Modeling and Automation of Industrial Processes

Modelação e Automação de Processos Industriais / MAPI

Stochastic Models

Stochastic Petri Nets & Queuing Networks

http://www.isr.tecnico.ulisboa.pt/~jag/courses/mapi2223

Prof. José Gaspar, rev. 2022/2023

Syllabus:

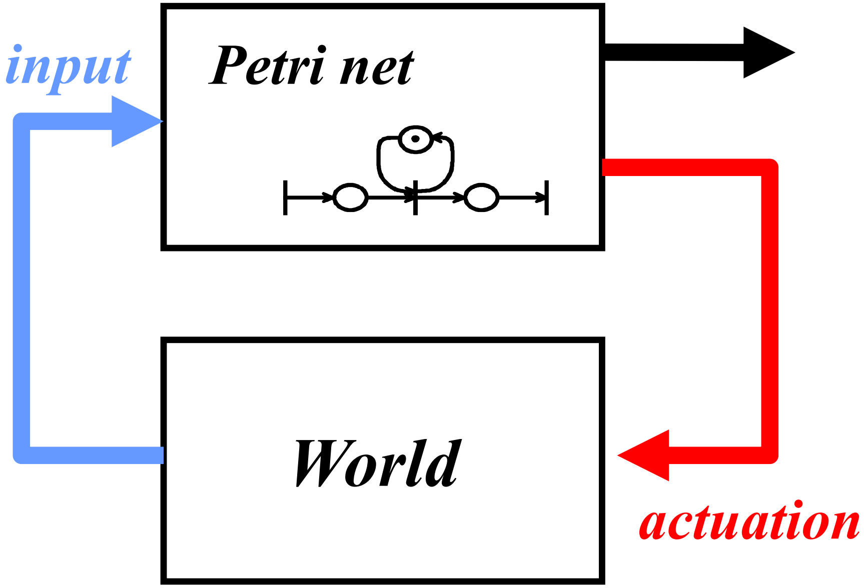
Chap. 2 – Discrete Event Systems

Chap. 3 – Stochastic Models

- Stochastic Timed Automata (STA)
- Stochastic Queueing Networks (SQN)
- Stochastic Petri Nets (SPN)
- Generalized Stochastic Petri Nets (GSPN)

Chap. 4 – Stochastic Analysis

Chap. 5 – Supervision of DESs



***Petri nets interacting with the world:** how to study the interaction along time?*

A1: Test the real system. There are many options. But may be impossible / undesirable, e.g. due to time, costs, security, …

A2: Study while designing. Given the dynamics

$\mu(k+1) = \mu(k) + D\,\mathbf{q}(\mathbf{k})$ and $D^{-}\,q(k) \leq \mu(k)$

characterize statistically the firing vector $\mathbf{q}(\mathbf{k})$.

Test the real system - May not be possible or not desired, e.g. in case of having security concerns or simply being in a too early system design stage. Otherwise, possible, and applied, in many real-world cases:

Interpreted Petri Nets (IPN) [Adamski'00] – Classic Petri net complemented with (i) input and output alphabets (sets) which are (ii) mapped to transitions and places.

Input Output Place Transition Petri Net (IOPT) [Gomes'07] – Classic Petri net complemented with communication channels and mappings of inputs and outputs to/from transitions and places.

Interpreted Petri Net for Embedded Systems (IPNES) [Krzywicki'21] – Classic Petri net completed with modelling of distributed systems involving synchronization and data exchange.

[Adamski'00] "From interpreted Petri net specification to reprogrammable logic controller design", M. Adamski & J. Monteiro, ISIE'2000 Vol. 1: 13-19, 2000
[Gomes'07] "The input-output place-transition petri net class and associated tools", L. Gomes et al., IEEE Int. Conf. on Industrial Informatics, Vol. 1: 509-514, 2007
[Krzywicki'21] "IPNES-Interpreted petri net for embedded systems", K. Krzywicki et al., Procedia Computer Science 192, 2021

Study while designing - Given the dynamics $\mu(k+1) = \mu(k) + D\, q(k)$ ***consider a statistical characterization of the firing vector*** $q(k)$.

Stochastic Timed Automata (STA) [Cassandras'08] – Timed ***automata*** where the time delays are modeled as random variables or probabilistic distributions.

Stochastic Timed Petri Net (STPN) [Zurawski'94] – T-timed ***Petri nets***, where the time delays are modeled as random variables or probabilistic distributions.

[Cassandras'08] Introduction to discrete event systems, Christos Cassandras and Stéphane Lafortune, Springer 2008 – chapter 6, section 6.4.

[Zurawski'94] "Petri nets and industrial applications: A tutorial." R. Zurawski, M. Zhou. IEEE Trans. on Industrial Electronics 41.6 (1994): 567-583. https://www.researchgate.net/publication/3217035

Stochastic Timed Automata

Stochastic Timed Automaton (STA) [Cassandras'08]

Definition. A *Stochastic Timed Automaton* is a six-tuple

$$(\mathcal{E}, \mathcal{X}, \Gamma, p, p_0, G)$$

where

$\mathcal{E}$	is a countable *event set*
$\mathcal{X}$	is a countable *state space*
$\Gamma(x)$	is a set of *feasible* or *enabled* events, defined for all $x \in \mathcal{X}$ with $\Gamma(x) \subseteq \mathcal{E}$
$p(x'; x, e')$	is a *state transition probability*, defined for all $x, x' \in \mathcal{X}, e' \in \mathcal{E}$, and such that $p(x'; x, e') = 0$ for all $e' \notin \Gamma(x)$
$p_0(x)$	is the pmf $P[X_0 = x]$, $x \in \mathcal{X}$, of the initial state X_0

and $G = \{G_i : i \in \mathcal{E}\}$ is a *stochastic clock structure*.

[Cassandras'08] Introduction to Discrete Event Systems, Christos Cassandras and Stéphane Lafortune, Springer 2008 – chapter 6, section 6.4 "Stochastic Timed Automata".

