Petri Nets Properties

Examples & Solutions

2022/2023 J. Gaspar

Example1: simple Petri net, properties?

(P, T, A, w, x_0)

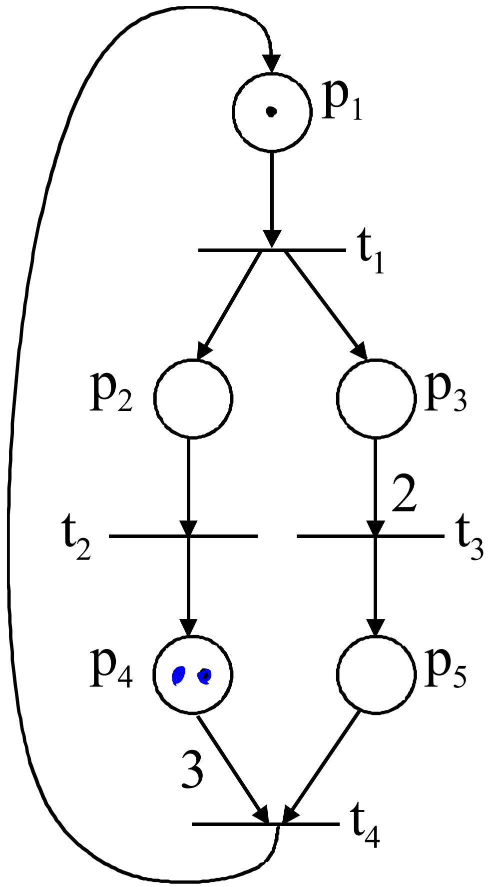
$P=\{p_1, p_2, p_3, p_4, p_5\}$

$T=\{t_1, t_2, t_3, t_4\}$

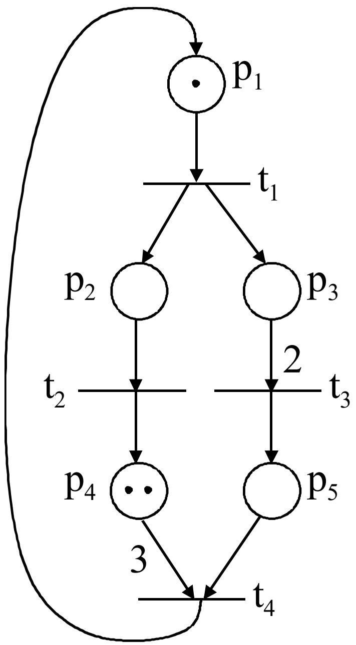
$A=\{(p_1, t_1), (t_1, p_2), (t_1, p_3), (p_2, t_2), (p_3, t_3), (t_2, p_4), (t_3, p_5), (p_4, t_4), (p_5, t_4), (t_4, p_1)\}$

$w(p_1, t_1)=1,$
$w(t_1, p_2)=1, w(t_1, p_3)=1,$
$w(p_2, t_2)=1, w(p_3, t_3)=2,$
$w(t_2, p_4)=1, w(t_3, p_5)=1,$
$w(p_4, t_4)=3, w(p_5, t_4)=1,$
$w(t_4, p_1)=1$

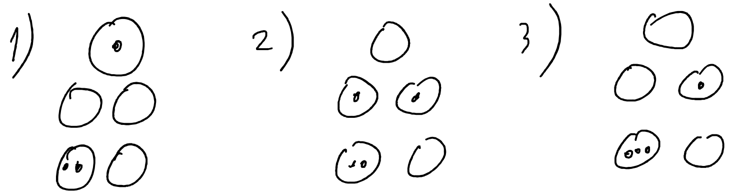
$x_0 = \{1, 0, 0, 2, 0\}$



1. *Reachability?*
2. *Coverability?*
3. *Safeness?*
4. *Boundness?*
5. *Conservation?*
6. *Liveness?*



Study of properties based on the Reachability tree



(1, 0, 0, 2, 0)

↓ t_1

(0, 1, 1, 2, 0)

↓ t_2

(0, 0, 1, 3, 0)

