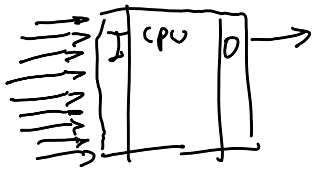
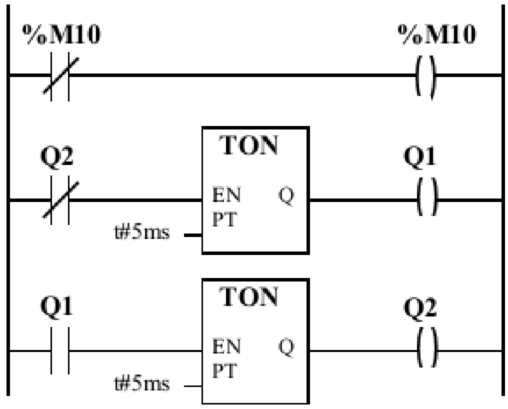
Sol: Ex2 2012/13 30.1.2013

Q1. *Majority circuit:* Implement a majority circuit with nine inputs using a standard PLC programming language. Name the inputs %i0.2.0 till %i0.2.8, and name the output %q0.4.0.

```
ACC:=0;
IF %i0.2.0 THEN ACC:=ACC+1; END_IF;
IF %i0.2.1 THEN ACC:=ACC+1; END_IF;
IF %i0.2.2 THEN ACC:=ACC+1; END_IF;
IF %i0.2.3 THEN ACC:=ACC+1; END_IF;
IF %i0.2.4 THEN ACC:=ACC+1; END_IF;
IF %i0.2.5 THEN ACC:=ACC+1; END_IF;
IF %i0.2.6 THEN ACC:=ACC+1; END_IF;
IF %i0.2.7 THEN ACC:=ACC+1; END_IF;
IF %i0.2.8 THEN ACC:=ACC+1; END_IF;
IF ACC>=5 THEN %q0.4.0:=True; ELSE %q0.4.0:=False; END_IF;
```

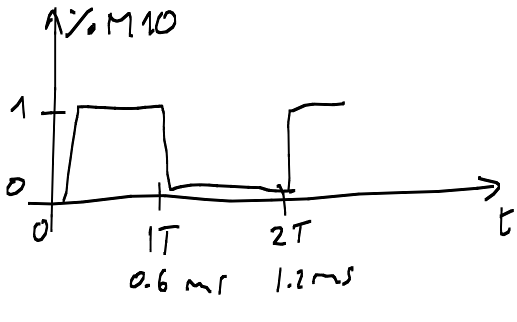


Q2. *Scan cycle:* Consider that the ladder diagram in the next figure is the single code run by a PLC, in a MAST section configured to be cyclic. The PLC input and output take **0.1msec+0.1msec** and each ladder instruction (contact read, coil write, timer) takes about **0.05msec**. At **t=0** the memory cells **%M10**, **Q1** and **Q2** have the logic value False. The timers have preset values of **5msec**. (i) Indicate the scan period of the PLC. (ii) Sketch the time response of **%M10** indicating clearly the time scale. (iii) Sketch the time response of **Q1**. (iv) Discuss whether **%M10** and **Q1** can or cannot be accurately replicated by an output **%q0.4.1**.

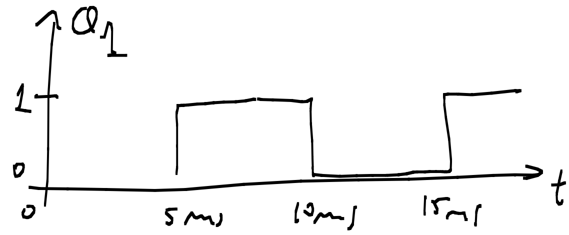


(i) $T = 0.1 + 8 \times 0.05 + 0.1 = 0.1 + 0.4 + 0.1 = 0.6$

(ii)