Example of a Stochastic Timed Automaton (STA) [Cassandras'08]

$$(\mathcal{E}, \mathcal{X}, \Gamma, p, p_0, G)$$

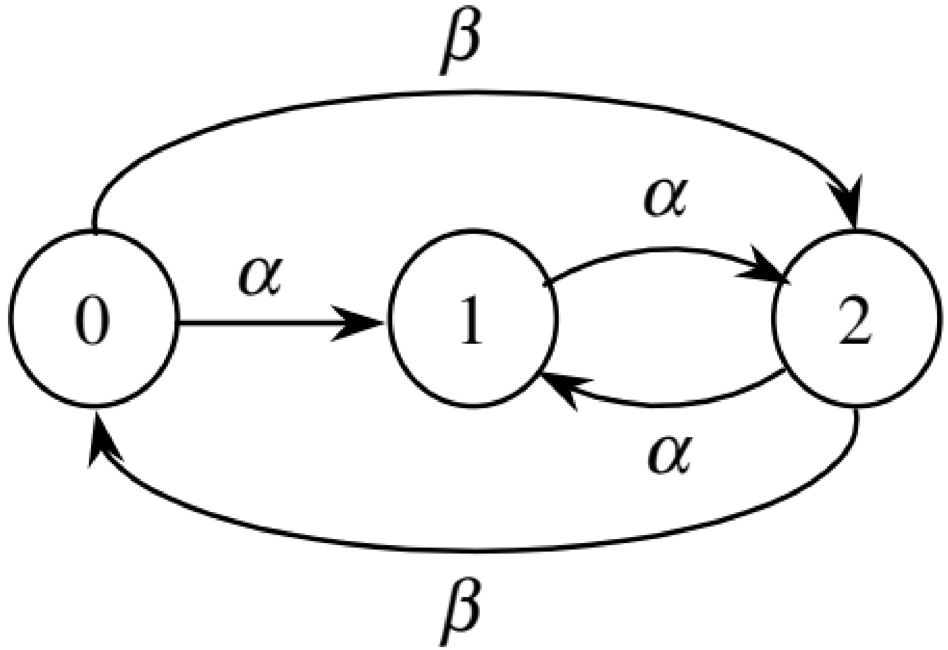
$$\mathcal{E} = \{\alpha, \beta\}, \quad \mathcal{X} = \{0, 1, 2\}$$

$$\Gamma(0) = \Gamma(2) = \{\alpha, \beta\}, \quad \Gamma(1) = \{\alpha\}$$

$$p(1; 0, \alpha) = 1, \quad p(2; 0, \beta) = 1$$

$$p(2; 1, \alpha) = 1$$

$$p(0; 2, \beta) = 1, \quad p(1; 2, \alpha) = 1$$



	α	β
0	1, p=1	2, p=1
1	2, p=1	–
2	1, p=1	0, p=1

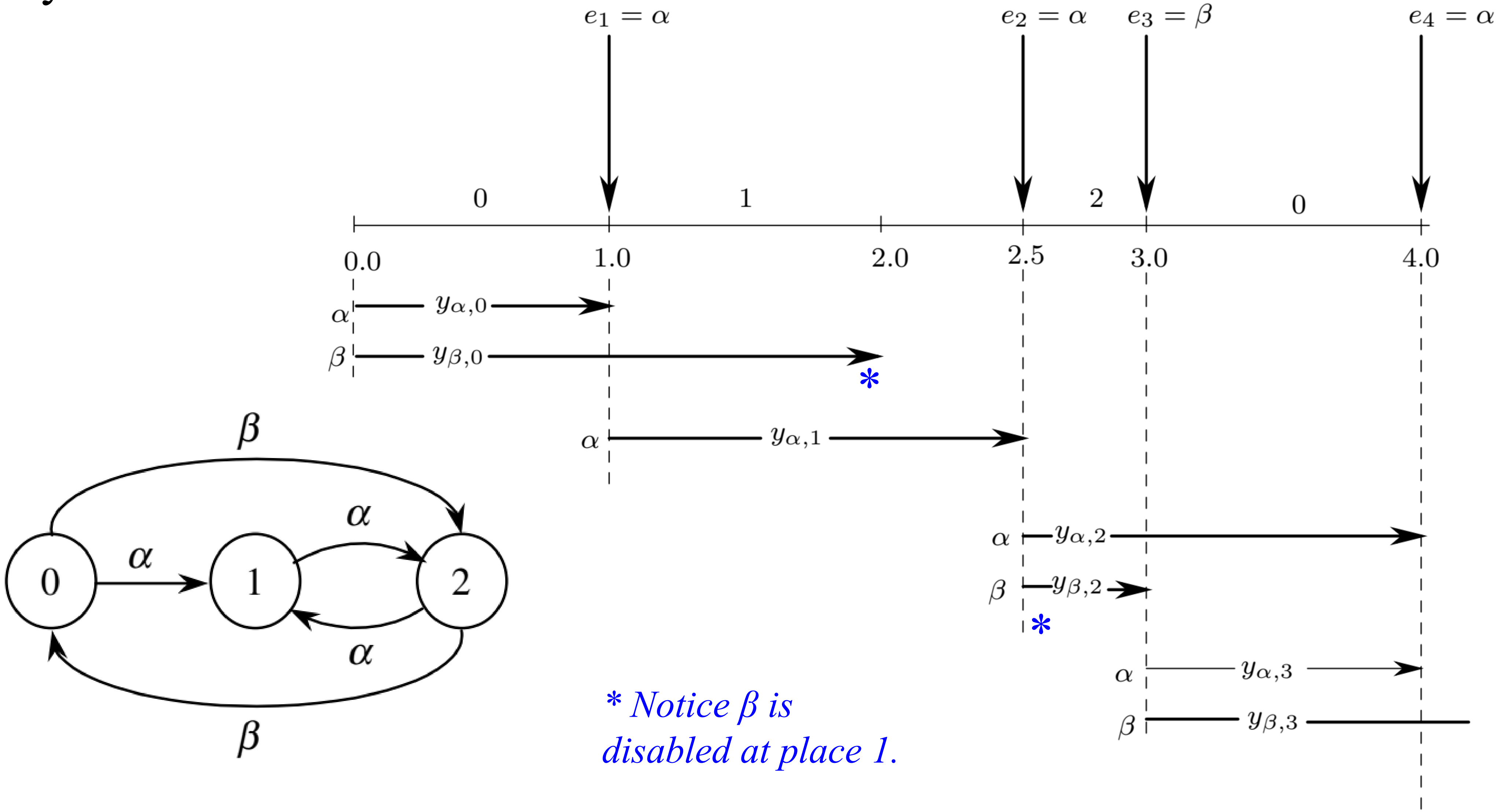
missing events in $\Gamma(.)$ *mean* $p = 0$.

Where is the stochastic nature?
Time and rate of events (next slide) …

Recall also the definition of Petri net $C = (P, T, I, O, \mu_0)$

[Cassandras'08] "Introduction to Discrete Event Systems", Christos Cassandras and Stéphane Lafortune, Springer 2008 – section 6.5 "The Generalized Semi-Markov Process".

