

Modeling and Automation of Industrial Processes

Modelação e Automação de Processos Industriais / MAPI

DES and Industrial Automation ***GRAF CET vs PN***

<http://www.isr.tecnico.ulisboa.pt/~jag/courses/mapi2122>

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Prof. José Gaspar, rev. 2021/2022

Syllabus:

Chap. 2b – Analysis of Discrete Event Systems

...

Chap. 2c - DESs and Industrial Automation

GRAFCET / Petri Nets Relation

Model modification

Tools adaptation

Analysis of industrial automation solutions by analogy with
Discrete Event Systems

...

Chap. 5 – Supervision of DESs

Some pointers to Discrete Event Systems

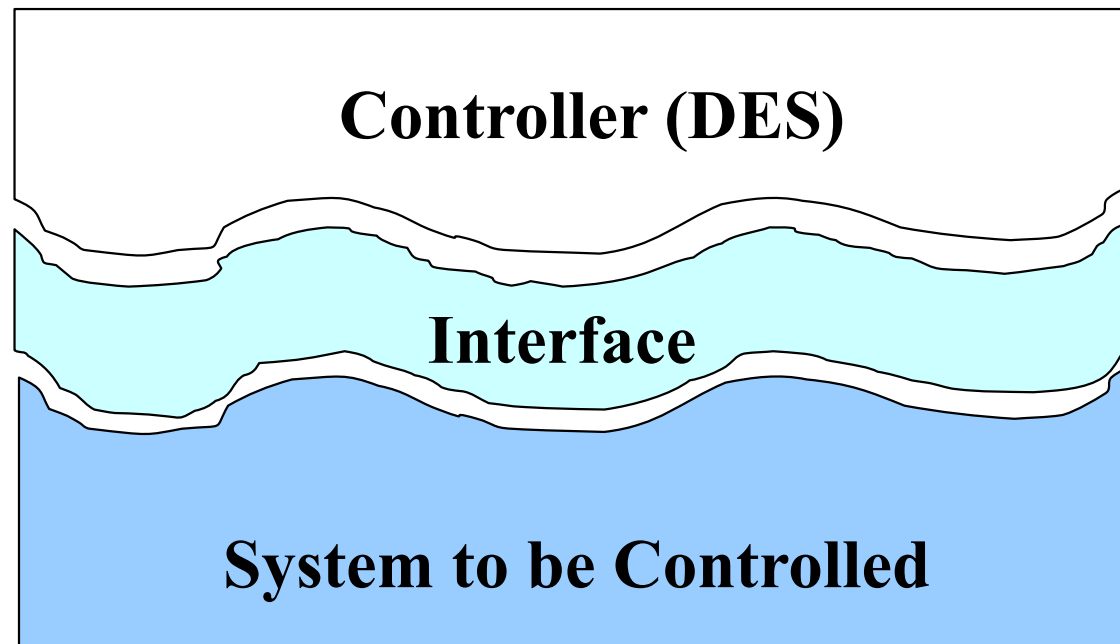
History: <http://prosys.changwon.ac.kr/docs/petrinet/1.htm>

Tutorial: <http://www.eit.uni-kl.de/litz/ENGLISH/members/frey/VnVSurvey.htm>
<http://vita.bu.edu/cgc/MIDEDS/>
<http://www.daimi.au.dk/PetriNets/>

Analysers,
and
Simulators: <http://www.ppgia.pucpr.br/~maziero/petri/arp.html> (in Portuguese)
<http://wiki.daimi.au.dk:8000/cpntools/cpntools.wiki>
<http://www.informatik.hu-berlin.de/top/pnk/download.html>

Bibliography: * **Petri Nets and GRAFCET: Tools for Modelling Discrete Event Systems**
R. DAVID, H. ALLA, New York : PRENTICE HALL Editions, 1992

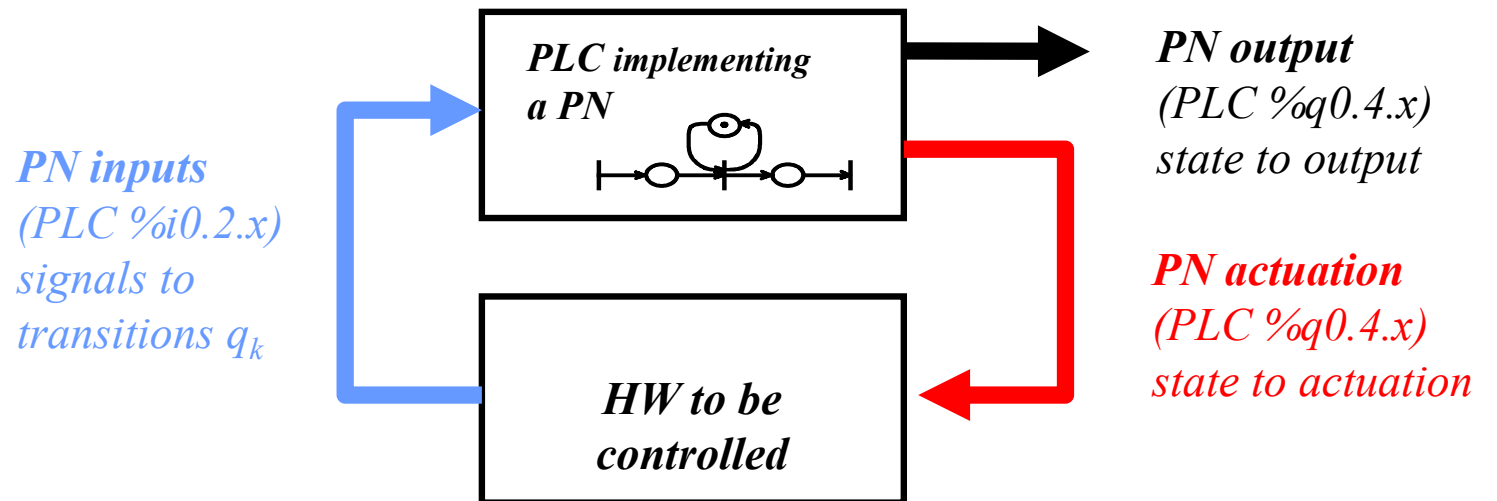
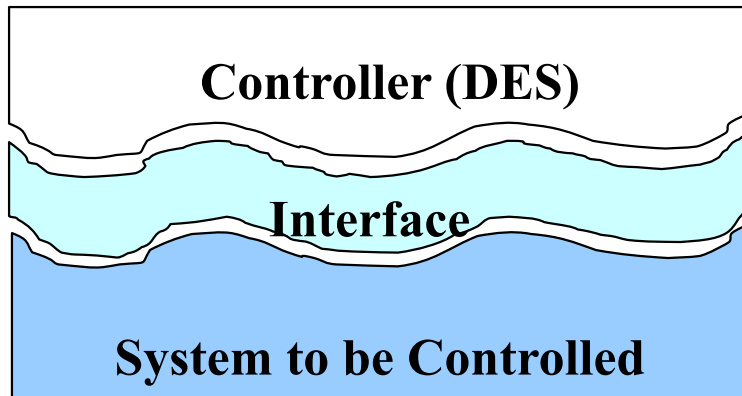
DES Implementation: Models