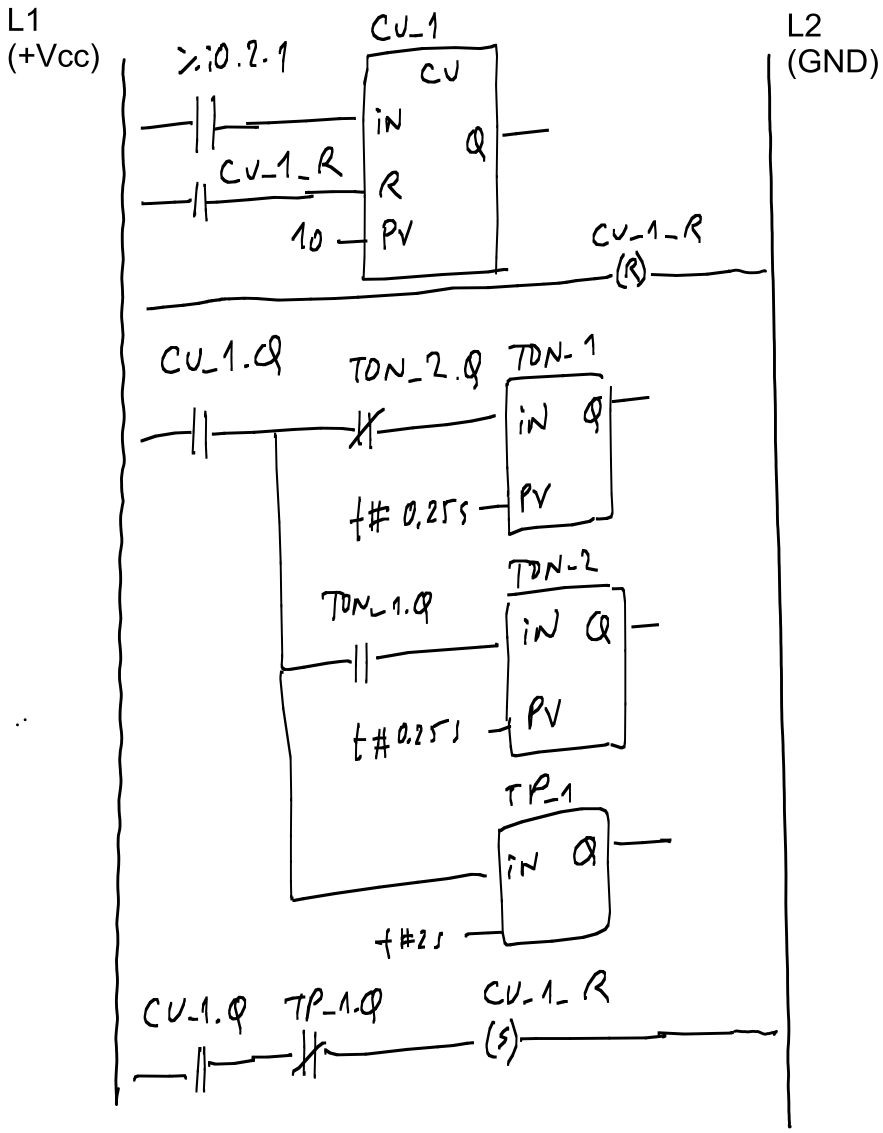
(iii)

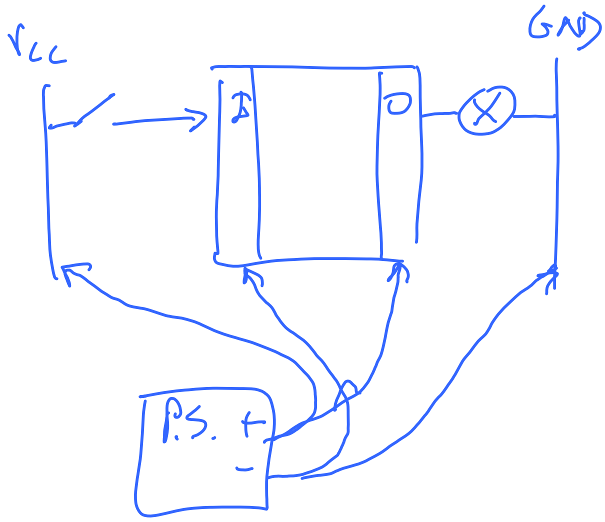


(iv) usually outputs are stable for $\Delta t \geq 4$ms, hence %M10 won't appear in the output