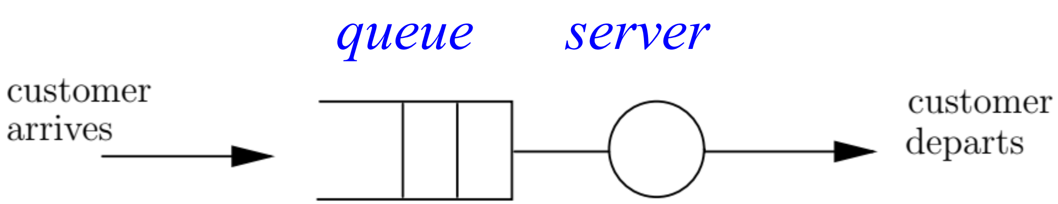
Events $\{\alpha, \beta\}$ have a stochastic nature, form a **stochastic clock**.
A generalized stochastic Markov process (GSMP) is generated by an STA:



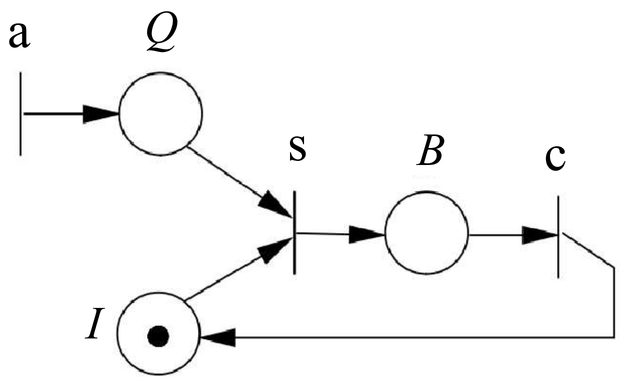
Stochastic Queuing Networks

Queueing system represented as a Petri net [Cassandras'08]

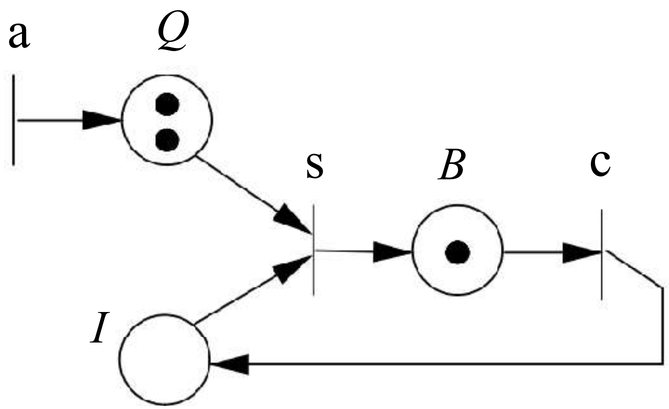
a)



b)



c)



a) Simple queueing system

b) Petri net model, initial state (0, 1, 0)

c) Petri net after firing $\{a, s, a, a, c, s, a\}$

Q	=	*#customers waiting*
B	=	*server busy*
I	=	*server idle*

The *A*/*B*/*m*/*K* notation, Kendall's notation

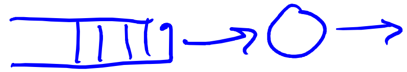
The **A/B/m/K** notation is a simple representation of a queueing system

A = interarrival time distribution
B = service time distribution
m = number of servers, m = 1,2,...
K = queue storage capacity, K = 1,2,...

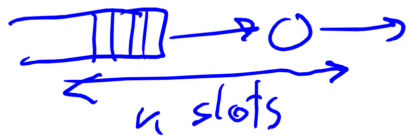
where distributions A and B:
G = General distribution
D = Deterministic i.e. fixed interarrival / service times
M = Markovian i.e. interarrival / service times exponentially distributed.

Examples:

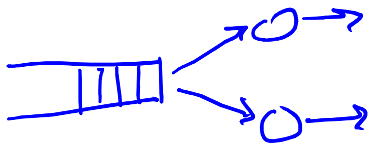
M/M/1 = interarrival and service times exponentially distributed, one server system, infinite storage.



M/M/1/K = interarrival and service times both exponentially distributed, one server, storage equal to K $< \infty$.

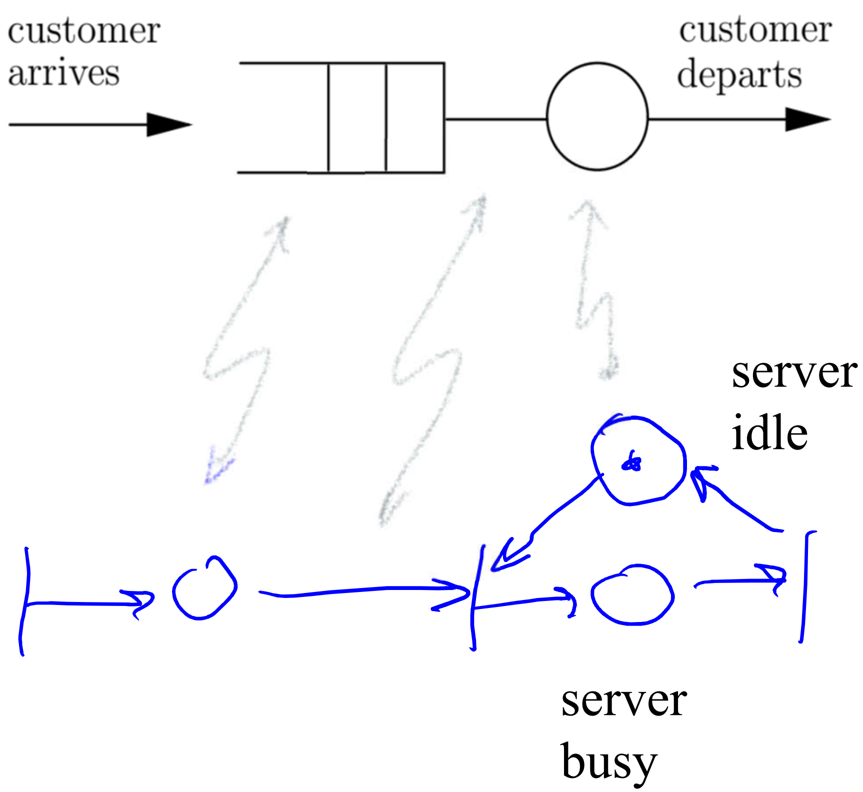


M/G/2 = interarrival times are exponentially distributed, service times have an arbitrary (general) distribution, two servers, infinite storage.