Besides *modelling the DES* it is required to

design models of the *System to be controlled* and
of the *Interface* to be used

DES Implementation: Petri Net and HW (input / output)



Given a Discrete Event System **how to implement it?**

1. Use a **GRAFCET**

- a) Less modeling ability
- b) Implementation in PLCs is straightforward
- c) **No analysis (or very scarce) methods available**

2. Use a **Petri Net**

- a) More modeling capacity
- b) **No direct implementation in PLCs** (therefore indirect or special software solutions required)
- c) Classical analysis methods available

(3. Use an **Automaton**)

Analysis of solutions

GRAFCET and Petri Nets

Similarities to exploit:

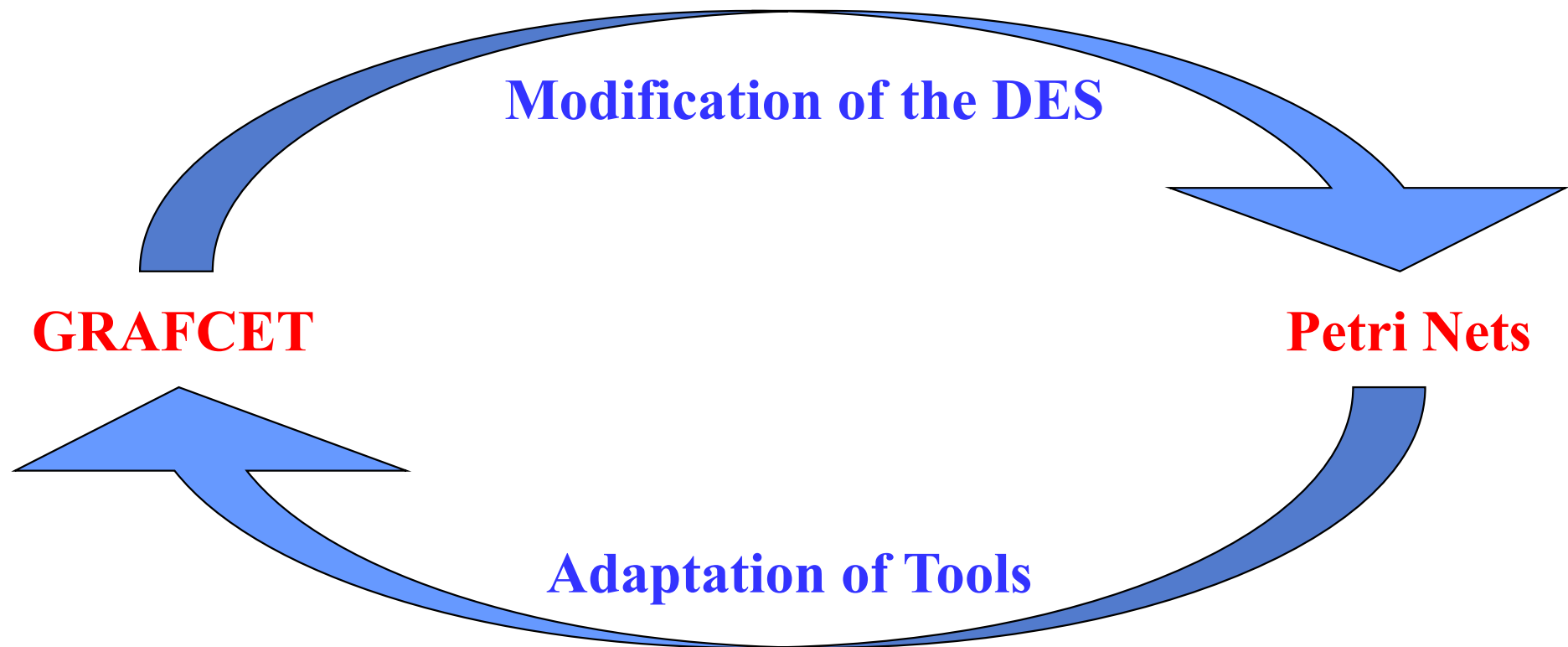
- a) Steps and Places are similar
- b) Transitions compose both tools
- c) Places can be used to implement **counters** (on marking changes) and **binary variables** (marked vs unmarked place)
- d) **Logic** functions can be rewritten resorting to the firing of transitions

Differences to be taken into account:

- a) Firing rules (mutual exclusion)
- b) **Conflicts**
- c) **Binary** activation of stages vs weights (>1) on transition input arcs
- d) Interface with the system to be controlled
- e) Activation functions