Q_1 can be seen if output is driven by transistors (if it is driven by relays, then Q_1 cannot be seen and/or can break after some short term the relays)

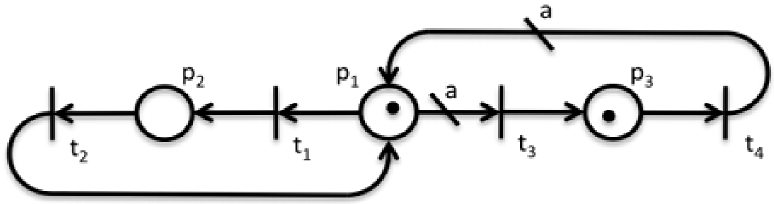
Q3. *Program:* Considering the PLC programming languages learnt in the course, implement the following logic function: After a counter reaches the counting of ten rising edge triggers in the PLC input %i0.2.1, a light connected to the output %q0.4.1 flashes at 2Hz, 50% duty cycle, during two seconds. The counter restarts only after ending the flashing of the light.





Q4. *PN properties:* Consider the Petri net graph shown in the next figure. Note that there are two arcs with generic non-negative weights 'a'.

See ex1 2010

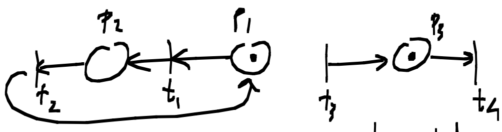


Let **a=0**.

a) Discuss the conservativeness and the boundness of the aforementioned Petri net, resorting to a reachability (sub)tree.

b) Discuss the liveness of each transition and the overall level of liveness for the Petri net.

a)



unbounded

non-conservative

(1,0)
$\downarrow t_1$
(0,1)
$\downarrow t_2$
(1,0)
dup

safe, i.e. 1-bounded
conservative

(1)
$\swarrow t_4$ $\searrow t_3$
(0) $\quad$ ~~(2)~~ (ω)
$\swarrow t_3$
(1)
dup

∴ the complete net is:
unbounded &
non-conservative

b)

t_1, t_2 are live L_4

t_3 is live L_4

t_4 is live L_4 (can fire ∞ times, provided t_3 fires ∞ times)

(1, 0, 1)
$\swarrow t_1$ $\downarrow t_3$ $\searrow t_4$
(0, 1, 1) $\quad$ (1, 0, 2) $\quad$ (1, 0, 0)
$\swarrow t_2$ $\downarrow t_3$ $\searrow t_4$

Let **a=1**.

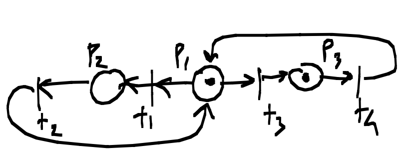
c) Discuss the conservativeness of the Petri net, for this case, and provide the weight vector.

d) Resorting to the Method of the Matrix Equations, study if and how the marking u=[1 1 1]' can be reached.

e) Build the reachability tree. Is the marking u=[0 2 0]' reachable?

f) Find the cycles of operation or place invariants, for this Petri net.

c)



$$D = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array}$$
$$D = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$w^T D = 0 \Leftrightarrow [w_1 \; w_2 \; w_3] D = 0$

$$\begin{cases} -w_1 + w_2 = 0 \\ w_1 - w_2 = 0 \\ -w_1 + w_3 = 0 \\ w_1 - w_3 = 0 \end{cases} \qquad \begin{cases} w_1 = w_2 \\ w_1 = w_3 \end{cases}$$