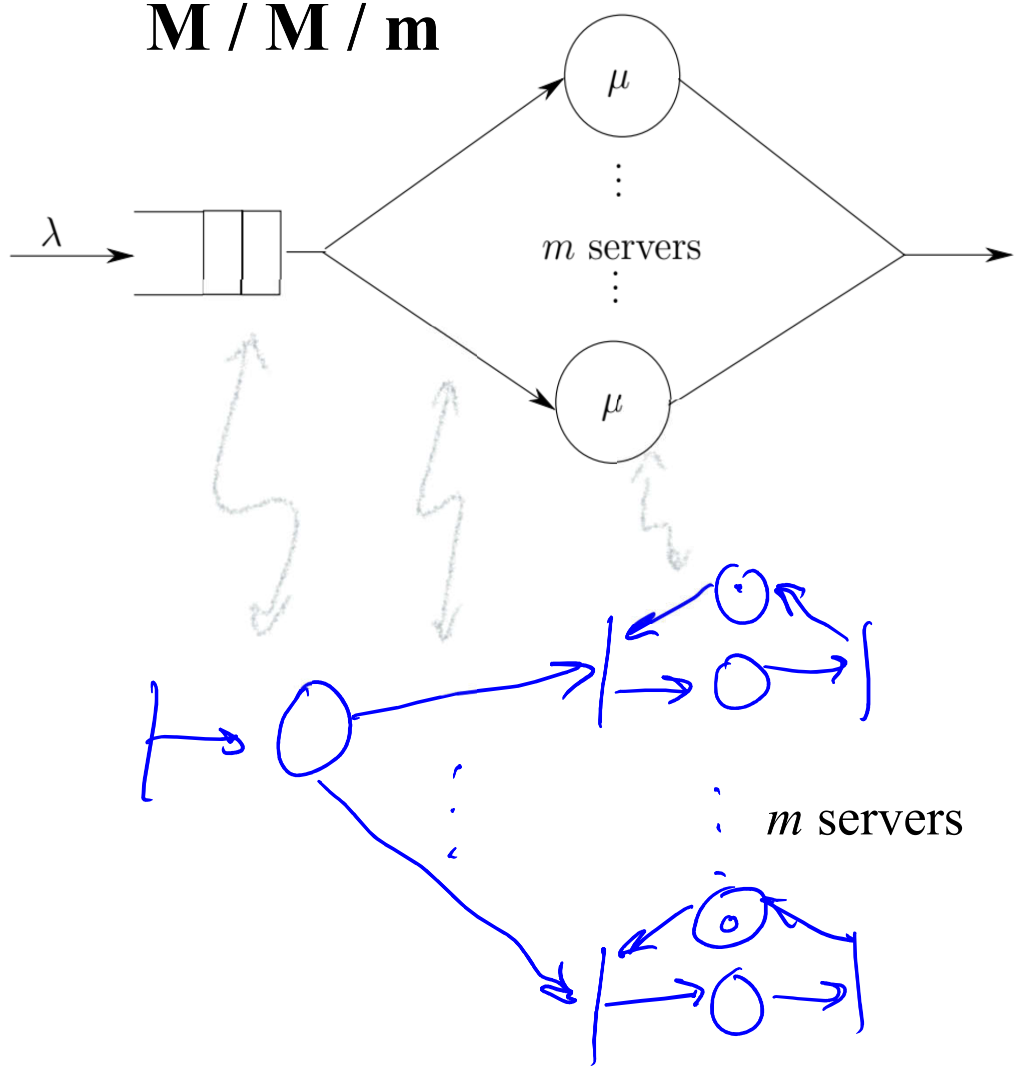


Conversion of a queueing system to a Petri net

M / M / 1

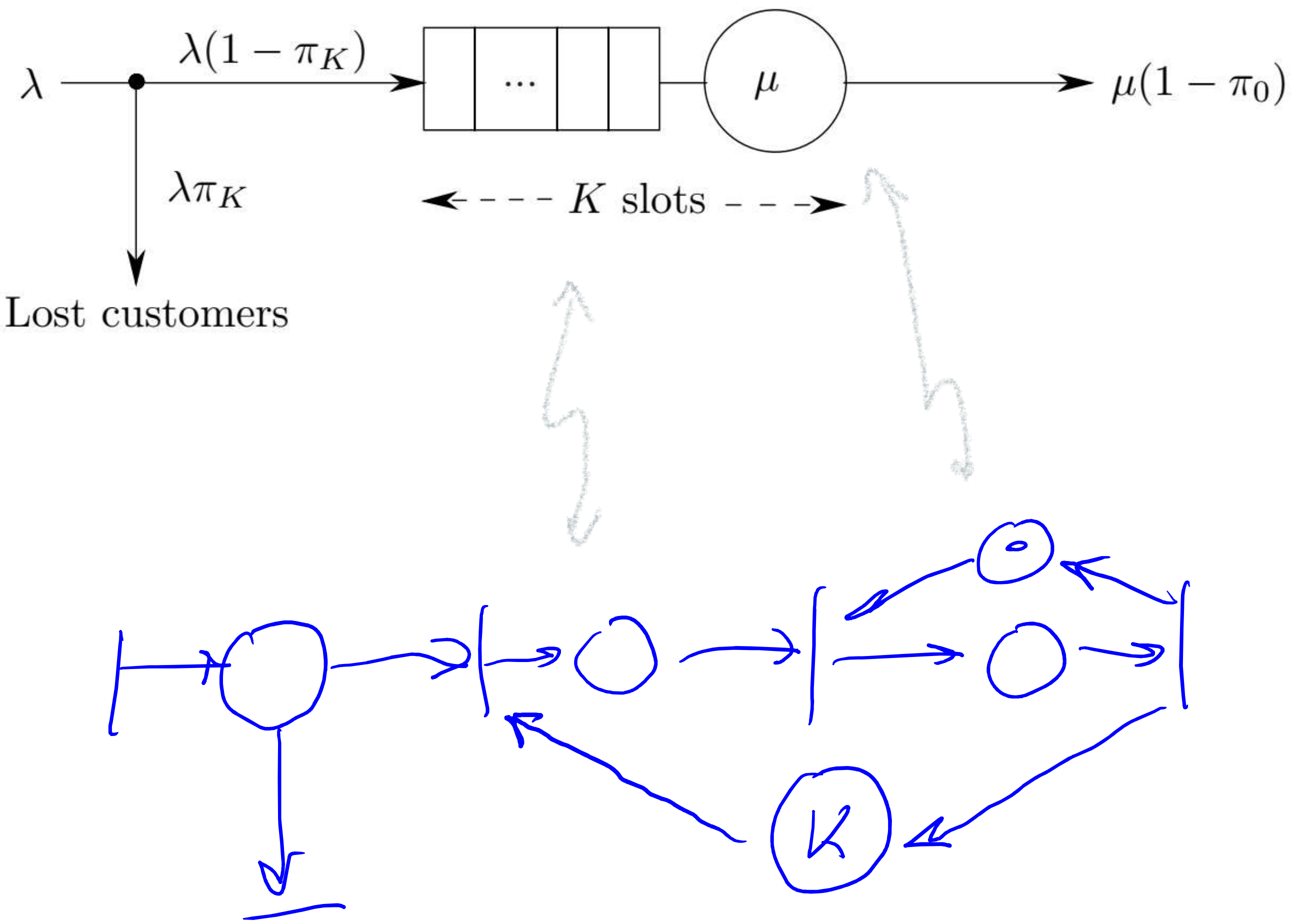


M / M / m

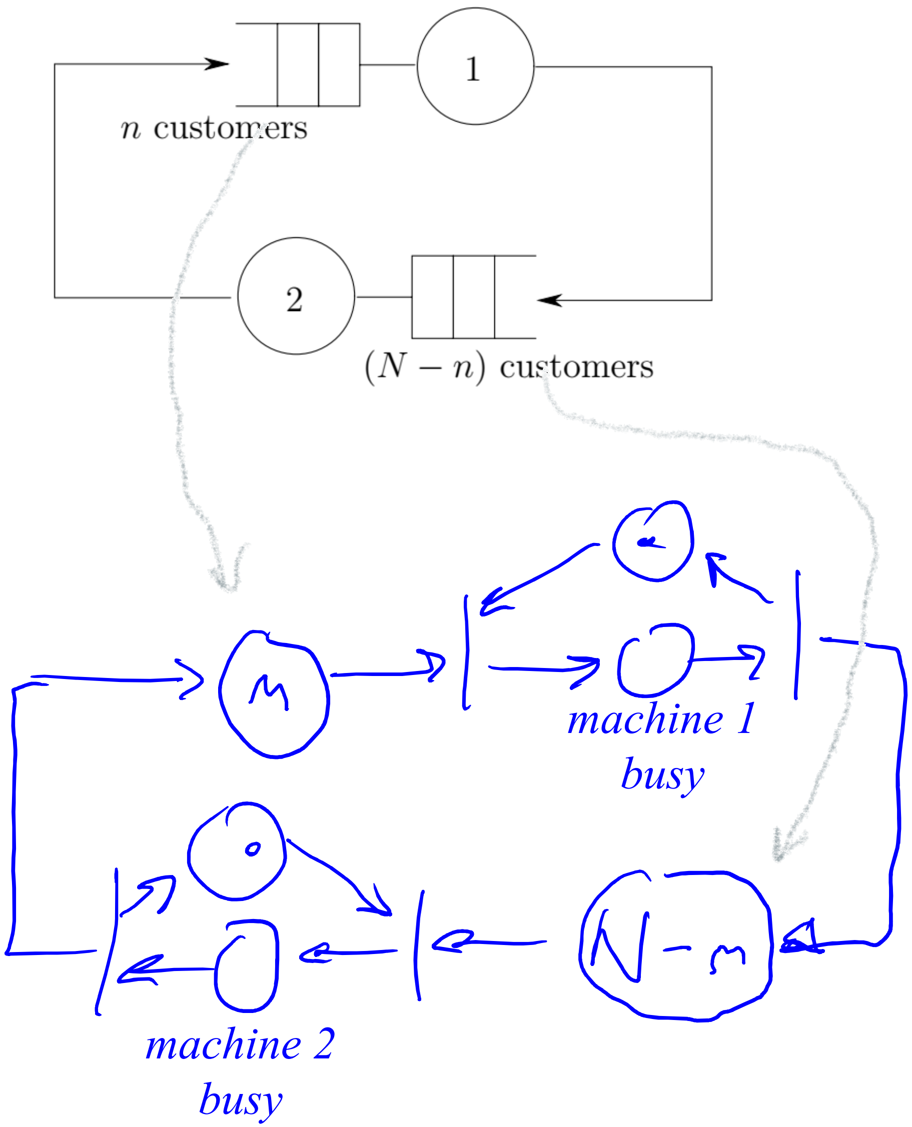


Conversion of a queueing system to a Petri net

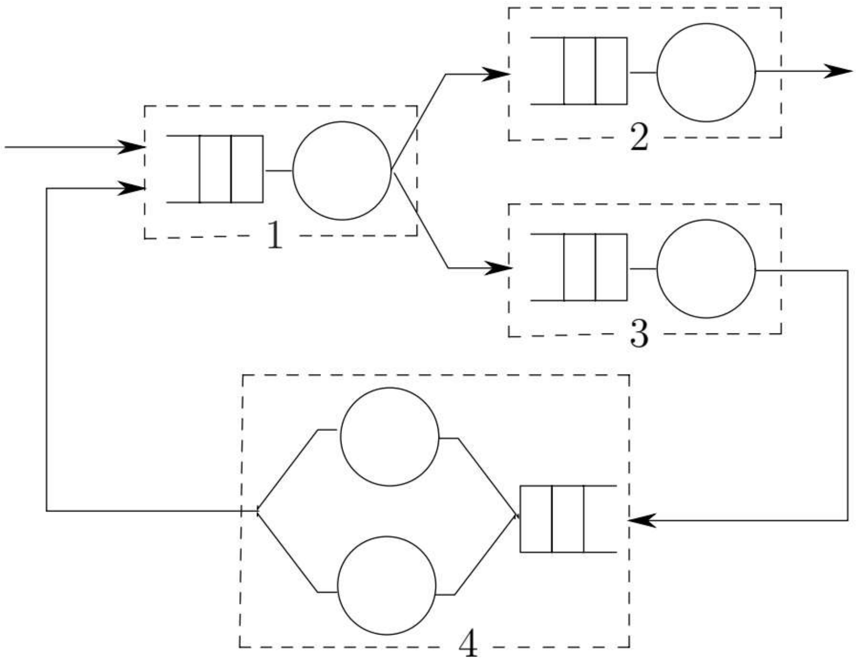
M / M / 1 / K

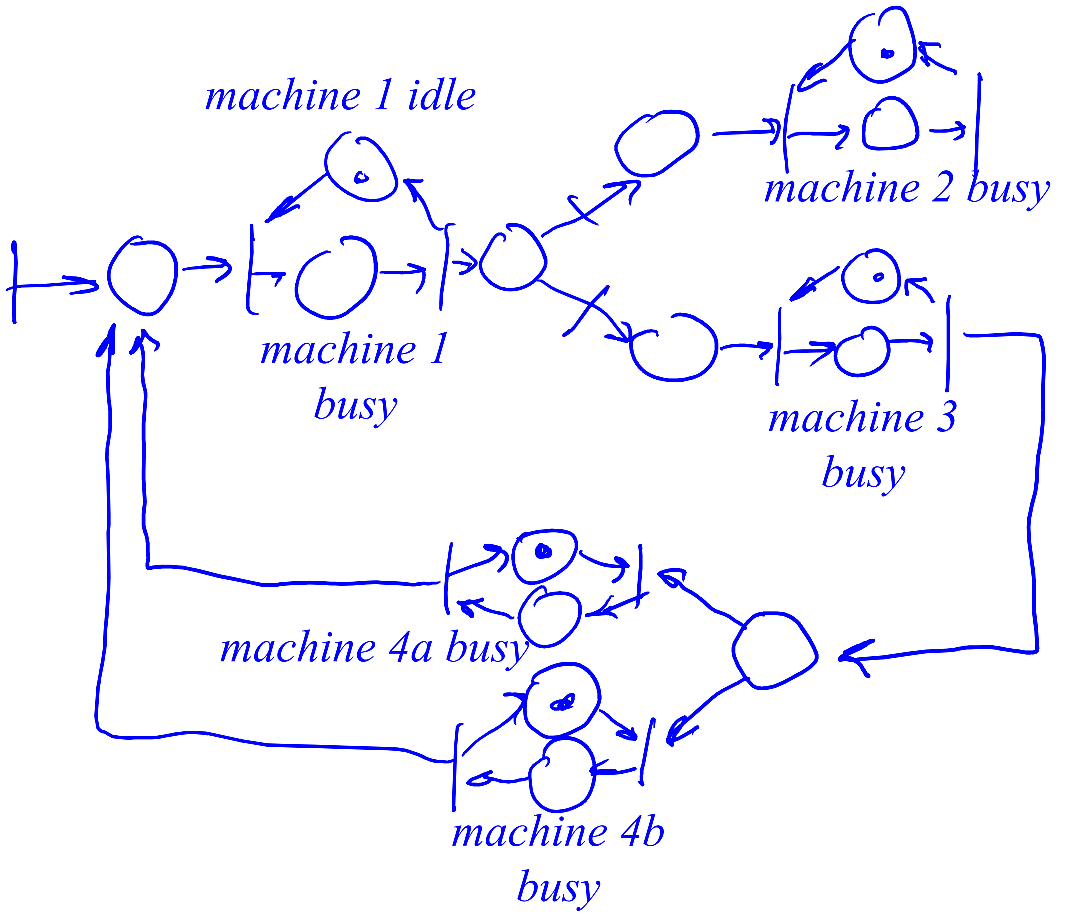


Closed queueing system serving **N customers** (M/M/2//N)



Queueing system including a closed loop





The Petri net shows the system can serve all incoming costumers. However, the number of costumers in the system can also become unbounded.