Implementation of DES using GRAFCET

Analysis

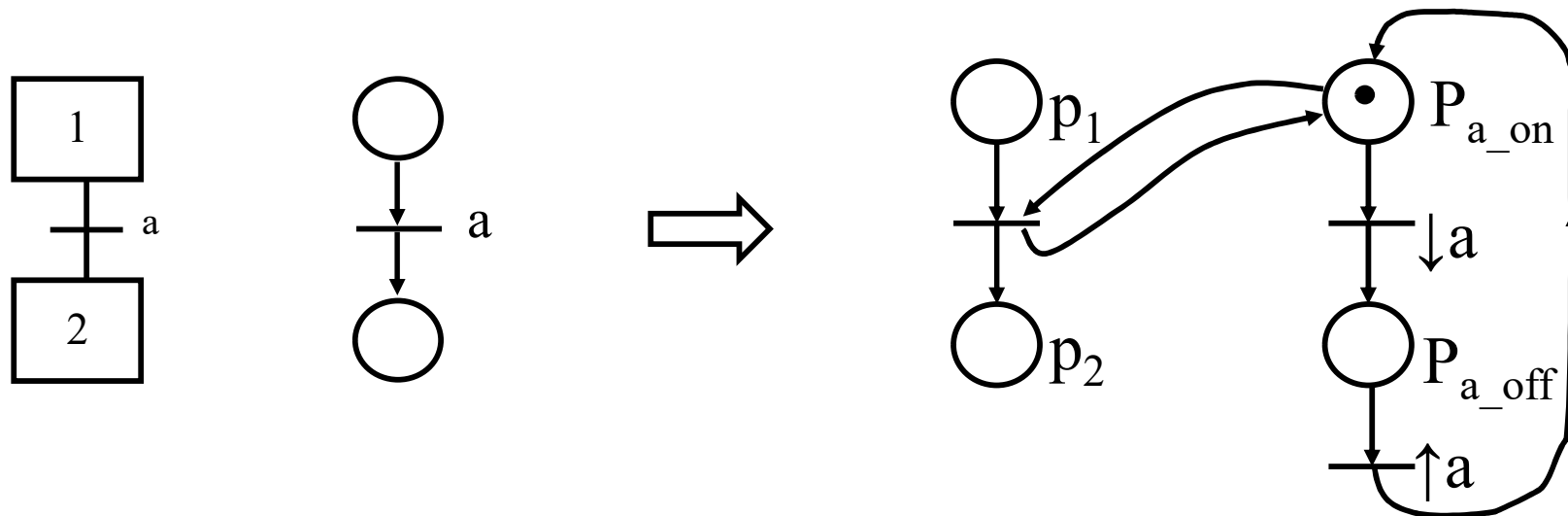


Analysis of solutions

GRAFCET → Petri Nets

Representation of variables active on **level**,

i.e. give a marking representation to an input active as a level

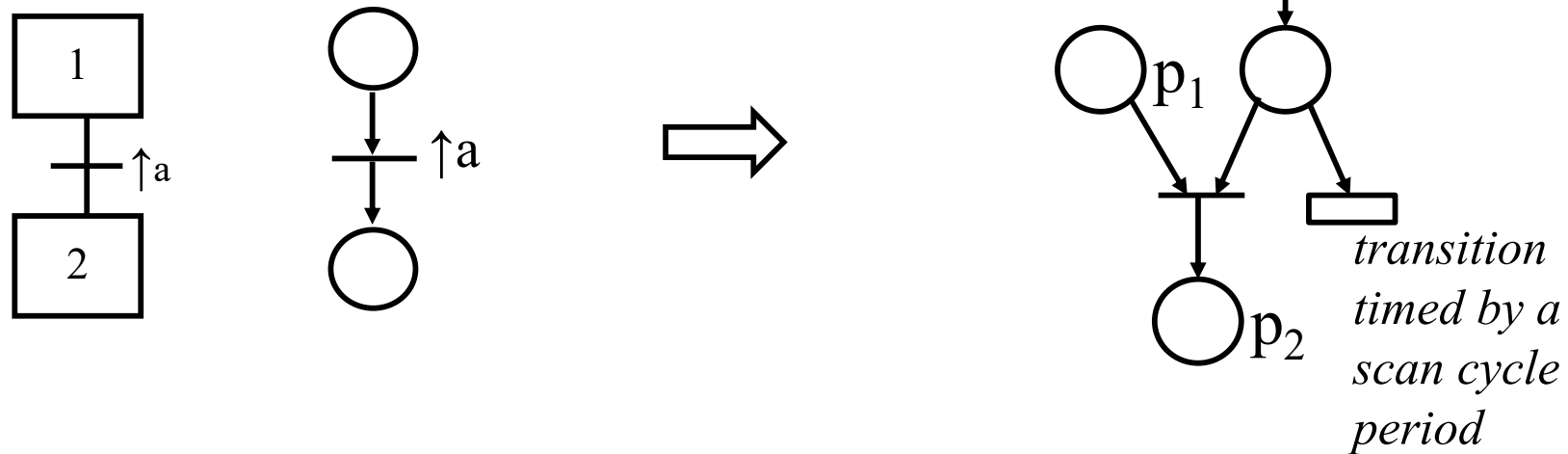


Analysis of solutions

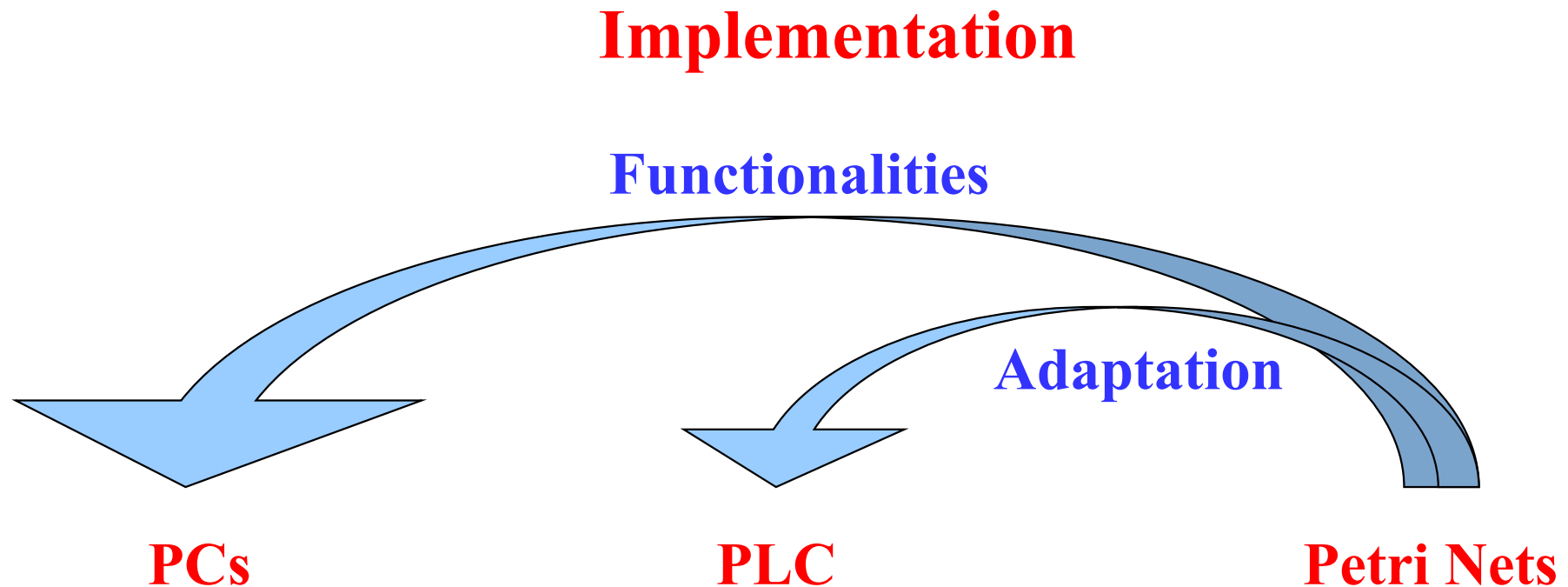
GRAFCET → Petri Nets

Representation of variables active at **edge**,

i.e. give a marking representation to an input active as a edge trigger



Implementation of DES using Petri Nets



Both solutions are valid
(but out of the scope of this course).

Analysis of solutions

Petri Nets → GRAFCET

Adaptation of Tools:

- 1) Reachability Tree → Reachability Graph
- 2) Method of the Matrix Equations
to describe the state evolution

Petri Nets → GRAFCET

Reachability Graph

Is a graph containing the **reachable markings**.

Is composed by two types of nodes:

- **terminal**
- **interior**

The **duplicated** nodes are not represented. *They become connected to the respective copies.*

The **symbol infinity (ω)** is not used in GRAFCET. *While in Petri nets ω is necessary to obtain finite trees, when a marking covers other(s), in GRAFCET the finite number of states renders ω unnecessary.*

Petri Nets → GRAFCET

Reachability Graph

Theorem - If a reachability graph has **terminal nodes** then the corresponding GRAFCET has **deadlocks**.

This reachability graph will be used to study the properties introduced in Chapter 6.

Petri Nets → GRAFCET

Reachable Set