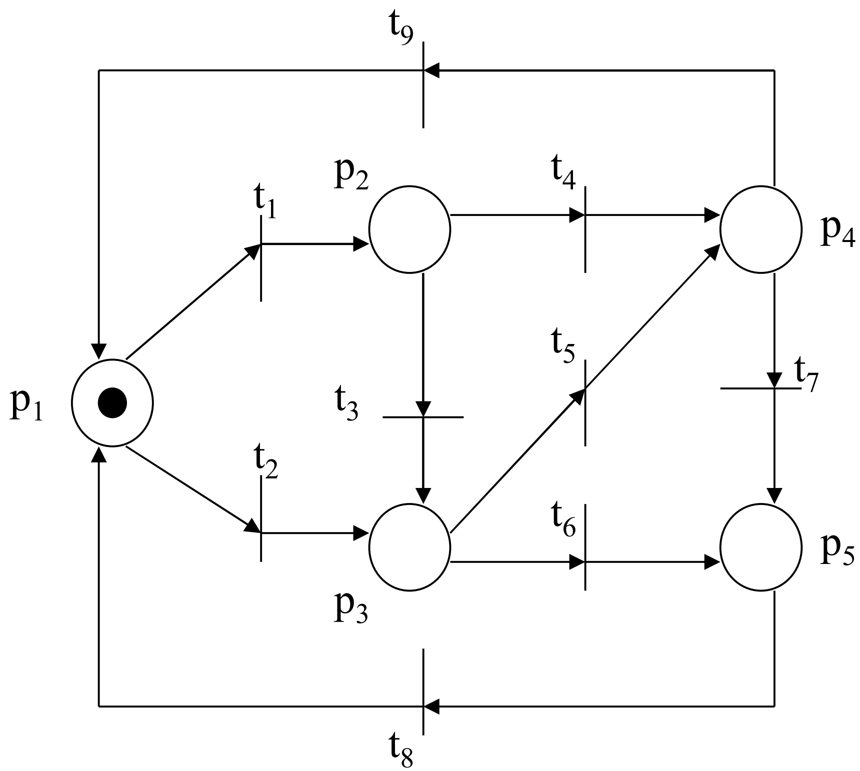
Term

1. Finite reachable set *(3 states)*
2. No state covers / is covered by another one
3. Is not safe (p_4 reaches 3 marks)
4. Net is 3-bounded
5. Net not strictly conservative
6. t_3 and t_4 are level ϕ, other ones are L_1

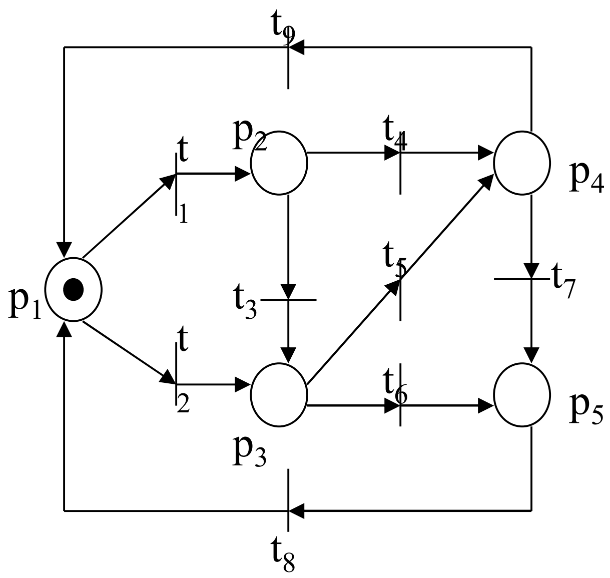
Example2: simple automation system modeled using PNs, properties?

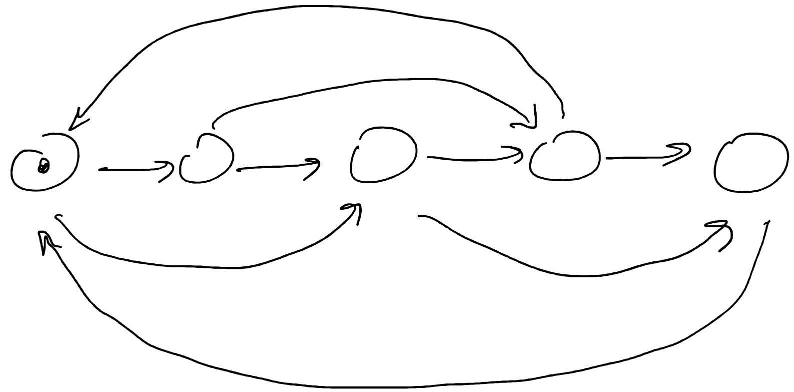
An automatic soda selling machine accepts
50c and $1 coins and
sells 2 types of products:
SODA A, that costs $1.50 and
SODA B, that costs $2.00.