Stochastic Petri Nets

Study while designing - Given the dynamics $\mu(k+1) = \mu(k) + D\, q(k)$ ***consider a statistical characterization of the firing vector*** $q(k)$.

Stochastic Timed Petri Net (STPN) – T-timed Petri nets, where the time delays are modeled as random variables or probabilistic distributions.

Stochastic Petri Net (SPN) – STPN where the time **delays are exponentially distributed** (time between events is a Poisson point process).

Generalized Stochastic Petri net (GSPN) – SPN models which **allow immediate transitions**. I.e. some transitions are immediate and some other have delays modeled as random variables.

[Zurawski'94] "Petri nets and industrial applications: A tutorial." R. Zurawski, M. Zhou. IEEE Trans. on Industrial Electronics 41.6 (1994): 567-583. https://www.researchgate.net/publication/3217035

Example of an STPN [Zurawski'94]

System setup :

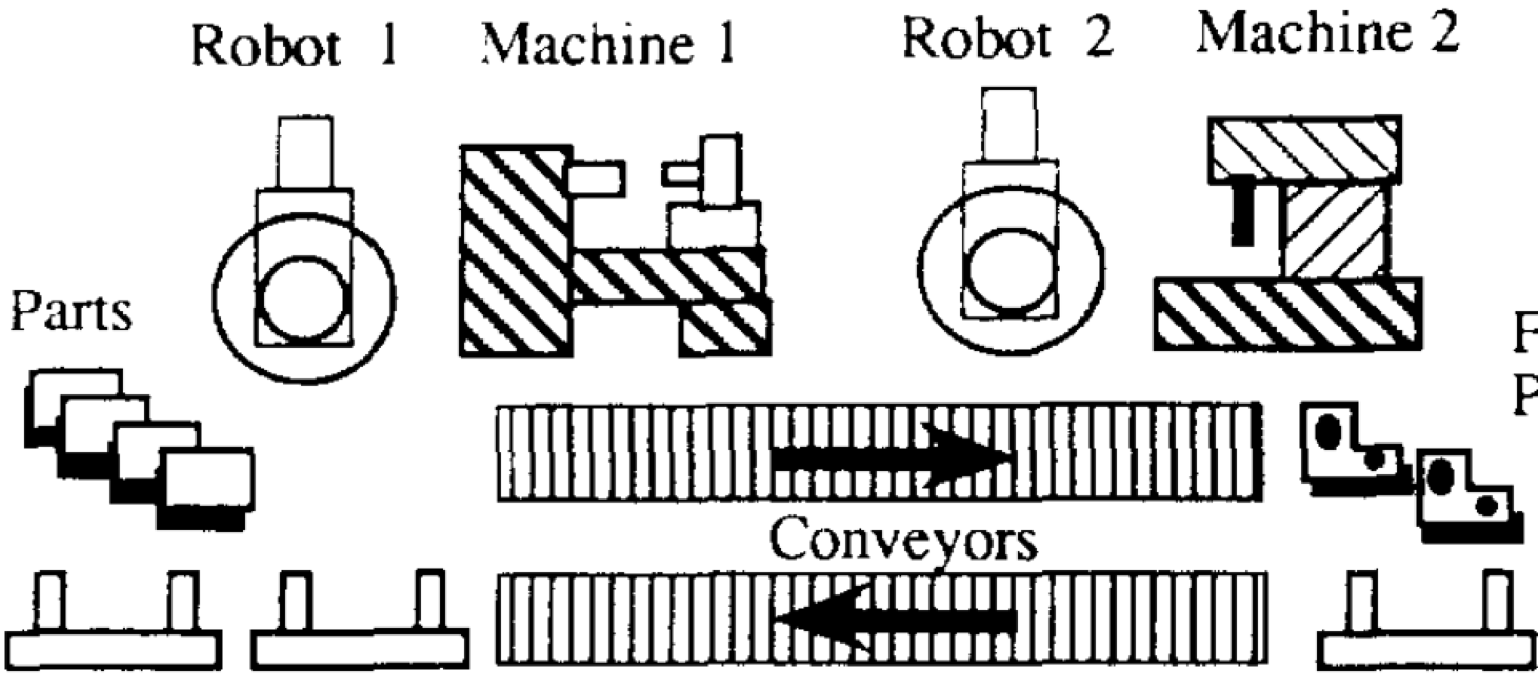
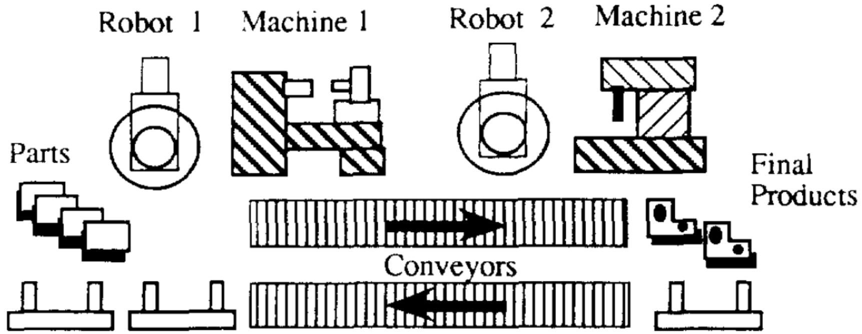
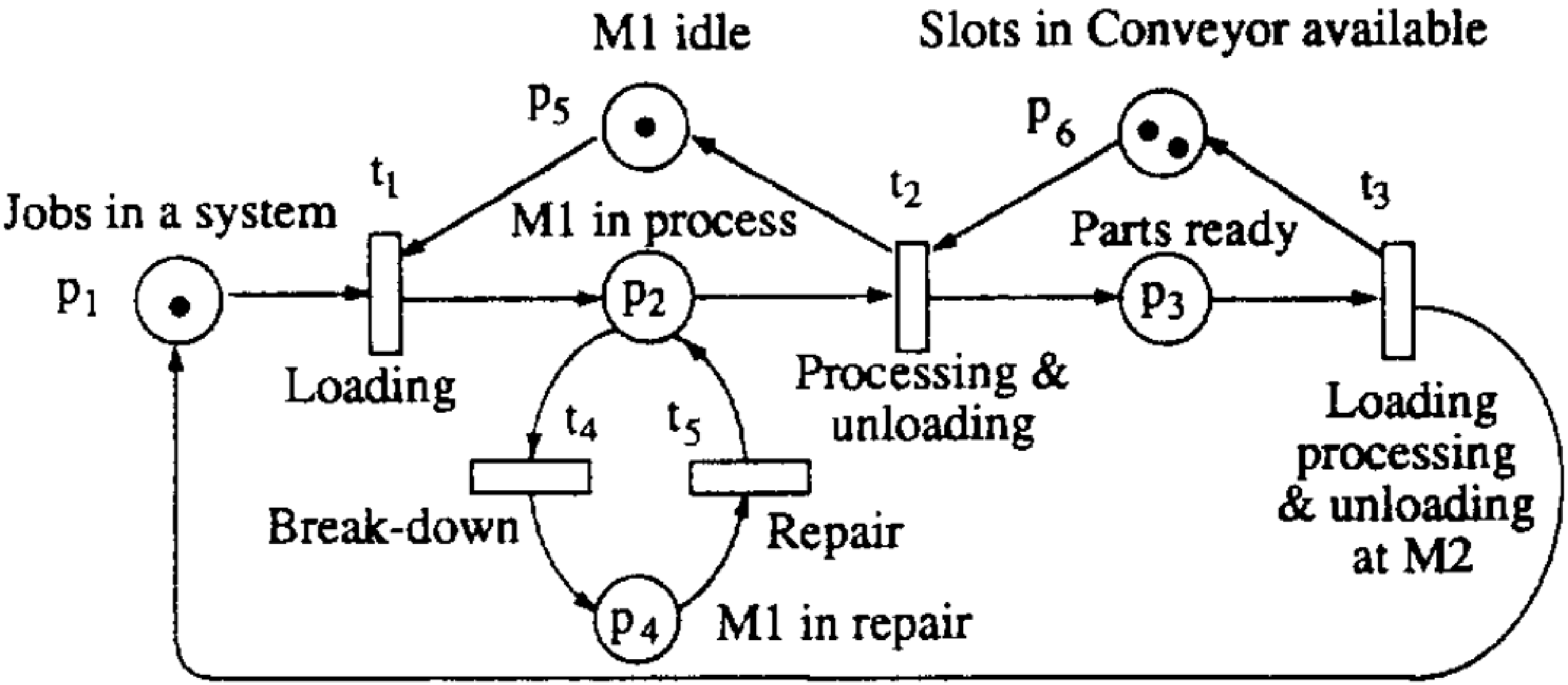