Given the GRAFCET $G=(S, T, I, O, \mu_0)$ with initial marking μ_0 , the set of all markings that are reachable is the **reachable set** $R(G, \mu_0)$.

Property usage: $\mu \in R(G, \mu_0)$

*Remark: the Reachable Set **is not infinite!***

*Given a GRAFCET with m steps it has **at most 2^m nodes.***

In some cases, m steps imply just m nodes.

Petri Nets → GRAFCET

Safeness, Boundedness and Limitation

The GRAFCET $G=(S, T, I, O, \mu_0)$ is always safe

The GRAFCET may become not safe if one uses some auxiliary elements such as counters or buffers.

If one uses these auxiliary elements, the analysis methods studied for Petri Nets can be used to assert specific safeness, boundedness and limitation properties.

Petri Nets → GRAFCET

Conservation

A GRAFCET $G=(S, T, I, O, \mu_0)$ is **strictly conservative** if for all $\mu' \in R(C, \mu)$

$$\sum_{p_i \in P} \mu'(p_i) = \sum_{p_i \in P} \mu(p_i).$$

A GRAFCET $G=(S, T, I, O, \mu_0)$ is **conservative** if there exist a weight vector **w without null elements**, for all $\mu' \in R(C, \mu)$ such that

$$\sum_{p_i \in P} w(p_i) \mu(p_i) = \text{constant}$$

Petri Nets → GRAFCET

Liveness of transitions: The transition t_j is live of

Level 0 - it can never be fired (dead transition).

Level 1 - if it is potentially firable, e.g. if there exist $m' \in R(C, \mu)$ such that t_j is enabled in μ' .

Level 2 - if, for each positive n , there exist a sequence of firings where occurs n firings of t_j .

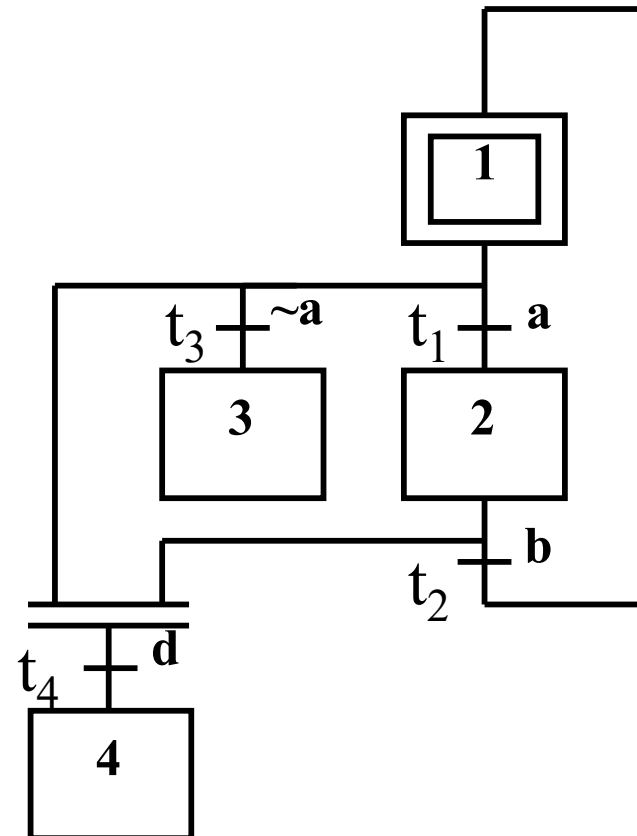
Level 3 - if there exist a sequence of firings where an infinite number of firings of t_j occurs.