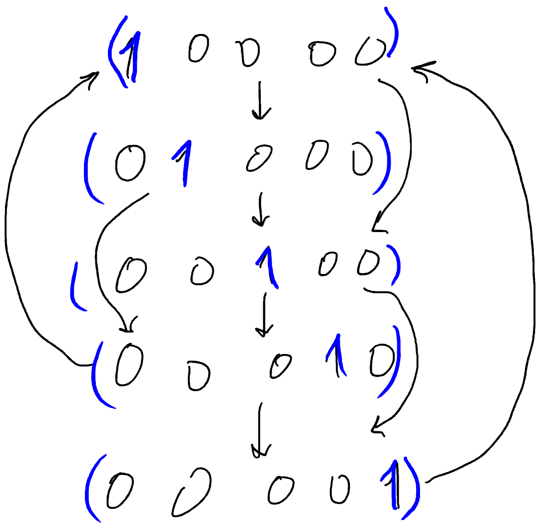
Assume that the money return operation is omitted.



p_1: machine with $0.00;
t_1: coin of 50 c introduced;
t_8: SODA B sold.

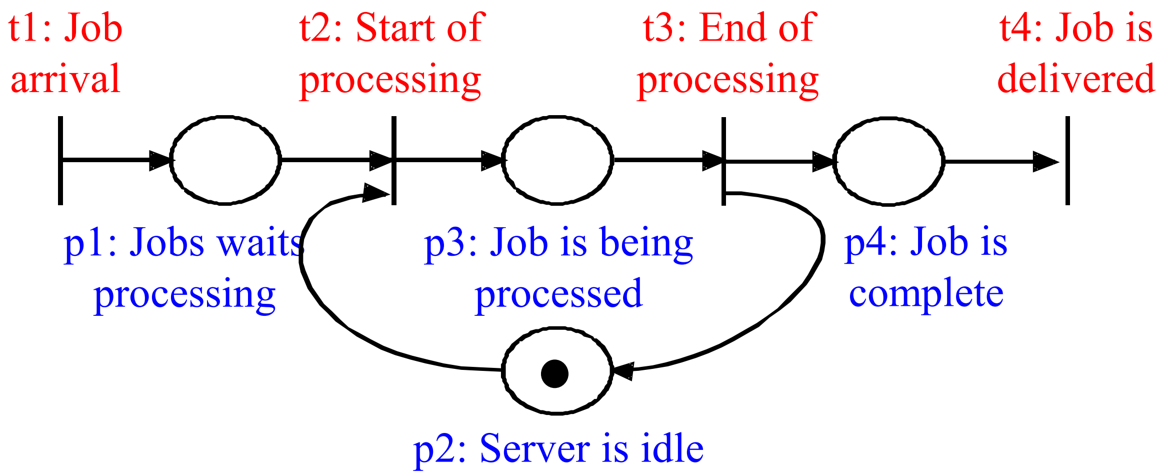






1. Finite reachable set (5 states)
2. no state covered by another one
3. Is safe
4. Is 1-bounded
5. Is conservative
6. Liveness, all transitions are L4

Example for the analysis of properties:



Event	Pre-conditions	Pos-conditions
t1	-	p1
t2	p1, p2	p3
t3	p3	p4, p2
t4	p4	-

Q: Exists ***conservation*** *?*

A: Find w such that $\mathbf{w^T.D=0}$

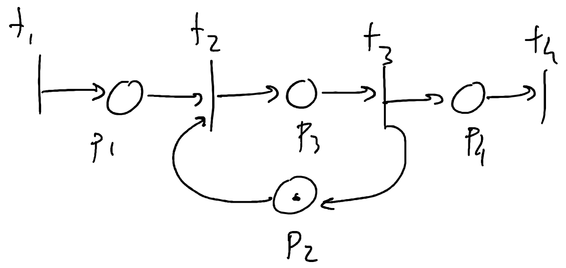
if $\exists w>0$ *then net is conservative*

else it is not conservative

$$D = \begin{bmatrix} 1 & -1 & & \\ & -1 & 1 & \\ & 1 & -1 & \\ & & 1 & -1 \end{bmatrix}$$

$$w^T = [w_1\ w_2\ w_3\ w_4] = ?$$

Q2: What changes if initial marking in p2 is zero?



Conservation

$w^T D = 0$

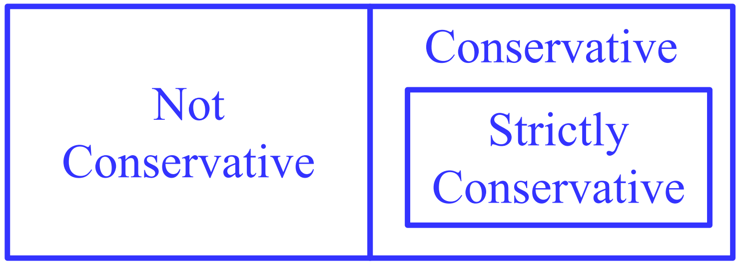
$\mu(k+1) = \mu(k) + D\,q(k)$

$w^T\mu(k+1) = w^T\mu(k) + w^T D\,q(k)$

equal; $w^T D = 0$; any

$D =$

	t_1	t_2	t_3	t_4
p_1	+1	−1		
p_2		−1	+1	
p_3		+1	−1	
p_4			+1	−1



$$\underbrace{\begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix}}_{w^T} \underbrace{\begin{bmatrix} 1 & -1 & & \\ & -1 & 1 & \\ & 1 & -1 & \\ & & 1 & -1 \end{bmatrix}}_{D} = 0$$

$$\begin{cases} w_1 = 0 \\ -w_1 - w_2 + w_3 = 0 \\ w_2 - w_3 + w_4 = 0 \\ -w_4 = 0 \end{cases} \qquad \begin{cases} w_1 = 0 \\ w_2 = w_3 \\ - \\ w_4 = 0 \end{cases}$$

e.g. $w = [0 \quad 1 \quad 1 \quad 0]$ //

problems

NOT conservative

Discrete Event Systems

Example of a simple automation system modeled using PNs

An automatic soda selling machine accepts

50c and $1 coins and

sells 2 types of products:

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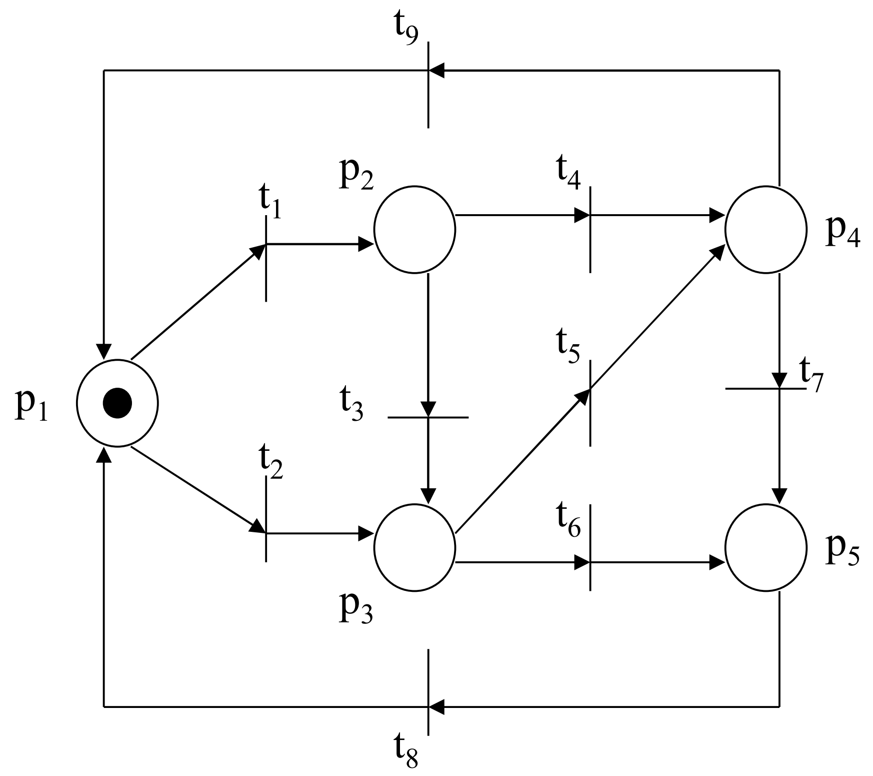
Assume that the money return operation is omitted.