Fig. 17. A production line consisting of two machines, two robots, and two conveyers.

[Zurawski'94] "Petri nets and industrial applications: A tutorial." R. Zurawski, M. Zhou. IEEE Trans. on Industrial Electronics 41.6 (1994): 567-583. https://www.researchgate.net/publication/3217035

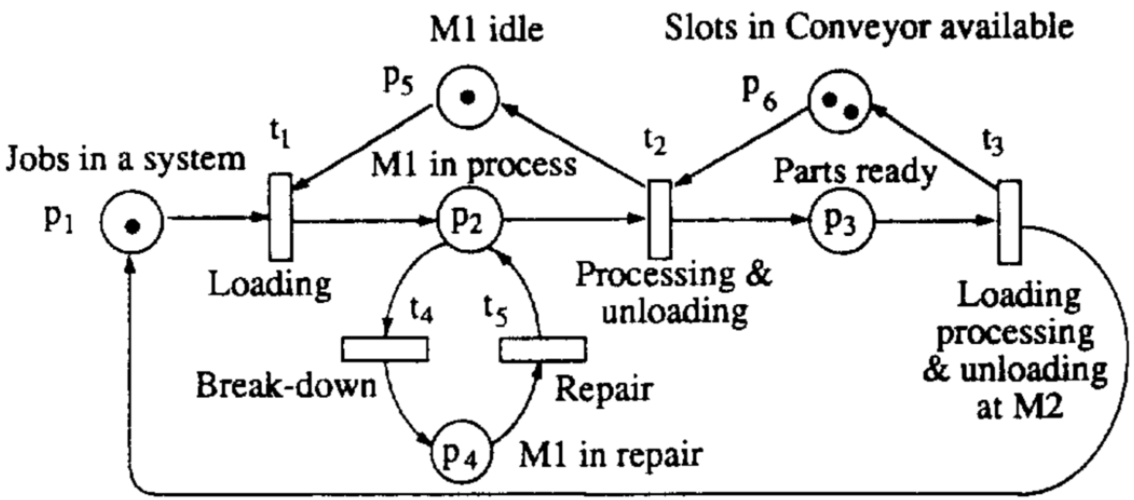
Example of an STPN





[Zurawski'94] "Petri nets and industrial applications: A tutorial." R. Zurawski, M. Zhou. IEEE Trans. on Industrial Electronics 41.6 (1994): 567-583

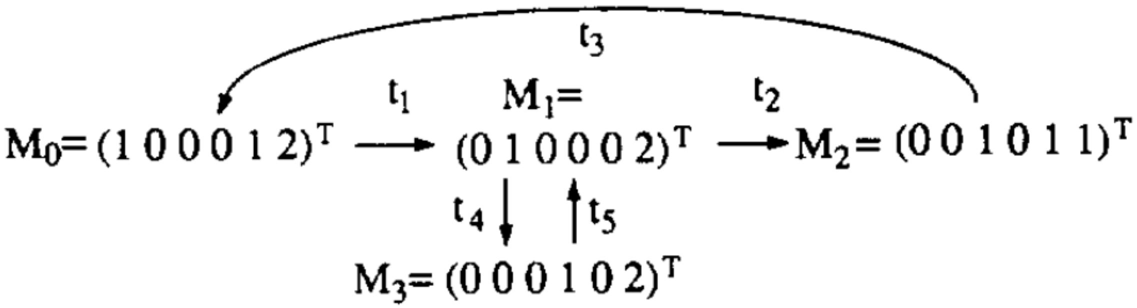
Example of an STPN



p1 Workpieces and pallets available
p2 M1 processing a workpiece
p3 Workpiece ready for processing at M2
p4 M1 in repair
p5 Conveyor slots available

