Modeling and Automation of Industrial Processes

Modelação e Automação de Processos Industriais / MAPI

Analysis of Discrete Event Systems
Running a Petri net with I/O

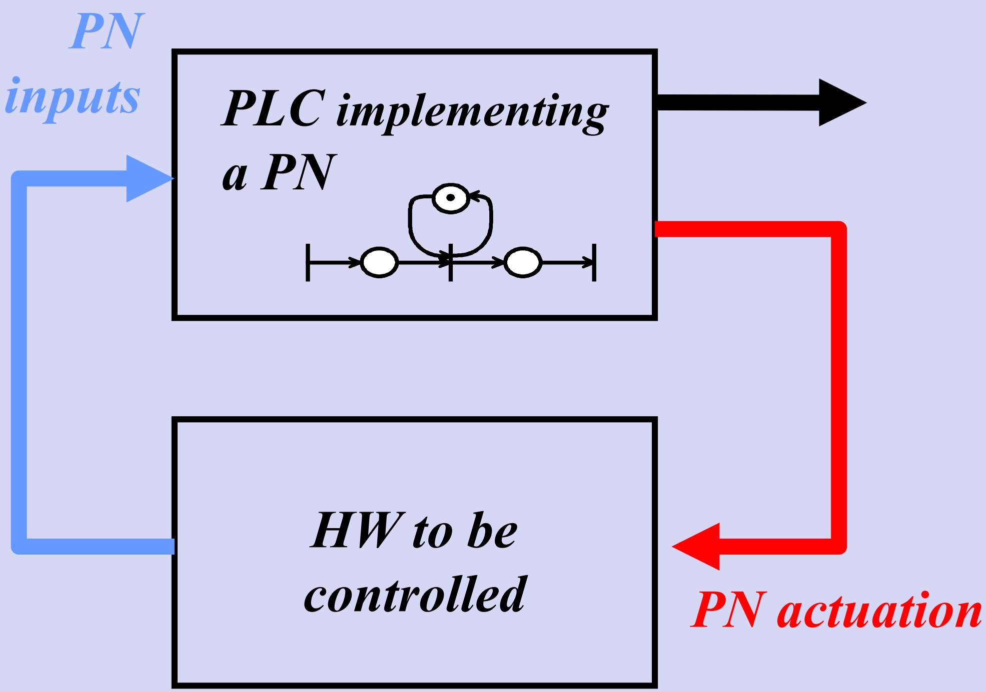
http://www.isr.tecnico.ulisboa.pt/~jag/courses/mapi22d

Prof. José Gaspar, rev. 2022/2023

Running a Petri net with HW inputs and outputs

Petri nets with input/output are a way of designing programs for Programmable Logic Controllers (PLCs).

Running a Petri net with I/O requires a system to interact with or, in other words, to **supervise**.

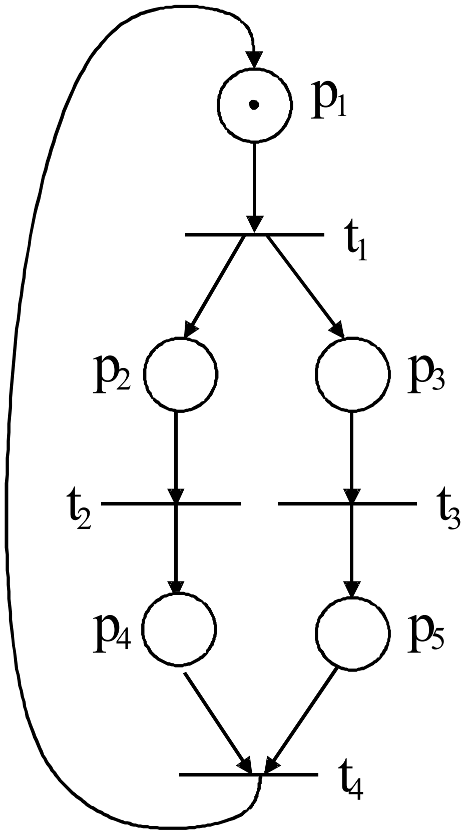


PN actuation outputs required to drive the system

PN inputs signals observed in the system and used to drive the Petri net.

http://www.isr.tecnico.ulisboa.pt/~jag/course_utils/pn_sim/PN_sim.html

Alternative definition (#3), how to build an Incidence Matrix, D ?



Petri net (P, T, D^-, D^+, μ_0), $D = D^+ - D^-$

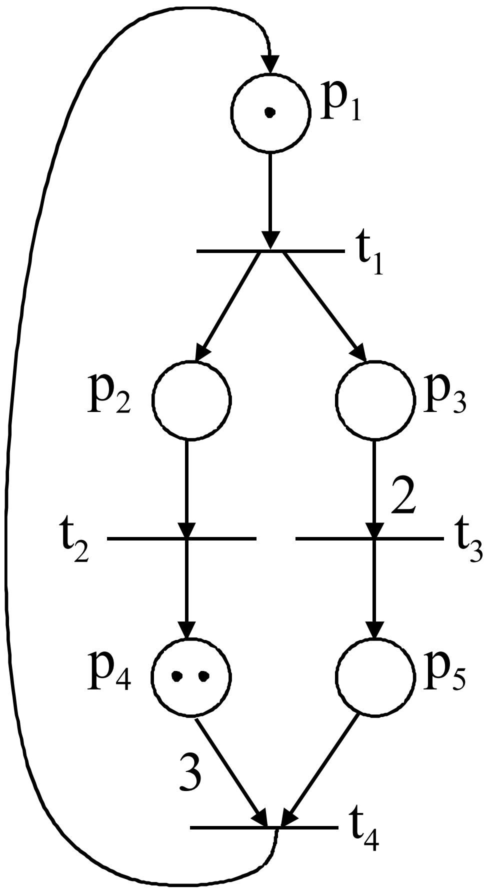
Set of places $P=\{p_1, p_2, p_3, p_4, p_5\}$

Set of transitions $T=\{t_1, t_2, t_3, t_4\}$

$$\begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{array} \quad \begin{array}{c} \quad t_1 \quad t_2 \quad t_3 \quad t_4 \\ D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \end{array}$$

Read the example marked by the arrows as "firing t_1 takes a mark from p_1 and adds a mark to p_2 and adds another mark to p_3".

Alternative definition (#3) of a Petri net, example with arc weights



(P, T, D^-, D^+, μ_0)

$P=\{p_1, p_2, p_3, p_4, p_5\}$

$T=\{t_1, t_2, t_3, t_4\}$

$$D = \begin{bmatrix} -1 & & & +1 \\ +1 & -1 & & \\ +1 & & -2 & \\ & +1 & & -3 \\ & & +1 & -1 \end{bmatrix}$$

$\mu_0 = \{1, 0, 0, 2, 0\}$

$$D^+ = \max(0, D) = \begin{bmatrix} & & & 1 \\ 1 & & & \\ 1 & & & \\ & 1 & & \\ & & 1 & \end{bmatrix}$$

$$D^- = -\min(0, D) = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 2 & \\ & & & 3 \\ & & & 1 \end{bmatrix}$$

Alternative definition (#3) of a Petri net

A marked Petri net is a *5-tuple* [Iordache06]

$$(\mathbf{P}, \mathbf{T}, \mathbf{D}^{-}, \mathbf{D}^{+}, \mu_0) \quad \text{or} \quad (\mathbf{P}, \mathbf{T}, \mathbf{Pre}, \mathbf{Post}, \mu_0)$$

where:

P	- set of places	
T	- set of transitions	
Pre	- pre conditions matrix	Pre : $P \times T \rightarrow \mathbb{N}$
Post	- post conditions matrix	Post : $P \times T \rightarrow \mathbb{N}$
μ_0	- initial marking	μ_0 : $P \rightarrow \mathbb{N}$

Note: $\mathbf{D} = \mathbf{D}^{+} - \mathbf{D}^{-} = \mathbf{Post} - \mathbf{Pre}$ is named the incidence matrix.

[Iordache06] "Supervisory control of concurrent systems: a Petri net structural approach", Marian Iordache and Panos J. Antsaklis, Birkhauser Boston, 2006

Analysis Methods, 2- MME

Method of the Matrix Equations (MME) of State Evolution

The dynamics of the Petri net state can be written in compact form as:

$$\mu(k+1) = \mu(k) + Dq(k)$$

where:

- μ (k+1) - marking to be reached
- μ (k) - initial marking
- q(k) - firing vector (transitions)
- D - incidence matrix. Accounts the balance of tokens, giving the transitions fired.