Level 4 - if for each $\mu' \in R(C, \mu)$ there exist a sequence s that enables the firing of t_j (live transition).

Petri Nets → GRAFCET

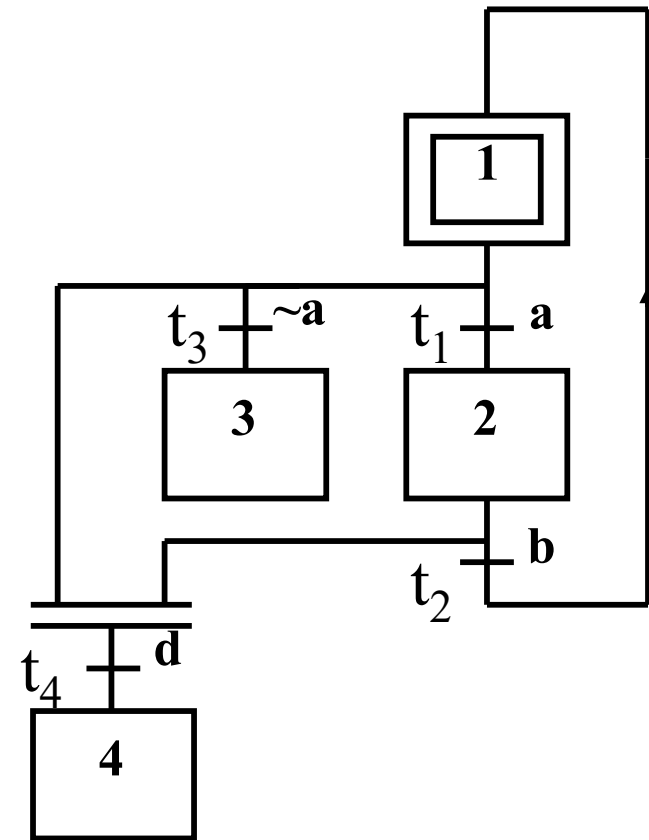
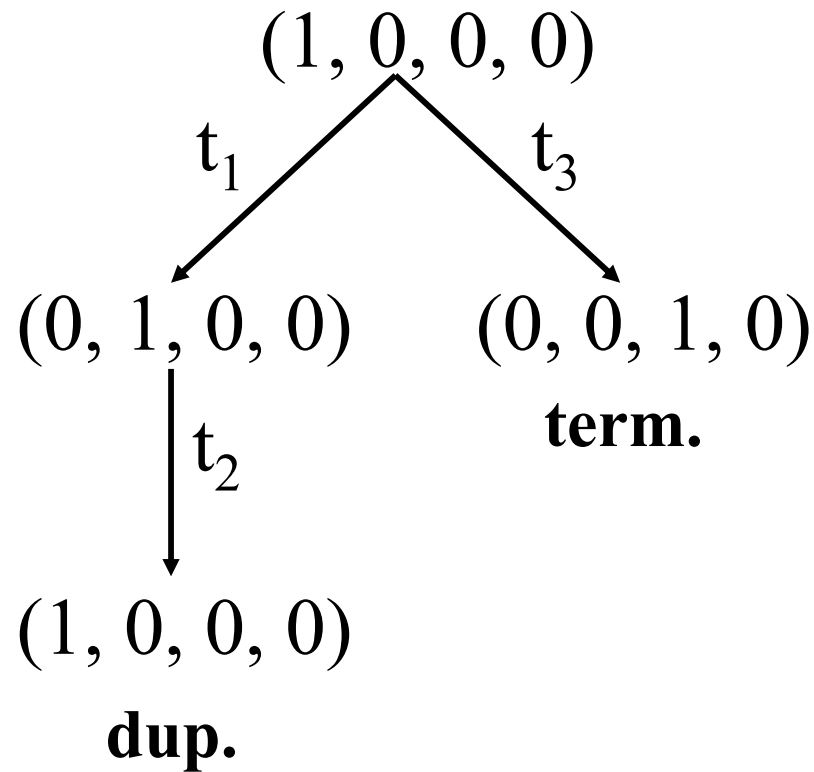
Example of GRAFCET

- t_1 and t_2 are level 3.
- t_3 is level 1.
- t_4 is level 0.



