gain 2)

polator with each band-edge filter therefore has the same overall gain in both numerical implementations, which can simplify the specification of loop filter parameters for timing recovery.

A.4 Matlab Code for Computing Filter Parameters

```matlab
% Compute fixed and floating-point parameters for band-edge filters
% Note: Numerator and denominator polynomials for the desired rational
% transfer functions are assumed to have been previously calculated
%
% Find the greatest magnitude among filter coefficients. For simplicity, a
% single value is determined for both filters. This is harmless in the
% baseband case (and of little relevance otherwise), as the symmetry in the
% frequency domain ensures that the same result would have been obtained
% with two independent scaling factors.
```
% '1' is included in the expression to
% saturate the result when all
% coefficients are smaller than 1,
% thus avoiding scaling by G<1.

G_fl = max([1; abs([Ah_be(2:length(Ah_be)); Bh_be; ...
          Al_be(2:length(Al_be)); Bl_be])]);

Gh_fl = G_fl; % Upper scaling factor
Gl_fl = G_fl; % Lower scaling factor

% Compute the vector of feedback (Fb) and feedforward (Ff) coefficients
% used by the filtering routines. In the chosen time-domain implementation
% it is preferable to work with the symmetrical denominator coefficients
% -a_i, i=1. Note that the scaling factor is stored in the position of a
% hypothetical coefficient a_0 in the denominator polynomials (which is
% omitted in practice because it would necessarily equal 1).

% Scale upper feedback coefficients
FbTIIh_fl = [Gh_fl; -Ah_be(2:length(Ah_be))/Gh_fl];
FfTIIh_fl = Bh_be/Gh_fl; % Scale upper feedforward coefficients

% Repeat for lower band-edge filter
FbTIIl_fl = [Gl_fl; -Al_be(2:length(Al_be))/Gl_fl];
FfTIIl_fl = Bl_be/Gl_fl;

% Compute fixed-point parameters. It turns out that the numerator
% coefficients of the band-edge filters have much smaller magnitude than
% those of the denominator, and direct conversion to Q15 format may yield
% insufficient precision. As a simple way of improving that precision, the
% numerator coefficients are multiplied by a constant (>1) before converting
% to Q15, thus introducing a DC gain in the filter responses.

% Find exponent of scaling factor
Exph = min([15; 15 - ceil(log2(Gh_fl))]);
Expl = min([15; 15 - ceil(log2(Gl_fl))]);
% Additional DC gain to be introduced in
% the fixed-point frequency response of band-edge filters
DCG_fx = 2;

% Convert feedforward and feedback coefficients to Q15 format. Note, however, that the scaling factor is converted to Qn, n<=15, and therefore n has to be supplied to the filtering routines as well.

FbTIIh_fx = round([FbTIIh_fl(1)*2^Exph; ... FbTIIh_fl(2:length(FbTIIh_fl))*2^15]);
FfTIIh_fx = round(DCG_fx*FfTIIh_fl*2^15);

% Repeat for lower band-edge filter
FbTIIl_fx = round([FbTIIl_fl(1)*2^Expl; ... FbTIIl_fl(2:length(FbTIIl_fl))*2^15]);
FfTIIl_fx = round(DCG_fx*FfTIIl_fl*2^15);

%---------------------------------------------------------------------------
% Regenerate transfer function polynomials in both fixed and floating point to plot the frequency response and make sure that it is the desired one.
%---------------------------------------------------------------------------

% Upper filter, floating point
Ah_fl = [1; -FbTIIh_fl(1)*FbTIIh_fl(2:length(FbTIIh_fl))];
Bh_fl = FbTIIh_fl(1)*FfTIIh_fl;

% Lower filter, floating point
Al_fl = [1; -FbTIIl_fl(1)*FbTIIl_fl(2:length(FbTIIl_fl))];
Bl_fl = FbTIIl_fl(1)*FfTIIl_fl;

% Upper filter, fixed point
Ah_fx = [1; -(FbTIIh_fx(1)/2^Exph)*(FbTIIh_fx(2:length(FbTIIh_fx))/2^15)];
Bh_fx = (FbTIIh_fx(1)/2^Exph)*(FfTIIh_fx/2^15);

% Lower filter, fixed point
Al_fx = [1; -(FbTIIl_fx(1)/2^Expl)*(FbTIIl_fx(2:length(FbTIIl_fx))/2^15)];
Bl_fx = (FbTIIl_fx(1)/2^Expl)*(FfTIIl_fx/2^15);
Bibliography