Analysis Methods, 2- MME

Method of the Matrix Equations (MME) of State Evolution

For a Petri net with n places and m transitions

$$\mu \in N_0^{\,n}$$

$$q \in N_0^{\,m}$$

$$\boxed{D = D^+ - D^-}, \quad D \in Z^{n\times m}, \quad D^+ \in N_0^{n\times m}, \quad D^- \in N_0^{n\times m}$$

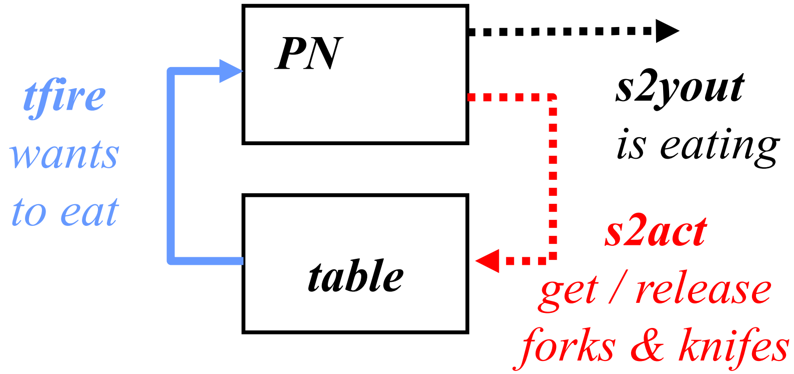
The enabling firing rule is $\boxed{D^- q \leq \mu}$

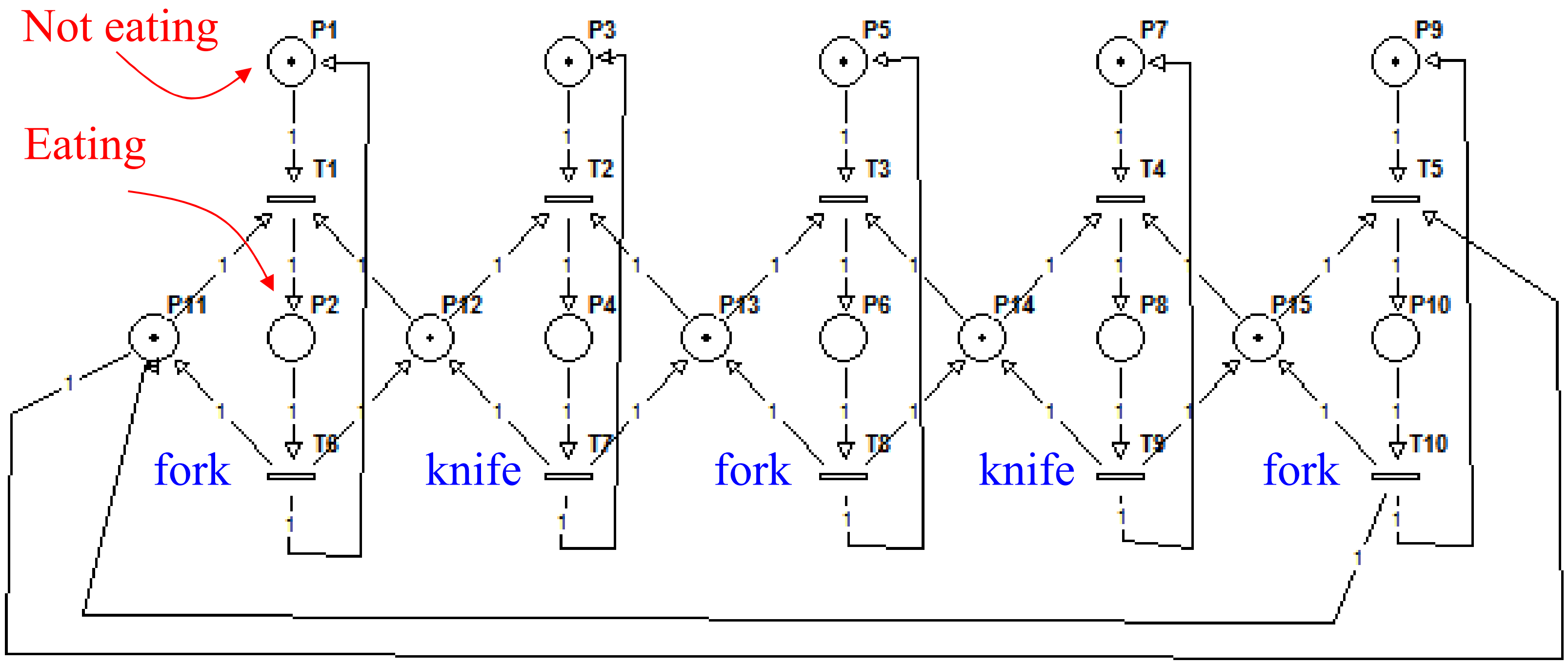
Can also be written in compact form as the inequality $\mu + Dq \geq 0$, interpreted element-by-element.

Note: unless otherwise stated in this course all vector and matrix inequalities are read element-by-element.

Example 1: Philosophers Dinner

This PN has inputs "Philosopher i wants to eat".

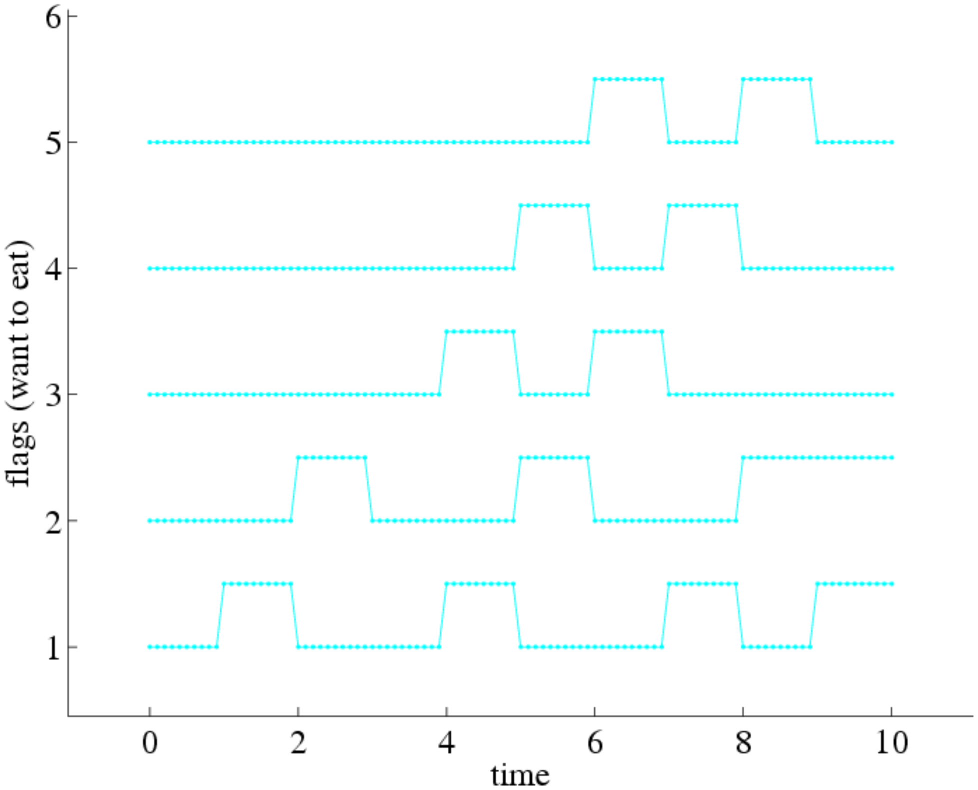




Philosopher1, Philosopher2, Philosopher3, Philosopher4, Philosopher5

Running a Petri net with HW inputs and outputs

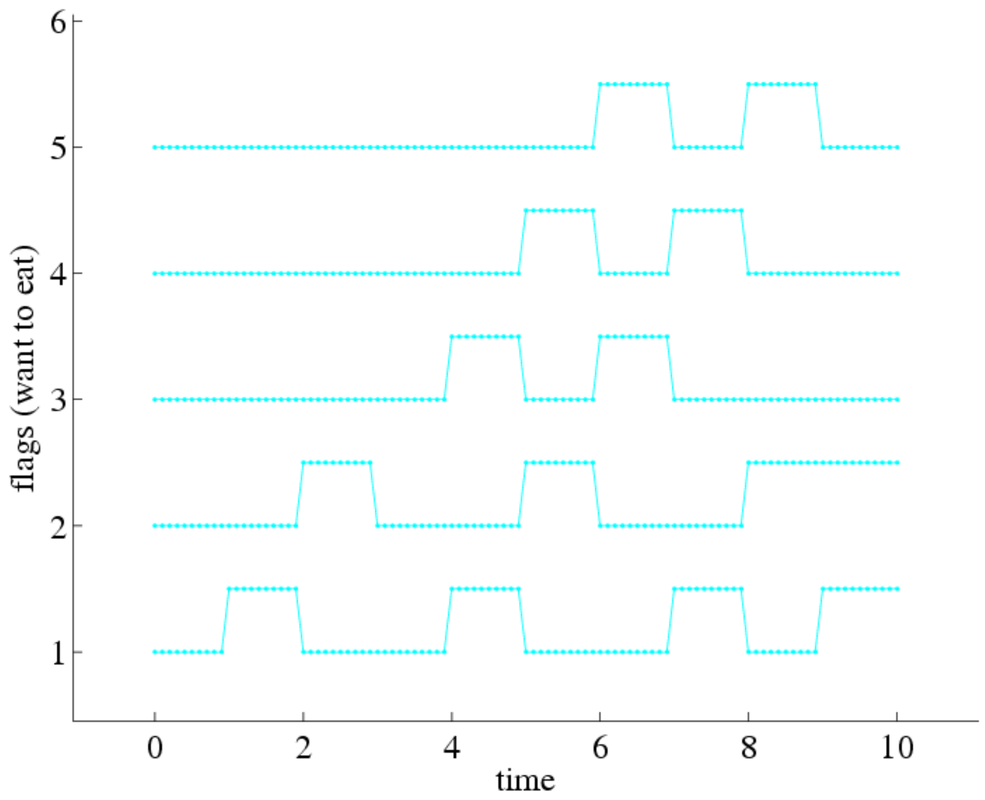
Example: Philosophers Dinner – input / events



