Modeling and Automation of Industrial Processes

MEEC / MEGE 2022/2023

1st Exam, 28th June 2023

Read all questions of the exam carefully before starting to answer.

- *Provide detailed justifications to all answers.*
- *The use of bibliographic material, either in paper or in digital format is allowed.*
- *Exchange of information with others is forbidden (e.g. voice, WiFi, Bluetooth, GPRS, WAP, …).*
- *Exam duration: 2 hours.*

-- Part 1 --

The first part of this exam is resolved in Moodle https://moodle.deec.tecnico.ulisboa.pt. The questions in Moodle have automatic evaluation. **Do not deliver paper resolution** to the questions of this part.

-- Part 2 --

The second part of this exam should be resolved in detail and justified. **Deliver your resolution on sheets of paper** to the questions in this part.

P2.1 *[Square wave]* The code provided in this question allows generating a square wave based on a sequence of pulses. Various wave forms are available depending on **your student number** written in variable **%MD0**. Consider the code shown below is the single code run by a PLC in a MAST section configured as cyclic.

a) Assume that the assignments denoted by the symbol "**:=**" are the single instructions consuming time, and that each one takes about **0.2msec**. The PLC input and output take **1msec + 1msec**. Note some scan cycles are faster than others. Compute the minimum scan cycle period and the maximum scan cycle period.

b) Given the program, indicate the possible values of **%MD2**. Can we replace **%MD2** by **%MD1**, in all places of the program, without effecting the results in **%MD4**?

```
(* sequence of 1sec pulses *)
TP_0( IN:=NOT(%m0), PT:=t#1s, Q=>%m0 );

(* output value on %MD190 *)
if RE(%m0) then
  if %MD2=0 then
    %MD2:=1;
  elsif GE(%MD2,100000) then
   (* found %MD2 >= 100000 *)
    %MD2:=1;
  else
    %MD2:=10*%MD2;
  end_if;
  %MD4:= MOD( %MD0/%MD2, 10 );
  %MD190:= %MD4 +48 +256*32;
end_if;
```

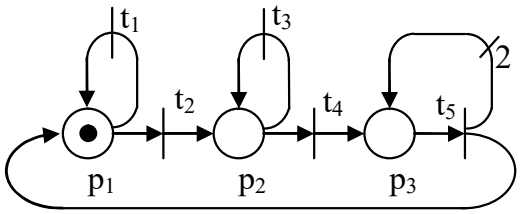
c) Write additional Structured Text code so that at the first scan cycle, and just in that scan cycle, the memory variable **%MD0** is set with **your student number** (5 or 6 digits).

d) Consider that **%MD0** contains your student number. Assume **%MD4** is zero at **t=0** (as usual in PLCs after boot). Plot the values of **%MD4** from **t=0** to **t=12** seconds, according to the program running. Say one possible application for the value stored in **%MD190**. Note: the division of a double integer is rounded to a double integer.

P2.2 *[Petri net properties]* This problem focus on the discrete event systems analysis tools studied in the course. Consider the Petri net shown in the next figure.

a) Discuss the **boudedness**, **safeness** and **coverability** properties resorting to a reachability tree. Discuss the **liveness** of each transition.

b) Using the method of matrix equations, classify the Petri net about its **conservation**.



P2.3 *[Supervision]* Consider a Petri net $C = (P, T, D^+, D^-, \mu_0)$ where D^+ and D^- denote pre and post incidence matrices, μ_0 is the initial state and

$$P = \{p_1, p_2, \ \dots \ , p_N\} \quad T = \{t_1, t_2, \ \dots \ , t_N\} \quad D^+ = \begin{bmatrix} \vec{0} & 1 \\ I_{N-1} & \vec{0} \end{bmatrix} \quad D^- = I_N$$

N is a natural number, $N \geq 3$
I_N is a $N \times N$ identity matrix

a) Let $\mu_0 = [1\ 1\ 0\ \cdots\ 0]^T$. Compute a supervisor, $\left(D_c, \mu_{c_0}\right)$ based in place invariants, that makes the net safe.

b) Ignore the supervisor designed in (a). Consider $N = 5$, $\mu_0 = [1\ 1\ 0\ 0\ 0]^T$, $L = [0\ 0\ 1\ 0\ 0]$, $b = [1]$, $R_1 = [0\ 0\ 0\ 1\ 0]$, $R_2 = [1]$. The linear constraint $L\mu_p \leq b$ implies safeness of any place(s)? Let $L_a = R_1 + R_2 L$ and $b_a = R_2(b+1) - 1$, prove that the solutions $\mu_p \in \mathbb{N}_0^5$ of $L_a\mu_p \leq b_a$ are also valid solutions for $L\mu_p \leq b$. Propose a supervisor respecting $L_a\mu_p \leq b_a$. Check the transitions involved in the supervisor. Suggest a supervisor respecting $L\mu_p \leq b$ and that t_3 is unobservable.

P2.4 *[Stochastic PN and Markov chains]* Consider a Petri net $C = (P, T, A, w, \mu_0)$ where

$$P = \{p_A, \ p_B\}, T = \{t_1, t_2, t_3, t_4\} \text{ and } \mu_0 = (2, 0).$$

There are 8 arcs all with unit weight:

$$A = \{(p_A, t_1), (t_1, p_B), (p_A, t_2), (t_2, p_A), (p_B, t_3), (t_3, p_A), (p_B, t_4), (t_4, p_B)\}.$$

a) Draw the Petri net. Indicate the reachable set of the Petri net.

b) Consider now the transition firing probabilities $Prob(t_1) = \alpha$, $Prob(t_2) = 1 - \alpha$, $Prob(t_3) = \beta$ and $Prob(t_4) = 1 - \beta$. Let $P_{20} = Prob(\mu = (2,0))$ denote a steady state probability, i.e. *state holding time*. If $\alpha = 1/4$ and $\beta = 1/2$ then what is the value P_{20}?

c) Consider again $\alpha = 1/4$ and $\beta = 1/2$, and, in addition, the total number of firings of t_1, t_2, t_3 and t_4 is 2800. Indicate the expected number of firings of transition t_1.

PS: Do not forget to identify all sheets of paper. Good luck, José Gaspar