SPATIAL PRIORS FOR PERFUSION AND TRANSIT TIME ESTIMATION IN PASL-MRI

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ABSTRACT
Maps of perfusion and Arterial Transit Time (ATT) can be measured quantitatively using non-invasive Pulsed Arterial Spin Labeling (PASL) techniques. This can be achieved by fitting a kinetic model to magnetization difference data images acquired at multiple inversion times (TI). Here, spatial information is incorporated into a Bayesian estimation method, based on a maximum a posteriori (MAP) criterion, which also incorporates a priori knowledge regarding the model physiological parameters. Two types of spatial priors were tested. Monte Carlo simulations showed reduced parameter estimation errors when including spatial information using a Total Variation prior. Furthermore, the feasibility of the method proposed here was demonstrated through the application to empirical data.

Index Terms— PASL, MRI, spatial prior, Bayesian.

1. INTRODUCTION
Arterial Spin Labeling (ASL) magnetic resonance imaging (MRI) techniques offer a non-invasive way of obtain perfusion measurements which are potentially quantitative. They consist on magnetically labeling the water molecules in the blood and then measuring the magnetization of the tissues after a certain time interval, the inversion time (TI). The magnetization difference, $\Delta M$, between a labeled image and a control image, as a function of TI, can be described by a kinetic model [1]. Perfusion information can be estimated by fitting the model to the data, acquired at multiple TI points.

In order to cope with the intrinsically low signal-to-noise ratio (SNR) of PASL data, averaging over large regions-of-interest (ROI) is often performed, yielding a single value for each physiological parameter [2]. However, spatial maps of the parameters are more informative from a clinical point of view. Extraction of information regarding the parameters can be accomplished with improved performance on a voxel-by-voxel basis by using a Bayesian estimation method [3]. This procedure has the advantage of incorporating knowledge about the physiological distributions of the parameters, helping guide the estimation to a more consistent and realistic solution [4]. When working with maps of PASL data, the spatial structure of the values of each parameter can also be incorporated as additional information, describing the belief that the value of the parameter in a voxel is correlated in some way with that in its neighbors. Considering a Bayesian approach, this spatial prior can be defined in different ways [5][6]. For example, Groves et al implemented a combined physiological and spatial Gaussian process and used the Euclidean distance between voxels to describe the spatial prior information [5]. Commonly used Euclidean distance priors are usually useful for images with slow transitions [7]. However, more drastic transitions in the parameter values are expected to occur at the boarders between different tissue types or arterial territories and also in discrete lesions exhibiting pathological values of the parameters. In other contexts, a Total Variation (TV) regularization [7][8] has been used, which is an edge preserving prior and is therefore more indicated in such cases.

In this work, we propose to investigate the performance of a spatial prior in a Bayesian framework for the estimation of perfusion and ATT maps based on the maximum a posteriori (MAP) criterion. This Bayesian method accounts for the amount of noise in the data and incorporates a priori knowledge of the physiological distributions of the multiple model parameters. Here, we additionally introduce a spatial prior taking into account the correlation between adjacent voxels in the parameter maps. We test two different types of spatial correlation, by considering a Total Variation (TV) regularization [7][8] and a common squared Euclidean distance between voxels. Both perfusion and ATT are estimated using both simulated and empirical data. With the simulated data, the proposed estimation approach is compared with a Bayesian method that does not account for the spatial regularization. In empirical data, we show the applicability of the method in the estimation of perfusion and ATT maps with real PASL data.

2. PROBLEM FORMULATION
Let us consider the following Additive White Gaussian Noise (AWGN) observation model,

$$y_v(t_p) = \Delta M(t_p, \theta_v) + \eta,$$  \hspace{1cm} (1)

where $y_v(t_p)$ is the observation at instant $t_p$ and voxel $v$, $\Delta M(t_p, \theta_v)$ is the magnetization difference and $\eta$ ~
\( \mathcal{N}(0, \sigma_y^2) \) is the noise with variance \( \sigma_y^2 \) (independent in time and space).

Here, we use a two-compartment kinetic model that describes \( \Delta M \) as a function of the model parameters of interest \( \theta = [f, \Delta t] \), where \( f \) is the perfusion and \( \Delta t \) is the arterial transit time (ATT) [9].

The MAP criterion for model parameter estimation can be formulated as the following optimization task

\[
\hat{\theta} = \arg \min_{\theta} E(y, t, \theta) ,
\]

where the energy function \( E(y, t, \theta) \) is given by

\[
E(y, t, \theta) = -\log [p(y|t, \theta)p(\theta)] .
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The distribution function \( p(y|t, \theta) \) models the acquisition process and the observations are assumed to be statistically independent along time and space. The distribution function \( p(\theta) \) represents the a priori knowledge of the parameters to be estimated. Here, both physiological and spatial priors are considered. The parameters \( \theta \) are defined with a multivariate Normal distribution \( \mathcal{N}(\vec{\theta}, C) \), where \( C = \text{diag}\{\sigma_1^2, \sigma_2^2, ..., \sigma_K^2\} \) is a diagonal covariance matrix (\( K \) is the number of unknown parameters). The uncertainty associated with the parameters is assumed to be known.