tu =

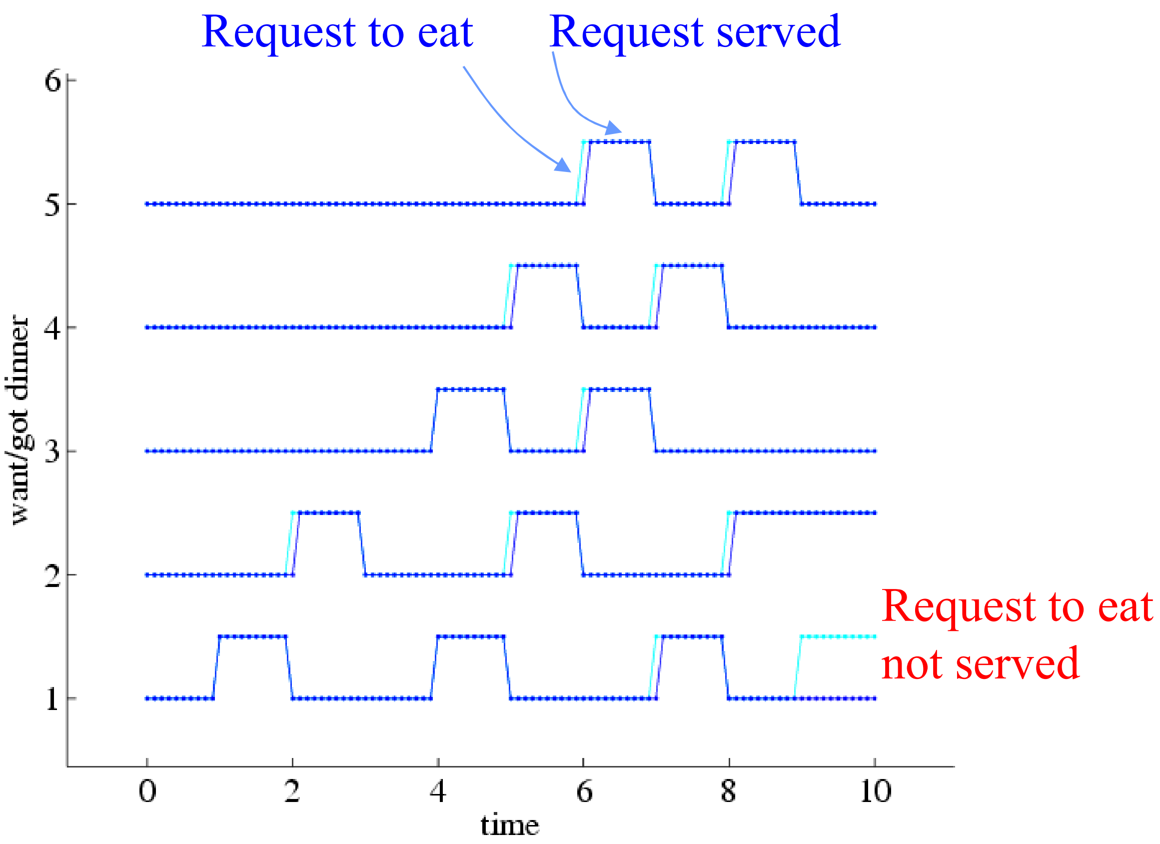
time	binary signals (events)				
0	0	0	0	0	0
1	1	0	0	0	0
2	0	1	0	0	0
3	0	0	0	0	0
4	1	0	1	0	0
5	0	1	0	1	0
6	0	0	1	0	1
7	1	0	0	1	0
8	0	1	0	0	1
9	1	1	0	0	0

Example: Philosophers Dinner – input / events




```
% first column = time in seconds
% next 5 columns = want to eat flags at time t
%

tu= [...
    0.0  want_to_eat( [] ) ; ...
    1.0  want_to_eat( 1  ) ; ...
    2.0  want_to_eat( 2  ) ; ...
    3.0  want_to_eat( []  ) ; ...
    4.0  want_to_eat( [1 3] ) ; ...
    5.0  want_to_eat( [2 4] ) ; ...
    6.0  want_to_eat( [3 5] ) ; ...
    7.0  want_to_eat( [4 1] ) ; ...
    8.0  want_to_eat( [5 2] ) ; ...
    9.0  want_to_eat( [1 2] ) ; ...
    ];


function y= want_to_eat(kid)
y= zeros(1,5);
for i=1:length(kid)
    y(kid(i))= 1;
end
```

Example: Philosophers Dinner – simulation



*Note: See this demo in the webpage *.*

Note2: Modern operating systems must work better than failing early like in this PN simulation. E.g. two programs requiring simultaneously much CPU and memory, the O.S. has managers that own the resources (CPU, memory, etc), queue the requests and in most cases even preempt the resources (CPU).

* www.isr.tecnico.ulisboa.pt/~jag/course_utils/pn_sim/PN_sim.html

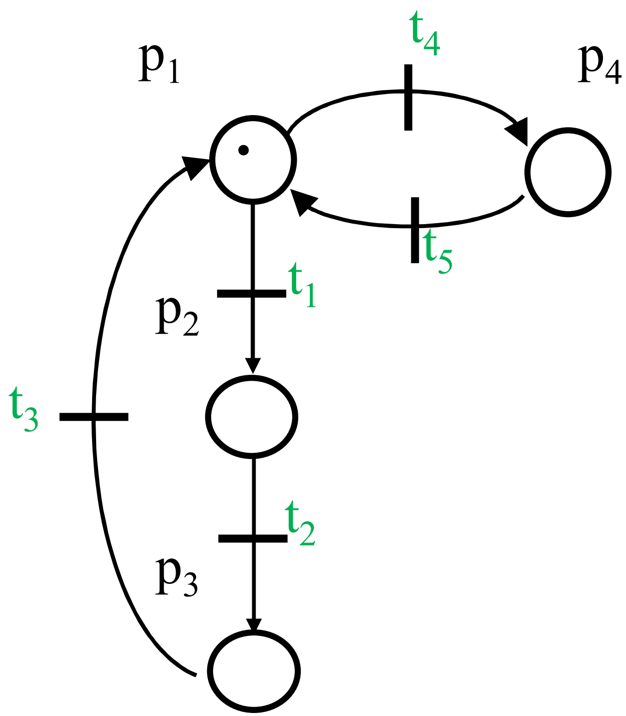
Running a Petri net with HW inputs and outputs

Running a generic Petri net

```
function [tSav, MPSav, youtSav]= PN_sim(Pre, Post, M0, ti_tf)
%
% Simulating a Petri net, using a SFC/Grafcet simulation methodology.
% See book "Automating Manufacturing Systems", by Hugh Jack, 2008
% (ch20. Sequential Function Charts)
%
% Petri net model:
%  M(k+1) = M(k) +(Post-Pre)*q(k)
%  Pre and Post are NxM matrices, meaning N places and M transitions

% 0. Start PN at state M0
%
MP=M0;
ti=ti_tf(1); tf=ti_tf(2); tSav= (ti:5e-3:tf)';
MPSav= zeros( length(tSav), length(MP) );
youtSav= zeros( length(tSav), length(PN_s2yout(MP)) );

for i= 1:length(tSav)

    % 1. Check transitions (update state)
    tm= tSav(i);
    qk= PN_tfire(MP, tm);
    qk2= filter_possible_firings(MP, Pre, qk(:));
    MP= MP +(Post-Pre)*qk2;

    % 2. Do place activities
    yout= PN_s2yout(MP);

    % Log all results
    MPSav(i,:)= MP';
    qkSav(i,:)= qk2';
    youtSav(i,:)= yout;

end
```

```
function qk2= filter_possible_firings(M0, Pre, qk)
% verify Pre*q <= M
% try to fire all qk entries

M= M0;
mask= zeros(size(qk));
for i=1:length(qk)
    % try accepting qk(i)
    mask(i)= 1;
    if any(Pre*(mask.*qk) > M)
        % exceeds available markings
        mask(i)= 0;
    end
end
qk2= mask.*qk;
```

Example 2: PN to PLC, *Loop or Wait*



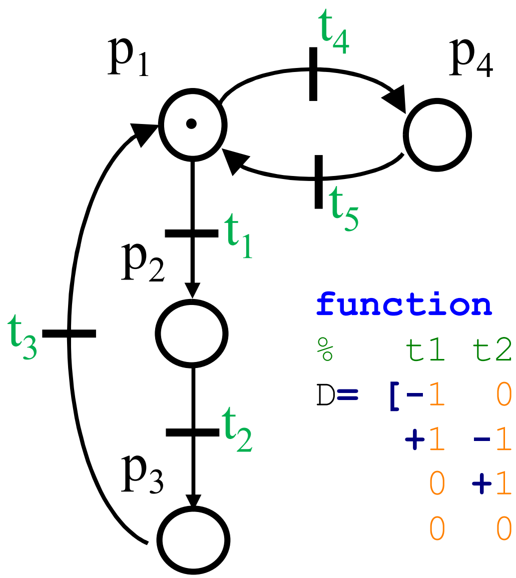
Application:

p1 – turn on output 1
p2 – turn on output 2
p3 – turn on output 3
p4 – wait

t1, t2, t3 – timed transitions
t4 – pressed button
t5 – released button

See code for this example in *http://users.isr.ist.utl.pt/~jag/course_utils/pn_to_plc/pn_to_plc.html*