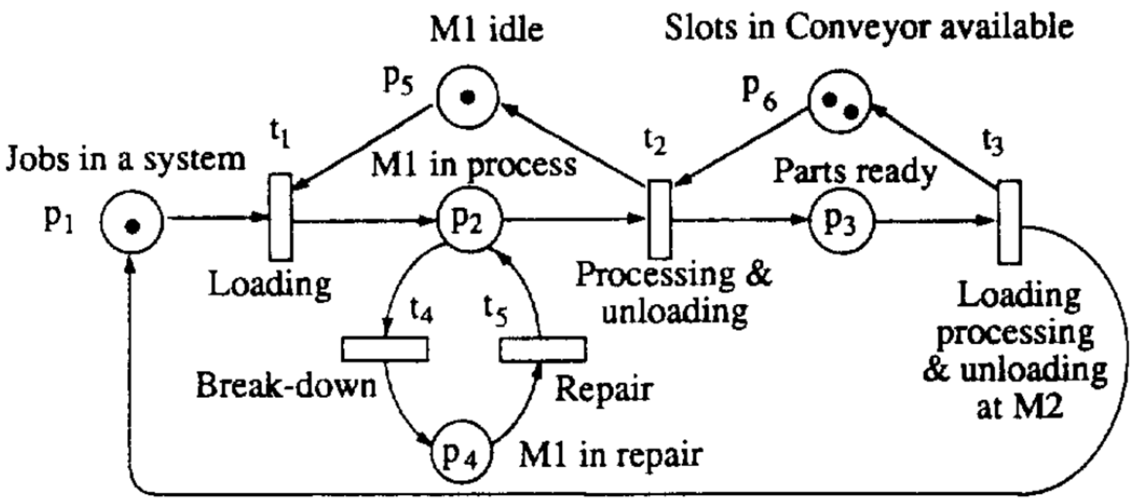
t1 Loading (R1 > M1)
t2 Processing and unloading (M1 > R1)
t3 Loading, processing and unloading (R2, M2)
t4 M1 breaks down
t5 M1 is repaired

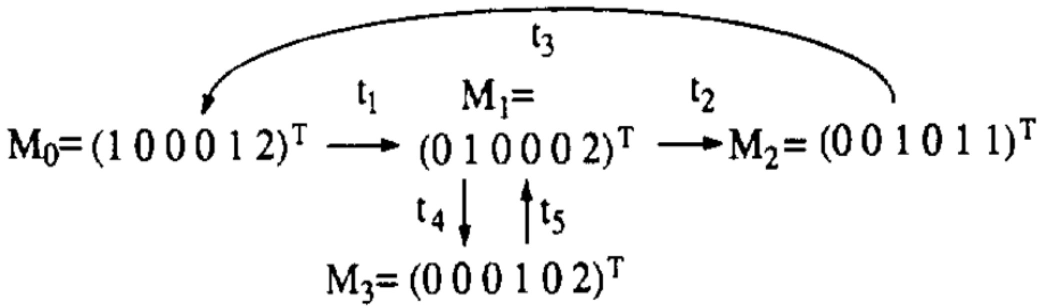
Reachability graph:



Q: The graph shows operation cycles. Can we tell state holding times (steady state statistics) given statistical information about the firing of transitions?

Example of an STPN





p1 Workpieces and pallets available
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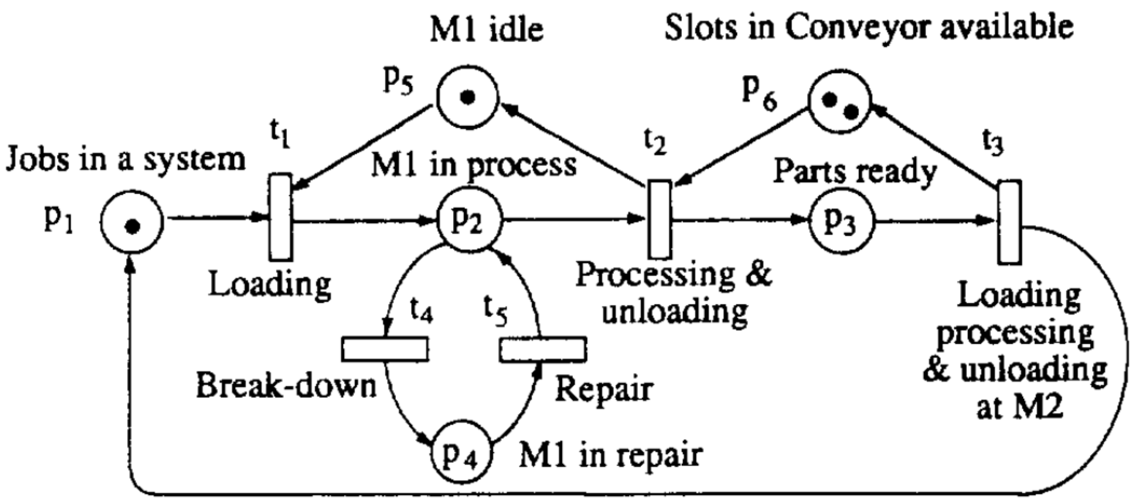
t1 Loading (R1 > M1)
t2 Processing and unloading (M1 > R1)
t3 Loading, processing and unloading (R2, M2)
t4 M1 breaks down
t5 M1 is repaired

Statistical information about transition firing:

$P(t_1) = 0.982591$
$P(t_2) = 0.122824$
$P(t_3) = 0.098259$
$P(t_4) = 0.089308$
$P(t_5) = 0.714460$

Example of an STPN




```
function main_test

% define the PN
D= [-1   0  +1   0   0
    +1  -1   0  -1  +1
     0  +1  -1   0   0
     0   0   0  +1  -1
    -1  +1   0   0   0
     0  -1  +1   0   0];
Dp=  D.*(D>0);
Dm= -D.*(D<0);
M0= [1 0 0 0 1 2]';

% simulate the PN
ti_tf= [0 50];
[~, M, ~]= PN_sim( Dm, Dp, M0, ti_tf );

% show stats places 1:4
fr= sum( M(:,1:4), 1 );
fr= fr / sum(fr);
fprintf(1, 'freq= ');
fprintf(1, '%.2f ', fr);
fprintf(1, '\n');

% output
% freq= 0.05 0.40 0.50 0.05
```

Statistical information about transition firing:

$P(t_1) = 0.982591$
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$P(t_5) = 0.714460$

```
function qk= PN_tfire(act, t)
% Possible-to-fire transitions
% act: 1xN : PN outputs
% t   : 1x1 : time
% qk : Mx1 : possible firing vector

x= [0.982591
    0.122824
    0.098259
    0.089308
    0.714460 ];

qk= round( rand(5,1) < x );
```

Can we compute `freq` *without running the simulation?*