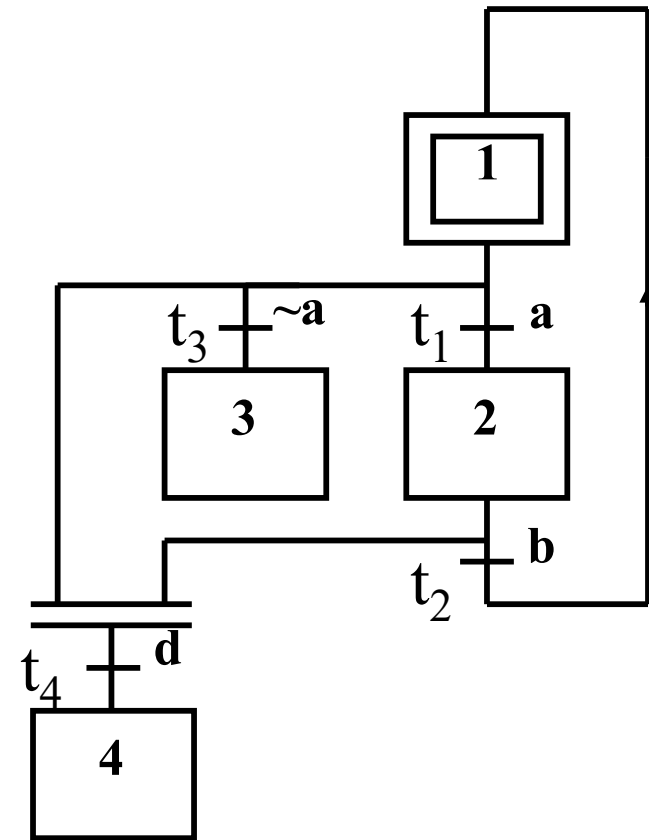
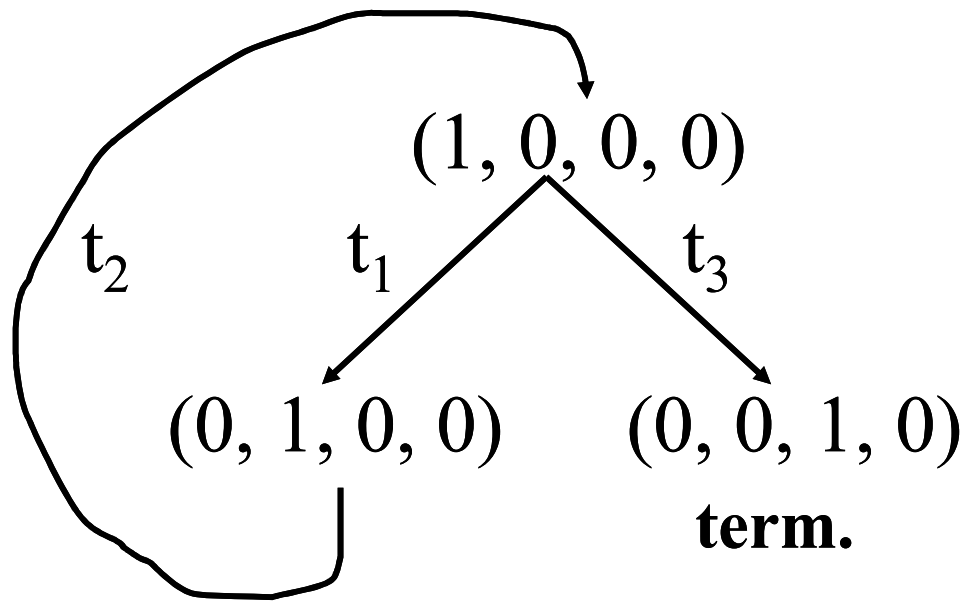
Petri Nets → GRAFCET

Example of GRAFCET



Petri Nets → GRAFCET

Example of GRAFCET



Strictly conservative.

Petri Nets → GRAFCET

Method of Matrix Equation (for the state evolution)

The evolution of a GRAFCET can be written in compact form as:

$$\mu' = \mu + Dq$$

where:

- μ' - Desired marking vector (column vector)
- μ - Initial marking
- q - Transition firing vector (column vector)
- D - Incidence matrix. Accounts for the token evolution as a consequence of transitions firing.

Petri Nets → GRAFCET

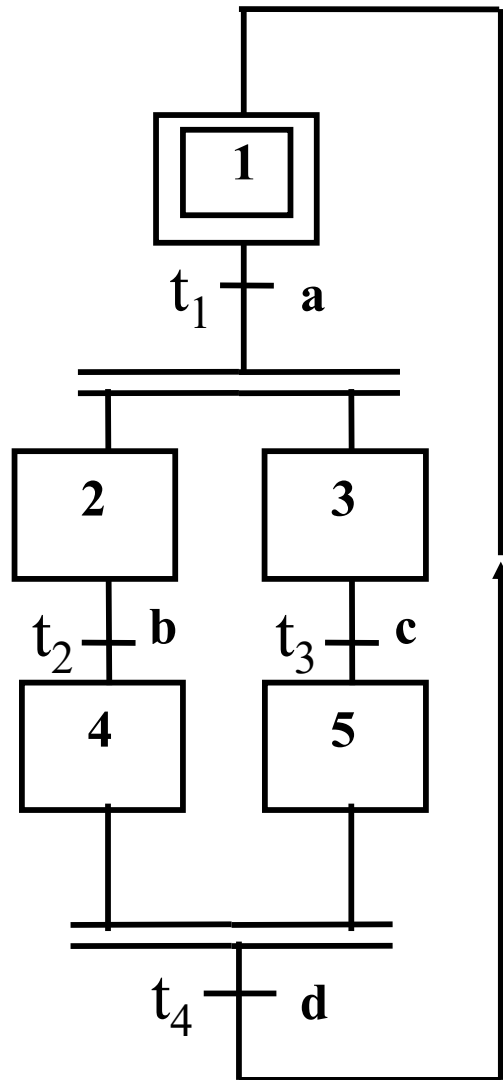
Additional problems that can be addressed resorting to the Method of Matrix Equations:

- **Reachability** (sufficient condition)

Theorem – if the problem of finding the vector of firings, for a GRAFCET without conflicts, from the state μ to the state μ' has no solution using the Method of Matrix Equations, then the problem of reachability of m' is impossible.

- **Conservation** – the conservation vector can be computed automatically.
- **Temporal invariance** – cycles of operation can be found.