Example 2: PN to PLC




```
function PN= define_petri_net
%    t1 t2 t3 t4 t5
D= [-1  0 +1 -1 +1
    +1 -1  0  0  0
     0 +1 -1  0  0
     0  0  0 +1 -1];

Pre  = -D.*(D<0);
Post=   D.*(D>0);

M0   = [1 0 0 0]';

% Petri net structure:
% 0.5 sec from p1..p3 to trans t1..t3
% col2=place, col3=trans

T = 0.5;
tt= [T 1 1; T 2 2; T 3 3];
PN= struct('pre',Pre, 'pos',Post, 'mu0',M0,
           'ttimed',tt);
```

```
function tst1_blink

PN          = define_petri_net;
input_map   = define_input_mapping;
output_map  = define_output_mapping;
ofname      = 'tst1_blink.txt';
plc_make_program( ofname, PN,
            input_map, output_map )
```

```
function inp_map= define_input_mapping
% input0 fires transition4
% negative input0 fires t5
inp_map= { ...
    0,         4 ;
    -(0+100), 5 ;
    };
```

```
function output_map= define_output_mapping
% map PN places 1..3 to the first output
bits
zCode= plc_z_code_helper('config_get');
output_map= { ...
    1, zCode.outpMin ; ...
    2, zCode.outpMin+1 ; ...
    3, zCode.outpMin+2 ;
    };
```

Example 2: PN to PLC

```
(* --- PNC: Petri net initialization --- *)

IF %MW100=0 THEN
 %MW201:=1; %MW202:=0; %MW203:=0; %MW204:=0;
 %MW100:=1;
END_IF;

(* --- PNC: Map inputs --- *)

%MW104 := BOOL_TO_INT( %i0.2.0 );
%MW105 := BOOL_TO_INT( NOT(%i0.2.0) );

(* --- PNC: Timed transitions --- *)

MY_TON_1(IN := INT_TO_BOOL(%MW201) (*BOOL*),
         PT := t#500ms (*TIME*),
         Q => timer_output_flag (*BOOL*),
         ET => my_time_1 (*TIME*));
%MW101:= BOOL_TO_INT(timer_output_flag);
MY_TON_2(IN := INT_TO_BOOL(%MW202) (*BOOL*),
         PT := t#500ms (*TIME*),
         Q => timer_output_flag (*BOOL*),
         ET => my_time_2 (*TIME*));
%MW102:= BOOL_TO_INT(timer_output_flag);
MY_TON_3(IN := INT_TO_BOOL(%MW203) (*BOOL*),
         PT := t#500ms (*TIME*),
         Q => timer_output_flag (*BOOL*),
         ET => my_time_3 (*TIME*));
%MW103:= BOOL_TO_INT(timer_output_flag);
```

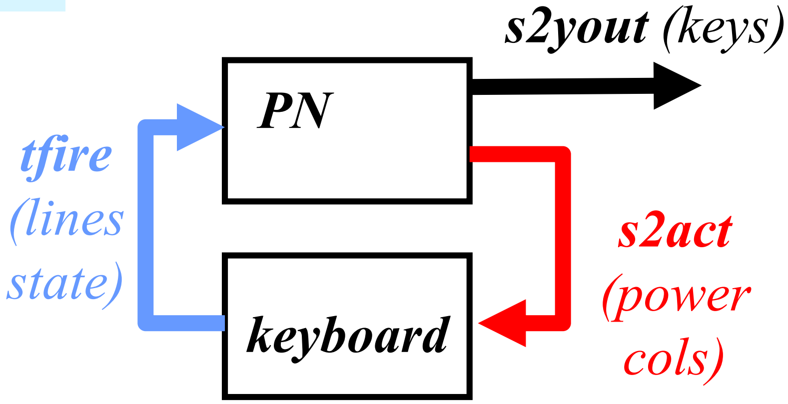
```
(* --- PNC: Petri net loop code --- *)

IF %MW101>0 AND %MW201>=1
THEN
 %MW201:=%MW201-1;
 %MW202:=%MW202+1;
END_IF;

IF %MW102>0 AND %MW202>=1
THEN
 %MW202:=%MW202-1;
 %MW203:=%MW203+1;
END_IF;

IF %MW103>0 AND %MW203>=1
THEN
 %MW203:=%MW203-1;
 %MW201:=%MW201+1;
END_IF;

IF %MW104>0 AND %MW201>=1
THEN
 %MW201:=%MW201-1;
 %MW204:=%MW204+1;
END_IF;

IF %MW105>0 AND %MW204>=1
THEN
 %MW204:=%MW204-1;
 %MW201:=%MW201+1;
END_IF;
```

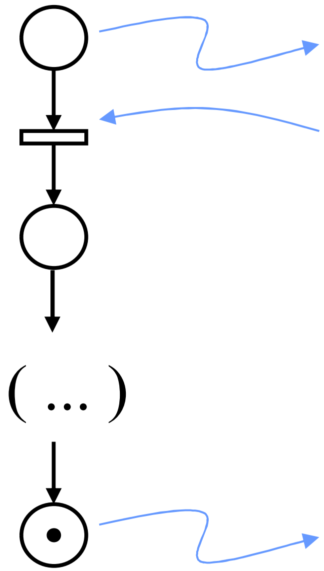
```
(* --- PNC: Output bits --- *)

IF INT_TO_BOOL(%MW201)
THEN SET(%q0.4.0);
ELSE RESET(%q0.4.0);
END_IF;
IF INT_TO_BOOL(%MW202)
THEN SET(%q0.4.1);
ELSE RESET(%q0.4.1);
END_IF;
IF INT_TO_BOOL(%MW203)
THEN SET(%q0.4.2);
ELSE RESET(%q0.4.2);
END_IF;
```

See code for this example in [*http://users.isr.ist.utl.pt/~jag/course_utils/pn_to_plc/pn_to_plc.html*](http://users.isr.ist.utl.pt/~jag/course_utils/pn_to_plc/pn_to_plc.html)

Example 3: Keyboard Reading

output = columns power
input = lines read





1. state to actuation:
   power kb columns
2. signals to transitions:
   wait signal on kb lines
3. state to output:
   key X is pressed

Code template (Matlab):

Main systems
a) `PN_sim.m`
b) `PN_device_kb_IO.m`

Interface functions
1) `PN_s2act.m`
2) `PN_tfire.m`
3) `PN_s2yout.m`

Running a Petri net with HW inputs and outputs

```
function lines= PN_device_kb_IO(act, t)

% Define 4x3-keyboard output line-values given actuation on the 3 columns
% and an (internal) time table of keys pressed
% Input:
%  act: 1x3 : column actuation values
%  t  : 1x1 : time
% Output:
%  lines: 1x4 : line outputs

global keys_pressed
if isempty(keys_pressed)
    % first column = time in seconds
    % next 12 columns = keys pressed at time t
    keys_pressed= [...
        0  mk_keys([]) ; 1  mk_keys(1)   ; ...
        2  mk_keys([]) ; 3  mk_keys(5)   ; ...
        4  mk_keys([]) ; 5  mk_keys(9)   ; ...
        6  mk_keys([]) ; 7  mk_keys([1 12]) ; ...
        8  mk_keys(12) ; 9  mk_keys([]) ; ...
        ];
end

% pressed keys yes/no
ind= find(t>=keys_pressed(:,1));
if isempty(ind)
    lines= [0 0 0 0]; % default lines output for t < 0
    return
end
keys_t= keys_pressed(ind(end), :);

% if actuated column and key pressed match, than activate line
lines= sum( repmat(act>0, 4,1) & reshape(keys_t(2:end), 3,4)', 2);
lines= (lines > 0)';
```

Keyboard simulator: generate line values given column values

```
function y= mk_keys(kid)
y= zeros(1,12);
for i=1:length(kid)
    y(kid(i))= 1;
end
```

Running a Petri net with HW inputs and outputs

Prototypes of the interfacing functions

The implementation of these functions is to be done by each group in the laboratory.

```
function act= PN_s2act(MP)

% Create 4x3-keyboard column actuation
%
% MP:  1xN : marked places (integer values >= 0)
% act: 1x3 : column actuation values (0 or 1 per entry)
```

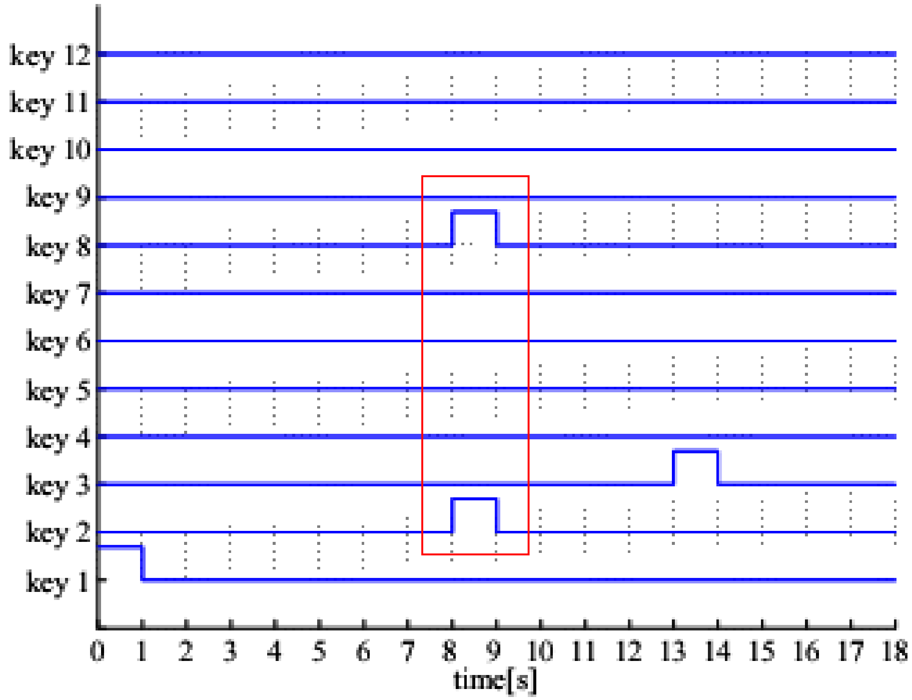
```
function qk= PN_tfire(MP, t)

% Possible-to-fire transitions given PN state (MP) and the time t
%
% MP: 1xN : marked places (integer values >= 0)
% t : 1x1 : time
% qk: 1xM : possible firing vector (to be filtered later with enabled
%            transitions)
```

```
function yout= PN_s2yout(MP)

% Show the detected/undetected key(s) given the Petri state
%
% MP: 1xN : marked places (integer values >= 0)
```