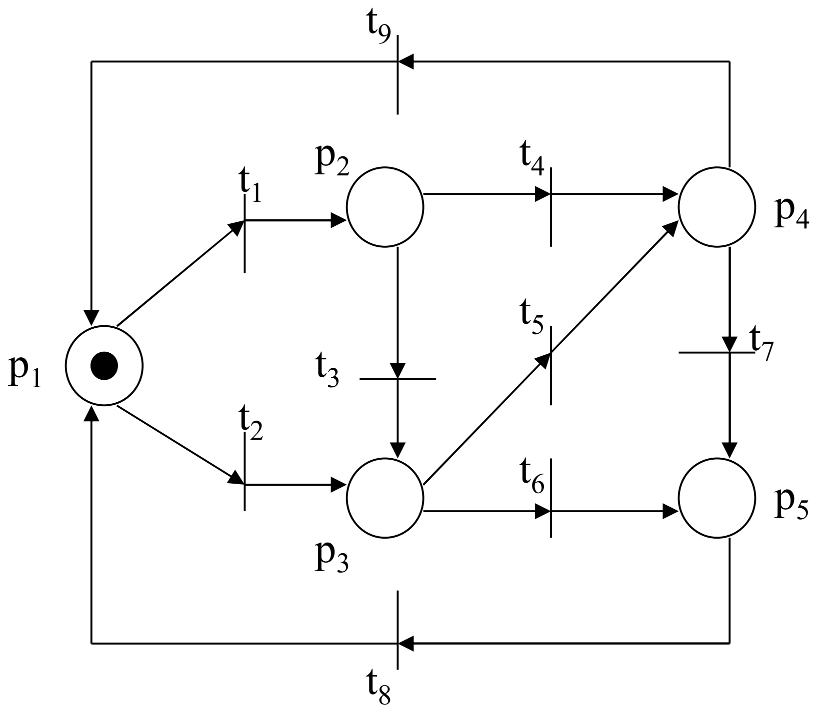


p_1: machine with $0.00;
t_1: coin of 50 c introduced;
t_8: SODA B sold.

Q: Are there transition firing vectors that make the Petri net return to the same state? In other words, does the Petri net have ***cycles of operation****?*

Discrete Event Systems

Example of a simple automation system modeled using PNs




```
D= [ -1  -1   0   0   0   0   0   1   1
      1   0  -1  -1   0   0   0   0   0
      0   1   1   0  -1  -1   0   0   0
      0   0   0   1   1   0  -1   0  -1
      0   0   0   0   0   1   1  -1   0 ]
```

Time invariance ? Find q such that. D.q=0

```
>> q= null( D, 'r' )
q =
    1   -1    1    0    1
   -1    1   -1    1    0
    1    0    0    0    0
    0   -1    1    0    1
    0    1    0    0    0
    0    0   -1    1    0
    0    0    1    0    0
    0    0    0    1    0
    0    0    0    0    1

>> q(:,1)= q(:,1)+q(:,4);
>> q(:,2)= q(:,2)+q(:,5);
>> q(:,3)= q(:,3)+q(:,4);
q =
    1    0    1    0    1
    0    1    0    1    0
    1    0    0    0    0
    0    0    1    0    1
    0    1    0    0    0
    1    0    0    1    0
    0    0    1    0    0
    1    0    1    1    0
    0    1    0    0    1
```

Note: there are more solutions; see function invar(D) of the SPNBOX toolbox