Example of GRAFCET



$$\mu' = \mu + Dq$$

$$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Conservation

$$w^T D = 0$$

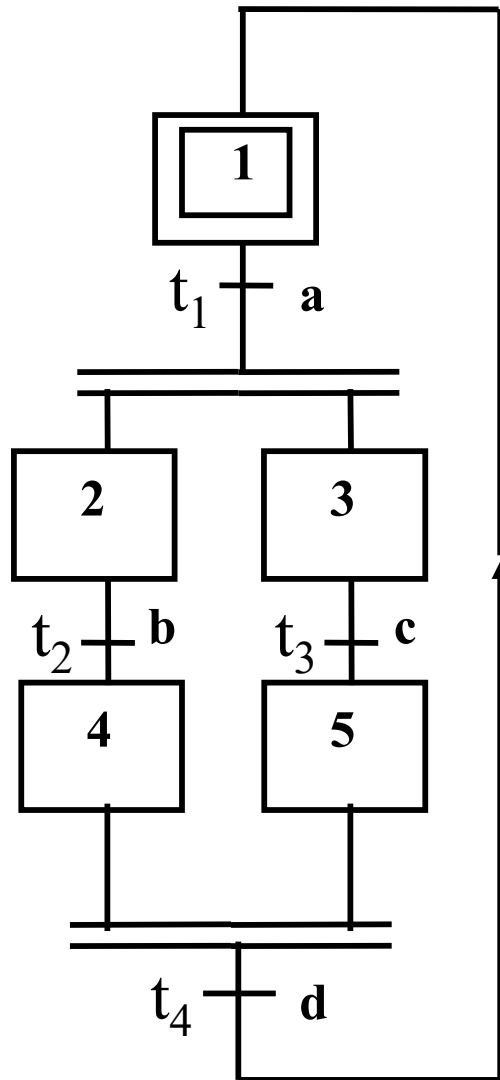
$$\begin{cases} -w_1 + w_2 + w_3 = 0 \\ -w_2 + w_4 = 0 \\ -w_3 + w_5 = 0 \\ w_1 - w_4 - w_5 = 0 \end{cases}$$

$$\begin{aligned} w_1 &= w_3 + w_4 \\ w_1 &= w_2 + w_5 \\ w_2 + w_3 &= w_4 + w_5 \end{aligned}$$

Solution:
Undetermined
set of equations

$$w = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Example of GRAFCET



$$\mu' = \mu + Dq$$

$$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad q = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{bmatrix}$$

Temporal invariance $Dq = 0$

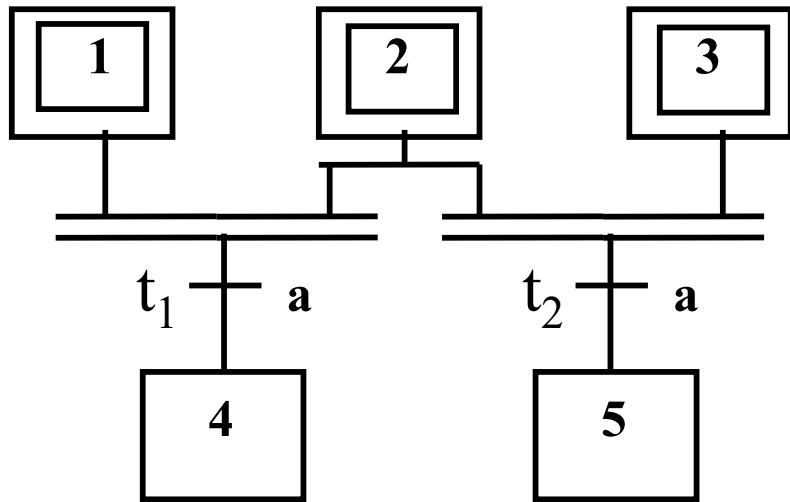
Solution:
Set of equations
with solution

$$\begin{cases} -\sigma_1 + \sigma_4 = 0 \\ \sigma_1 - \sigma_2 = 0 \\ \sigma_1 - \sigma_3 = 0 \\ \sigma_2 - \sigma_4 = 0 \\ \sigma_3 - \sigma_4 = 0 \end{cases}$$

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 1.$$

Example of GRAFCET

$$\mu' = \mu + Dq$$



$$\mu' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 & 0 \\ -1 & -1 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$q = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$$

Set of Equations is impossible
therefore the marking is not reachable...

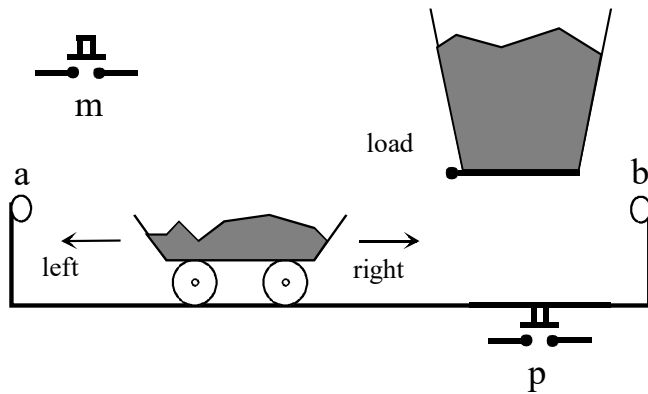
WRONG !

The method fails if there are conflicts!

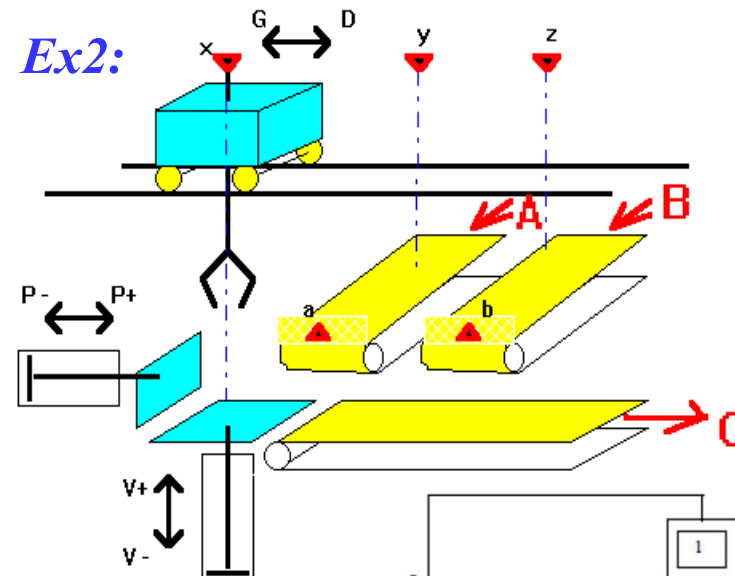
$$\left\{ \begin{array}{l} 0 = 1 - \sigma_1 \\ 0 = 1 - \sigma_1 - \sigma_2 \\ 0 = 1 - \sigma_2 \\ 1 = \sigma_1 \\ 1 = \sigma_2 \end{array} \right.$$