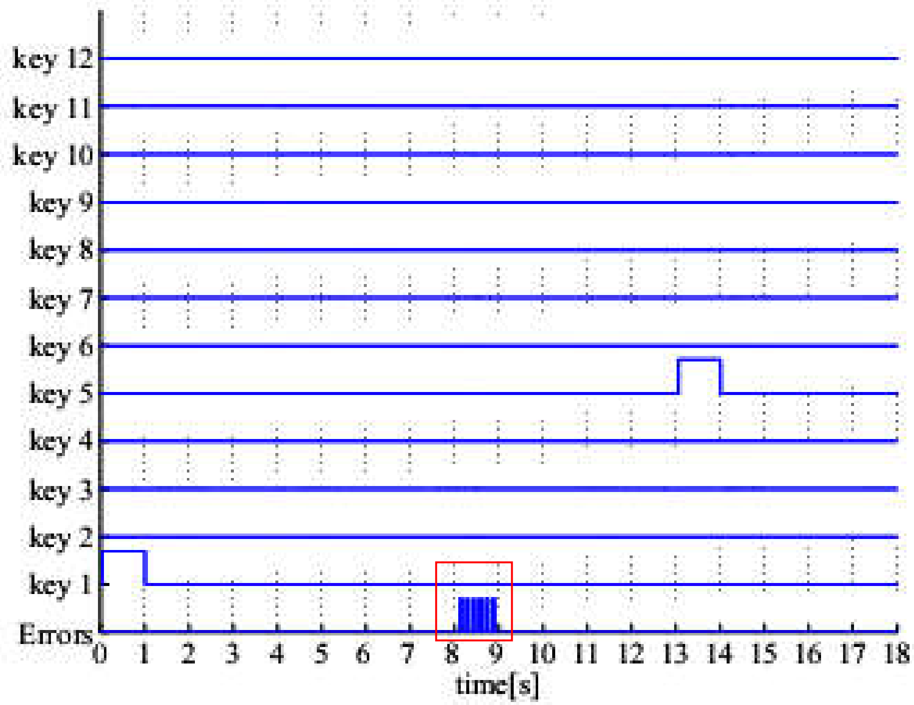
Running a Petri net with HW inputs and outputs

Laboratory assignment: detect keys pressed by the user and just accept those keys when there are not multiple keys pressed at the same time.

Keys pressed



Keys accepted



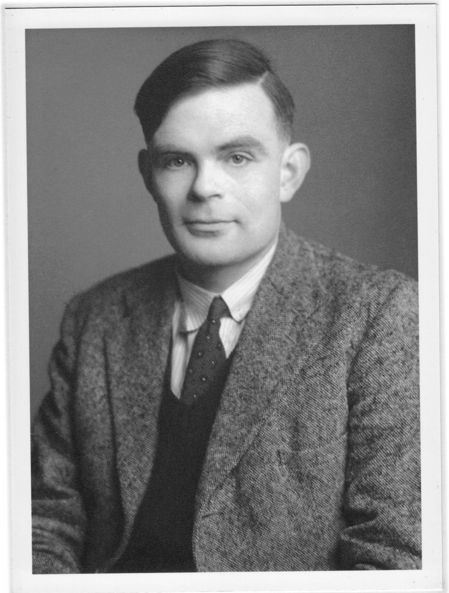
Industrial Automation

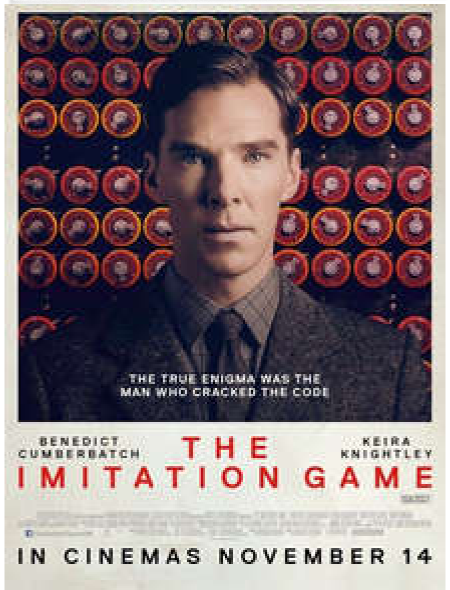
(Automação de Processos Industriais)

Discrete Event Systems:
Turing Machines, *Busy Beaver*

http://www.isr.tecnico.ulisboa.pt/~jag/courses/mapi22d

Prof. José Gaspar, 2022/2023





Simple ways to learn more about
Alan Turing

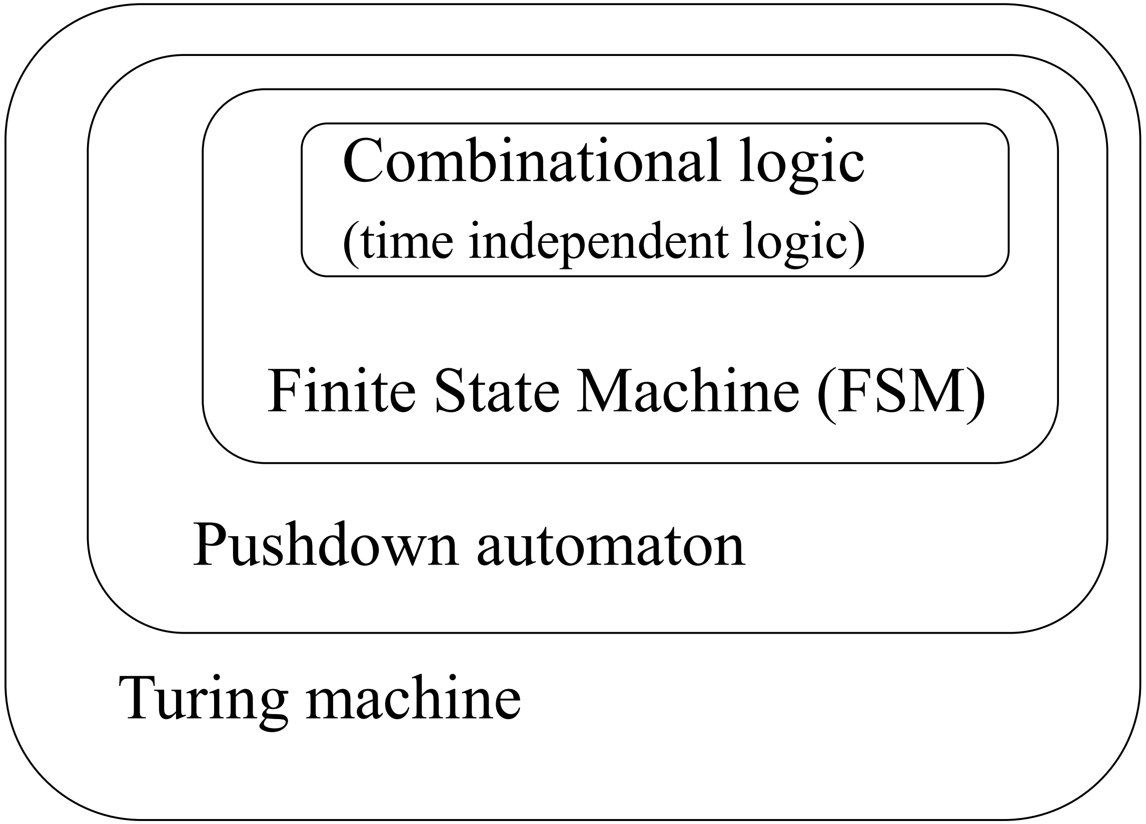
Check 2^{nd} world war history:

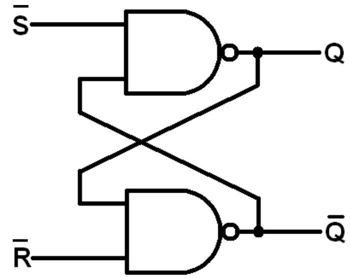
- Cryptanalysis of the ***Enigma*** Machine
- The British ***Bombe*** (Turing)

See the movie (2014):