The energy function (3) can be rewritten as the sum of three distinct terms,

\[
E(y, t, \theta) = E_Y(y, t, \theta) + E_P(\theta) + E_S(\theta) .
\]

The first is called the data fidelity term and is given by the posterior function, \( -\log [p(y|t, \theta)] \), which can be written as

\[
E_Y(y, t, \theta) = \frac{1}{2} \sum_{v} \frac{1}{\sigma_y^2} \sum_{p} (y_{p,v} - \Delta M_{p,v}(t_{p,v}, \theta_{v}))^2 ,
\]

The second and the third terms describe the prior physiological and spatial knowledge of the parameters (see Figure 1). The physiological prior term is given by

\[
E_P(\theta) = \frac{1}{2} \sum_{v} \sum_{k} (\theta_{k,v} - \theta_{0,k,v})^2 ,
\]

The first type of spatial prior term considered is the quadratic form of the euclidian distance (QFED) between the parameter values in neighboring voxels

\[
E_S(\theta) = \frac{1}{2} \sum_{v} \sum_{k} (\beta_{k,v} \sum_{n} (\theta_{k,v} - \theta_{k,v,n})^2 ,
\]

where \( \beta_{k,v} \) is a normalization spatial prior parameter and \( \theta_{k,v,n} \) is the value of the parameter \( k \) of voxel \( v \), in \( n \) horizontal and vertical adjacent neighbors. We consider \( n = 1, 2, 3, 4 \), known as four-element neighborhood [7].

The second type of spatial prior term considered is a TV prior described by the energy function

\[
E_S(\theta) = \frac{1}{2} \sum_{v} \sum_{k} \beta_{k,v} \sum_{n} (\theta_{k,v} - \theta_{k,v,n})^2 .
\]

The complexity and the computational cost of determining the first and second derivatives required for the task of optimizing the energy function (8) are extremely high. Therefore, a variation of the TV method, called Iteratively Reweighted Norm (IRN) [10], is used. The IRN approach consists of an iterative process whereby, at each iteration, a weighted form of the QFED is taken as the spatial prior energy function. An automatic hyper-parameter is also considered to guarantee the correct imposition on the strength of the spatial prior.

In the estimation procedure, the optimization is accomplished by the Levenberg-Marquardt algorithm. Both the Jacobian and Hessian matrix are determined at each iteration of the algorithm which allows the determination of the parameters simultaneously on all voxels.

3. EXPERIMENTAL RESULTS

Here, results with synthetic and real data are presented.

Monte Carlo Simulations

To test the performance of our proposed method, Monte Carlo simulations were performed. A 2D test object with a realistic brain mask of 700 voxels was considered: a segmentation into gray matter and white matter was considered for the perfusion maps and three arterial territories (anterior, medial and posterior) were considered for the ATT maps. For each condition tested, 15 synthetic datasets were generated at each voxel, yielding a total of 10500 runs.

Noise was added as a fraction of the maximum signal generated by the mean values of the parameters of all voxels:

\[
\sigma_Y = \gamma \times \max[\Delta M(t, \theta_0)] .
\]

and three different levels of noise were considered (\( \gamma = 10, 100 \) and 150\%).

For the estimation of the model parameters, three different types of prior information were used: i) physiological prior
In this work, a Bayesian framework was implemented in order to obtain quantitative brain maps of perfusion and arterial transit time from a time series of label-control image pairs of PASL data. In order to improve the parameter estimation from a kinetic model, spatial prior information was incorporated in addition to physiological prior information, and two different types of spatial priors were compared.

Monte Carlo simulations showed that using a Total Variation based spatial prior generally produces more accurate results than a Euclidean distance based spatial prior or the use of no spatial information. The proposed method was also applied to empirical PASL data to show the applicability of the algorithm. The results showed a good performance of the spatial prior in the identification of the expected perfusion and arterial brain regions.

In this work, the prior map of the parameter $\Delta t$ was considered to be homogeneous for all the voxels at each slice. In order to improve the estimation of the ATT maps, a priori knowledge of the arterial regions in the brain could be incorporated into the spatial prior.

**4. CONCLUSIONS**

at the perfusion maps, we observe that the TV does not have a strong effect in their visual aspect relative to the maps obtained with the physiological prior only. Moreover, we observe very smooth maps when obtained with the QFED prior relative to when they were obtained with the other two priors, probably because the relatively abrupt transition between gray and white matter is not well supported by the QFED prior.

**Real Data**

In order to demonstrate the applicability of the proposed Bayesian method, the maps of the physiological parameters perfusion and ATT were estimated from real multi-inversion time PASL data. The PASL data was collected from seven healthy volunteers on a 3T Siemens system. The acquisition slab contained nine contiguous axial slices, positioned parallel to the AC-PC line, with a resolution of $3.5 \times 3.5 \times 5.0 \text{mm}^3$. The magnetization difference (tag-control pairs) was sampled at a uniform set of inversion time points in the interval $[0.2; 2.4] \text{s}$, in steps of $2 \text{s}$, with 8 repetitions for each inversion time (total of 96 points). For each $\Delta M$ map, the noise was measured as the standard deviation of data inside a background region with a reasonable number of voxels.

The estimated maps of perfusion and ATT obtained with the three types of priors are presented in Figures 4 and 5, respectively. In general, all the estimation methods used were able to identify the expected brain regions on the perfusion and ATT maps. However, the absence of a spatial prior in the estimation procedure produced noisier maps than in the other two cases, which may impair a clear identification of the relevant regions. As expected from the simulated data, the perfusion maps estimated using the QFED based prior were smoother than the maps obtained with the physiological prior only and the TV based prior.
Fig. 4. Maps of estimated parameter $f (s^{-1})$ for 4 brain slices of subject 3, obtained with using the 3 types of priors tested: physiological prior only (top), QFED based prior (medial) and TV based prior (bottom).

Fig. 5. Maps of estimated parameter $\Delta t (s)$ for 4 brain slices of subject 3, obtained with using the 3 types of priors tested: physiological prior only (top), QFED based prior (medial) and TV based prior (bottom).

5. REFERENCES