Homework: see in previous chapters GRAFCET examples, convert them to Petri nets and study their properties (ignore the IO)

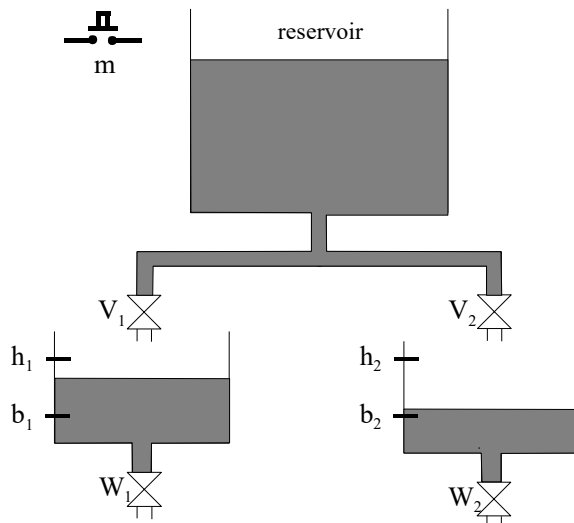
Ex1:



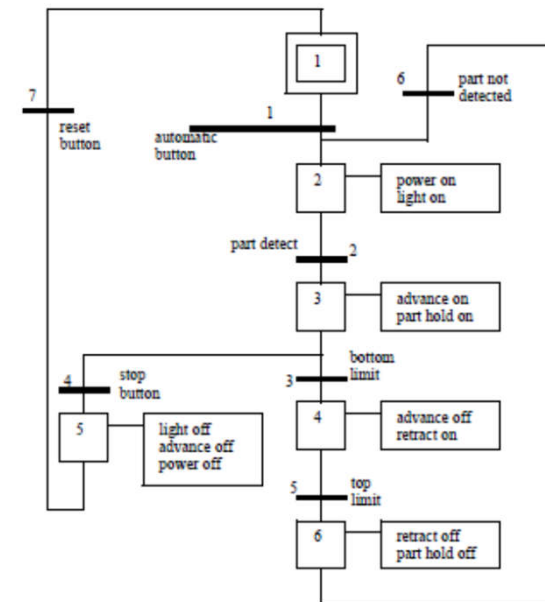
Ex2:



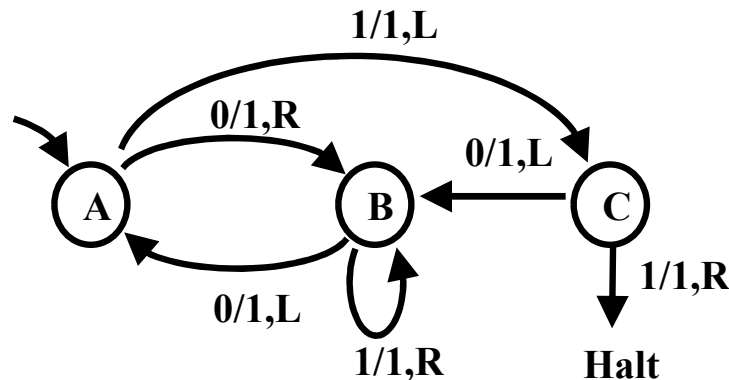
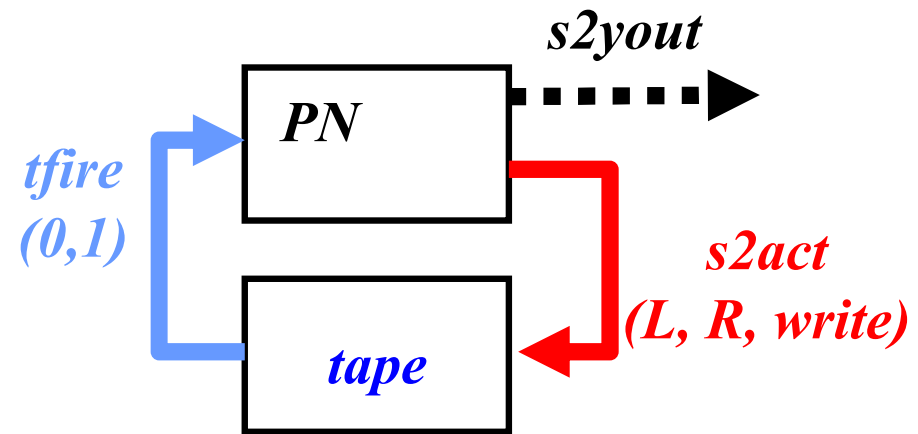
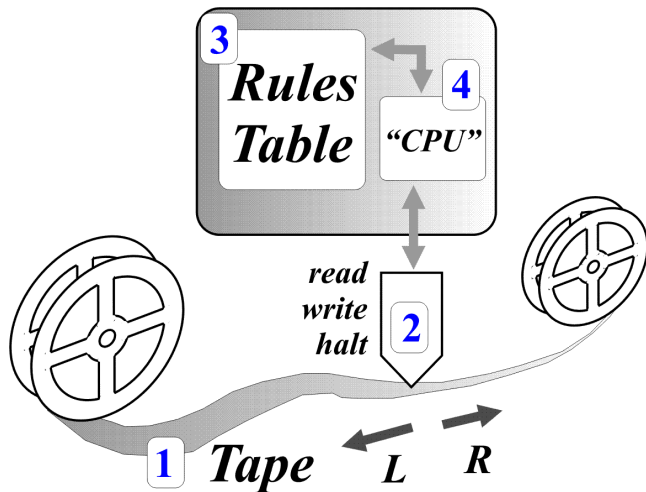
Ex3:



Ex4:



Homework 2: See in previous chapters Finite State Machines (FSMs), convert them to Petri nets and study their properties. Note that outputs at transitions can also be represented.



outputs = tape left, right, write
input = tape read (one bit, i.e. 0 or 1)

(see slides for chapter 7)