- The Imitation Game
- *Trailer to see in the weekend ;)*

Automata theory





Current state	Input SR	Next state
xx	11	11
xx	10	10
xx	01	01
xx	00	xx

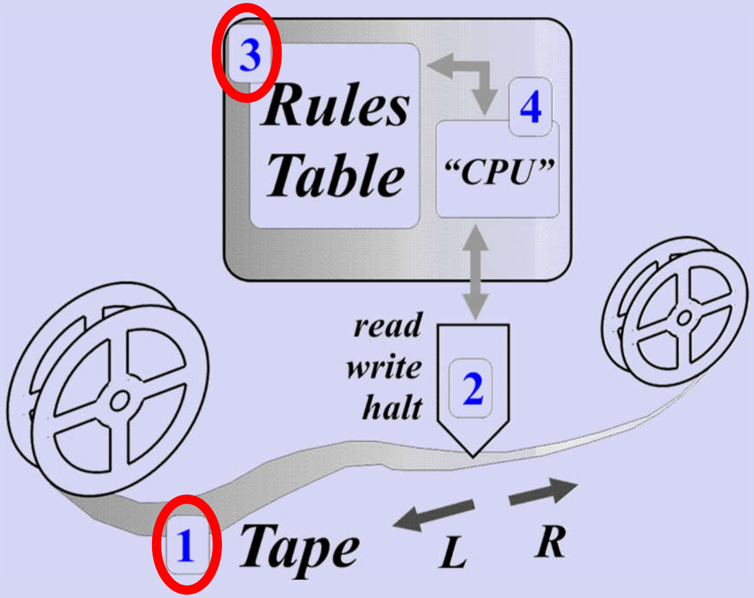
SR latch is an FSM example. The input *(S,R)=(0,0)* keeps the **memorized** value

$$(Q,\overline{Q}) = (x,x)$$

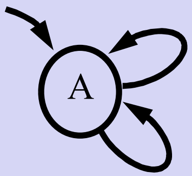
How many different states can the SR latch show?

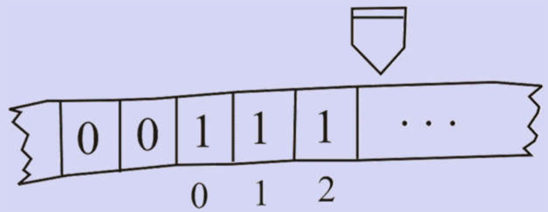
Turing machine (TM)

Components:
(1) Infinite length magnetic **Tape**
(2) Read/Write head
(3) Rules table, e.g. an FSM
(4) State register



Example of a simple Rules table, namely an **FSM**. Using this FSM the TM writes forever ones into the tape. Read the FSM as "if 0 or 1 is read from the tape, then write 1 to the tape, move tape to the left and continue in state A".



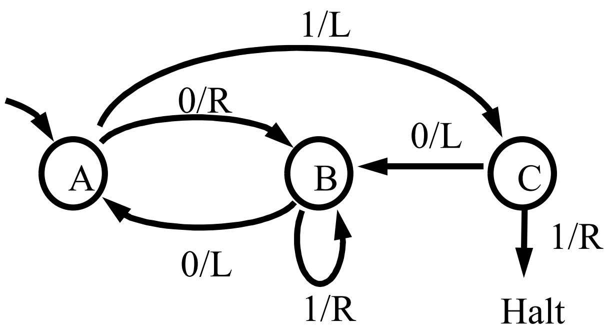


Note: a TM is not just an FSM; for example, it contains also an **infinite memory**.

Turing machine example: *Busy Beaver*

The objective is to fill the TM tape with ones, as many as possible, using a rules table (FSM) with a minimum number of states. By definition of *Busy Beaver*, the TM must halt (stop) some time after starting.

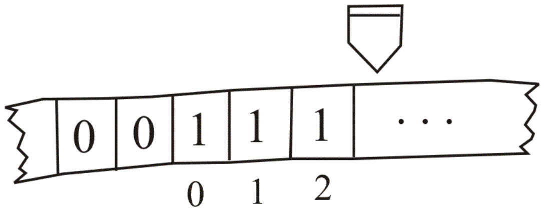
One implementation of the **3**-states **2**-symbols Busy Beaver is:



Current state	Input	Action R/W	Action L/R/N	Next state
A	0	write1	right	B
A	1	write1	left	C
B	0	write1	left	A
B	1	write1	right	B
C	0	write1	left	B
C	1	write1	null	halt

Turing machine in Matlab:

(1) **tape** and
(2) read/write head

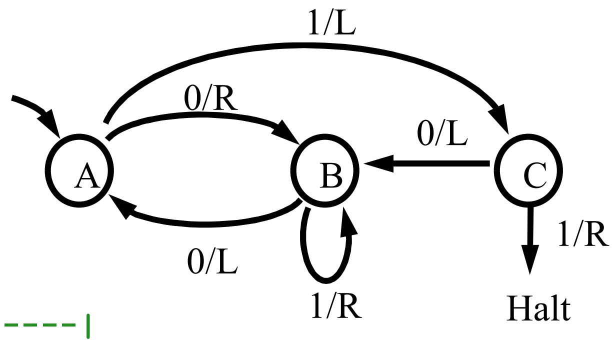



```
function TM_reset
global TMT
TMT= struct('pos',0,
            'val',[],
            'valNeg',[]);
```

```
function ret= TM_tape( op, arg1 )
% Tape for a Turing machine. Basic operations:
%   read/write and move Left/Right/None
global TMT; if isempty(TMT), TM_reset; end
switch op
    case 'reset', TM_reset;
    case 'left',  TMT.pos= TMT.pos+1;
    case 'right', TMT.pos= TMT.pos-1;
    case 'null_move' % do nothing
    case 'read', % 1st call may need tape
        realloc_if_needed( TMT.pos );
        if TMT.pos>=0, ret= TMT.val( TMT.pos+1 );
        else           ret= TMT.valNeg( -TMT.pos );
        end
    case 'write', % 1st call may need tape
        realloc_if_needed( TMT.pos );
        if TMT.pos>=0, TMT.val( TMT.pos+1 )= arg1;
        else           TMT.valNeg( -TMT.pos )= arg1;
        end
    otherwise, error('inv op')
end
```

Turing machine in Matlab:

(3) rules table, **FSM** of a *3-state Busy Beaver*




```
function FSM= def_BusyBeaver3
% FSM has four columns:
%   curr_state, true_false_condition, actions, next_state
%          |- T/F cond -----|     |- write action and move action ----|
FSM= {
    'A',  'TM_tape("read")==0',  'TM_tape("write",1); TM_tape("right");',  'B';
    'A',  'TM_tape("read")==1',  'TM_tape("write",1); TM_tape("left");' ,  'C';
    'B',  'TM_tape("read")==0',  'TM_tape("write",1); TM_tape("left");' ,  'A';
    'B',  'TM_tape("read")==1',  'TM_tape("write",1); TM_tape("right");',  'B';
    'C',  'TM_tape("read")==0',  'TM_tape("write",1); TM_tape("left");' ,  'B';
    'C',  'TM_tape("read")==1',  'TM_tape("write",1); TM_tape("null_move");', 'halt';
    };
```

Current state	Input	Action R/W	Action L/R/N	Next state
A	0	write1	right	B
A	1	write1	left	C
B	0	write1	left	A
B	1	write1	right	B
C	0	write1	left	B
C	1	write1	null	halt

Alternative, more compact, representation:

```
function FSM= def_BusyBeaver3
tbl= {'A01RB', 'A11LC', ...
      'B01LA', 'B11RB', ...
      'C01LB', 'C11NH'};
FSM= convert_table_to_list( tbl );
```

Turing machine in Matlab:

(4) state register, `curr_state` for **running** the machine

Recall the first line of the table:

```
FSM{1,:} =
'A'
'TM_tape("read")==0'
'TM_tape("write",1); TM_tape("right");'
'B'
```

and read it as "if current state is A and tape read is zero, then write 1 to the tape, move tape right, and the next state is B".

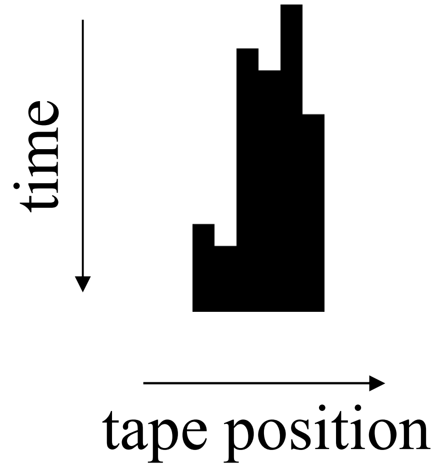
```
function TM_run
TM_tape( 'reset' );
FSM= TM_ini( 'BusyBeaver3' );
curr_state= FSM{1,1};
while ~strcmpi(curr_state, 'halt')
    for i=1:size(FSM,1)
        if strcmpi(FSM{i,1}, curr_state) ...
                && eval( FSM{i,2} )
            % found state and true condition
            eval( FSM{i,3} );
            % curr_state <- next state
            curr_state= FSM{i,4};
            break;
        end
    end
end
```

Download the complete implementation from:
http://isr.tecnico.ulisboa.pt/~jag/course_utils/Turing_Machines_sim/Turing_Machines_sim.html

Turing Machine *Busy Beaver*: simulation results

3-state Busy Beaver:
a0 -> b1r   a1 -> h1r
b0 -> c0r   b1 -> b1r
c0 -> c1l   c1 -> a1l

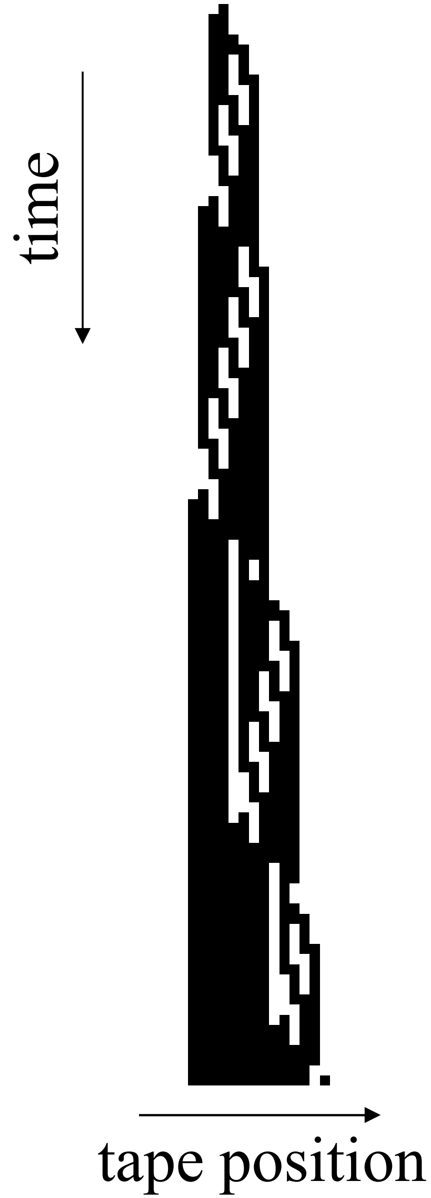
halts after **21 time steps**
fills **6 ones**



4-state Busy Beaver:
a0 -> b1r   a1 -> b1l
b0 -> a1l   b1 -> c0l
c0 -> h1r   c1 -> d1l
d0 -> d1r   d1 -> a0r

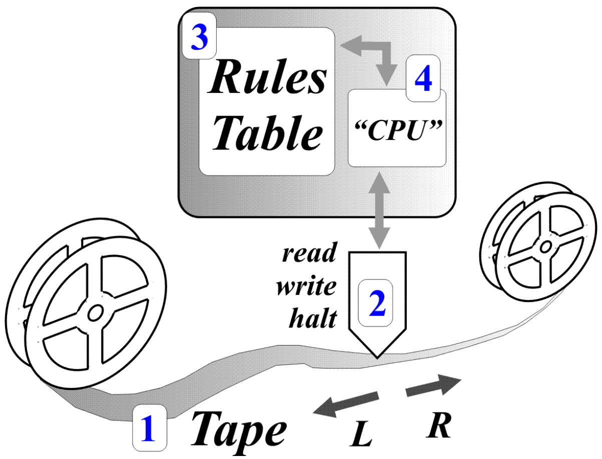
halts after **107 time steps**
fills **13 ones**

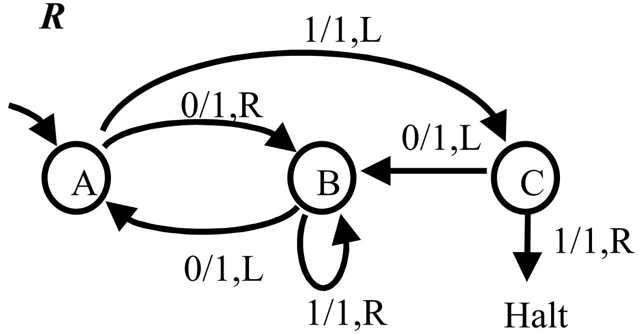
States	*Halts after n time steps*	*Fills m ones in the tape*
2	**6**	**4**
3	**21**	**6**
4	**107**	**13**
5	**47,176,870 ?**	**4098 ?**
6	**$> 7.4 \times 10^{36534}$**	**$> 3.5 \times 10^{18267}$**

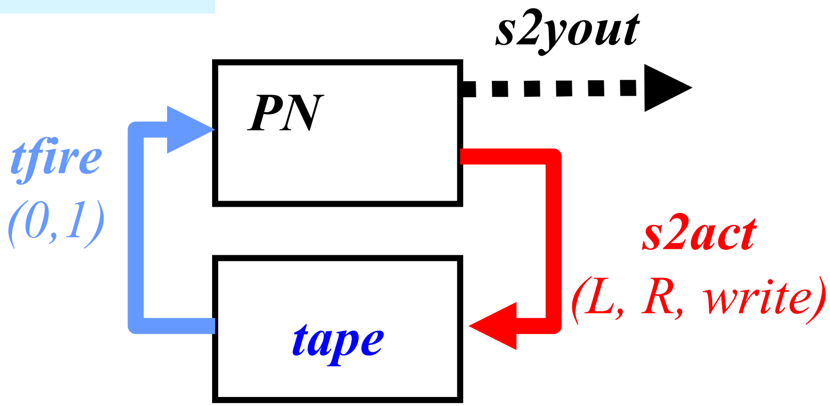


http://www.catonmat.net/blog/busy-beaver/
http://www.logique.jussieu.fr/~michel/bbc.html
(competition results visited Nov 2016)

Example 4: Busy Beaver FSM as PN







outputs = tape left, right, write

input = tape read (one bit, i.e. 0 or 1)

Code template (Matlab):

Main systems

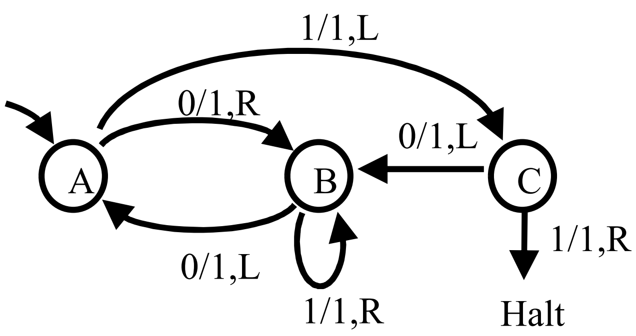
a) PN_sim.m (as before)
b) TM_tape.m (see Turing)

Interface functions

1) PN_s2act.m
2) PN_tfire.m
3) PN_s2yout.m

Turing machine, Busy-Beaver 3states 2symbols,
graph vs **table**, see input / output :

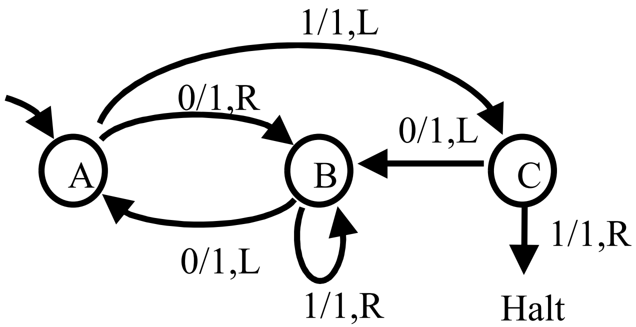
outputs = tape left, right, write (1)
input = tape read (one bit, i.e. 0 or 1)



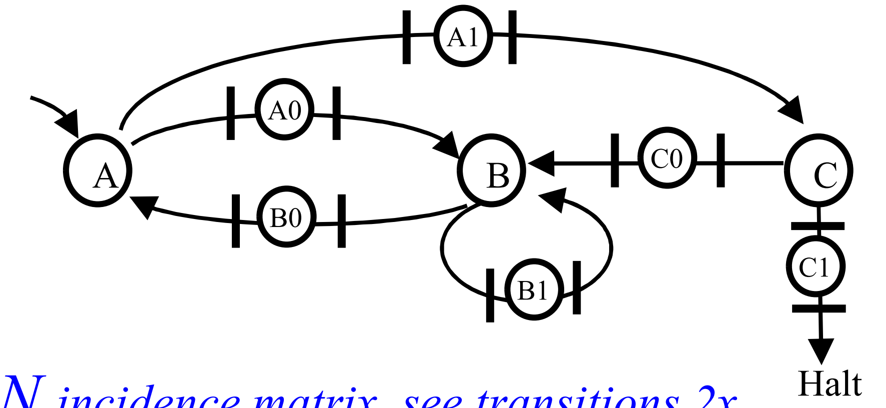
Current state	Input	Action R/W	Action L/R/N	Next state
A	0	write1	right	B
A	1	write1	left	C
B	0	write1	left	A
B	1	write1	right	B
C	0	write1	left	B
C	1	write1	null	halt

Busy-Beaver is a FSM with outputs in the arcs (not the "places"), hence it is a Mealy machine (not a Moore machine). How to represent as a Petri net just with outputs in its places?

FSM



PN each arc of the FSM completed with 1 place and 2 transitions



1. FSM table

	0	1
A	B	C
B	A	B
C	B	H
H	-	-

2. PN incidence matrix, see transitions 2x number of places A, B, C

	0A	1A	0B	1B	0C	1C
A	-1	-1	+1			
B	+1		-1	-1+1	+1	
C		+1			-1	-1
H						+1

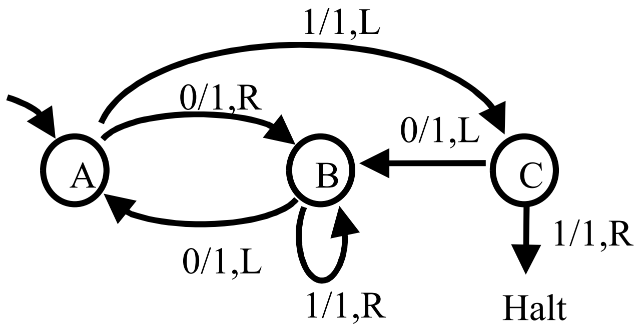
3. FSM table + outputs

	0	1
A	B/1,R	C/1,L
B	A/1,L	B/1,R
C	B/1,L	H/1,R
H	-	-

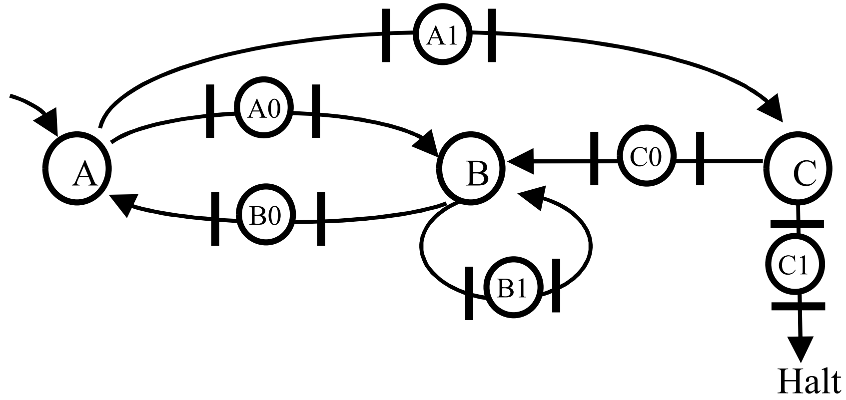
4. PN incidence matrix with outputs at the arcs

	Output	0A	0Ax	1A	1Ax	0B	0Bx	1B	1Bx	0C	0Cx	1C	1Cx
A	-	-1		-1			+1						
A0	1,R	+1	-1	+1	-1								
A1	1,L												
B	-		+1			-1		-1	+1		+1		
B0	1,L					+1	-1						
B1	1,R							+1	-1				
C	-				+1					-1		-1	
C0	1,L									+1	-1		
C1	1,N											+1	-1
H	-												+1

Turing-Machine Busy-Beaver: PN shown in previous slide, here implement **Input / Output**



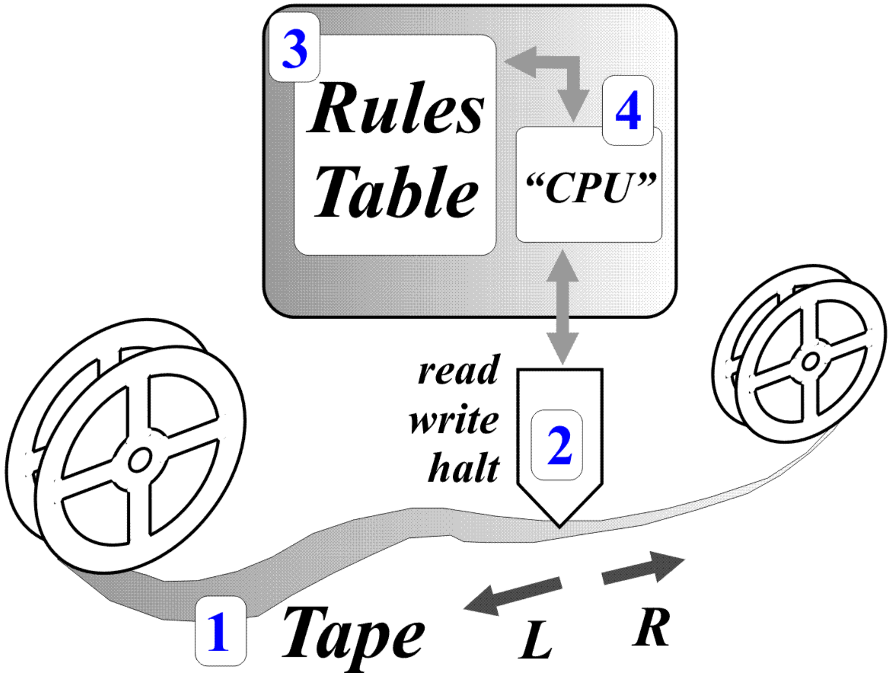
1	A	-
2	A0	1,R
3	A1	1,L
4	B	-
5	B0	1,L
6	B1	1,R
7	C	-
8	C0	1,L
9	C1	1,N
10	H	-




```
% TM_tape.m : left, right, read, write
% at A,B,C, read bit & activate transitions
% at A0,B1 move right
% at A1,B0,C0 move left


function act= PN_s2act( MP )

act= TM_tape('read');

if max(MP([2 3  5 6  8 9]))>0
    TM_tape('write',1);
end

if MP(3)>0 || MP(5)>0 || MP(8)>0
    TM_tape('left');
elseif MP(2)>0 || MP(6)>0
    TM_tape('right')
else
    % do nothing
end
```

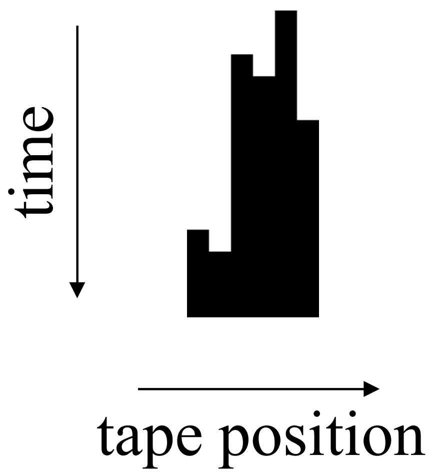
```
function qk= PN_tfire( act, t )

qk= ones(1,12);

if act  % read 1 from tape
    qk([1 5 9])= 0;
    qk([3 7 11])= 1;
else
    qk([1 5 9])= 1;
    qk([3 7 11])= 0;
end
```

Turing Machine Busy Beaver: simulation results



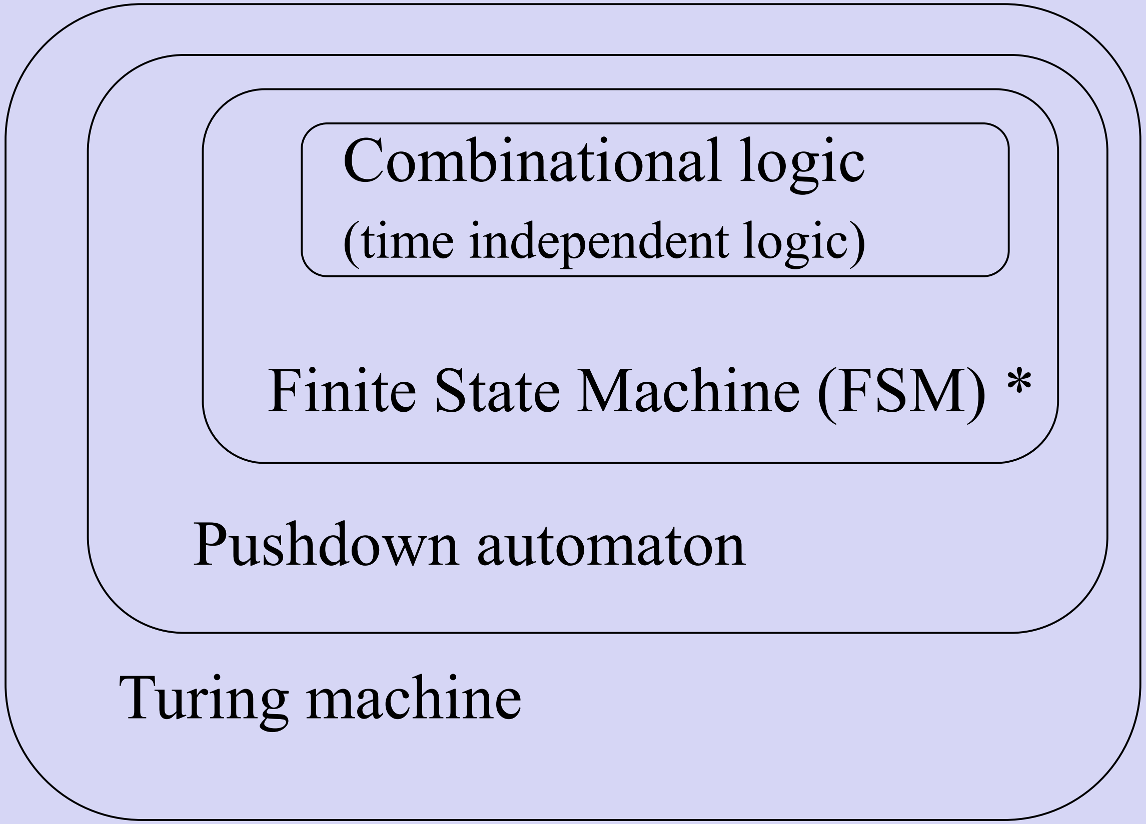
3-state Busy Beaver:
a0 -> b1r a1 -> h1r
b0 -> c0r b1 -> b1r
c0 -> c1l c1 -> a1l

halts after **21 time steps**
fills **6 ones**



Code in *http://www.isr.tecnico.ulisboa.pt/~jag/course_utils/Turing_Machines_sim/Turing_Machines_sim.html*

Automata theory



* *Time dependency, **memory**, is an essential component for automata.*
*Petri nets will introduce another essential component: **parallelization**.*