∴ is strictly conservative

e.g. $w_1 = w_2 = w_3 = 1$

d)

$\mu' = \mu + Dq$

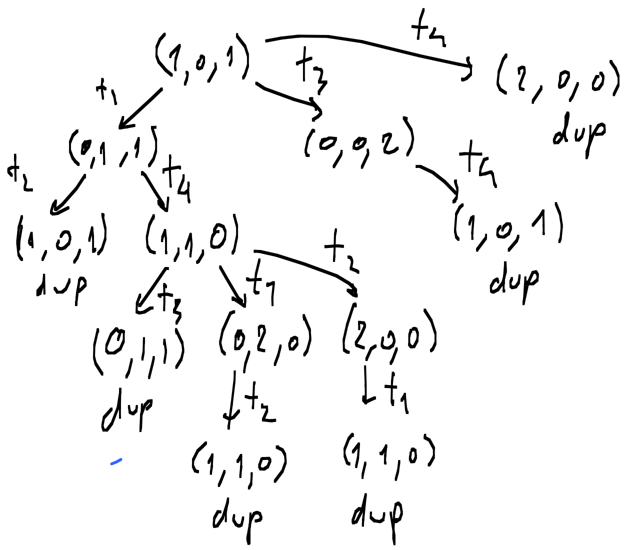
$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + Dq$$

$$\begin{cases} -q_1 + q_2 - q_3 + q_4 = 0 \\ +q_1 - q_2 = 1 \\ q_3 - q_4 = 0 \end{cases} \qquad \begin{cases} q_1 - q_2 = 0 \\ q_1 - q_2 = 1 \end{cases}$$

impossible

∴ u is not reachable

e)



Yes $x=[0\ 2\ 0]'$ is reachable

e.g.

$$(1,0,1) \xrightarrow{t_1} (0,1,1) \xrightarrow{t_4} (1,1,0) \xrightarrow{t_1} (0,2,0)$$

f)

$x^T D = 0$ (done before) $x = [1\ \ 1\ \ 1]^T$ all places can participate in the cycles

cycles $Dq=0$

$$\begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = 0$$

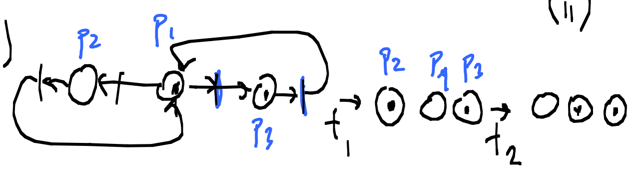
only 2 eqs relevant $(D(1,:) = -D(2,:) - D(3,:))$

$$\begin{cases} q_1 - q_2 = 0 \\ q_3 - q_4 = 0 \end{cases} \quad \begin{cases} q_1 = q_2 \\ q_3 = q_4 \end{cases}$$

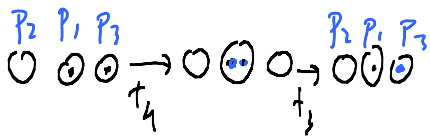
e.g. (i) $q_1 = q_2 = 1$ and $q_3 = q_4 = 0$ or

(ii) $q_1 = q_2 = 0$ and $q_3 = q_4 = 1$

(i)



(ii)



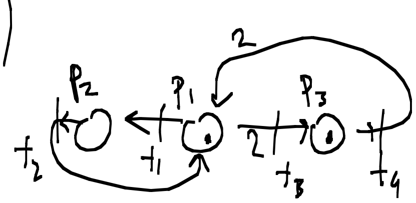
$$\left(\mathrm{null}(D^T) = \mathrm{null}\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad \begin{cases} x_1 = x_2 \\ x_2 = x_3 \end{cases} \right)$$

Let **a=2**.

g) Discuss the following statement "This Petri net is strictly of level 3".

h) Discuss the liveness levels for a=0 and a greater or equal to 2.

g)



All transitions are Level 4 ($\forall \mu$ $\exists q$ such that t_i fires)

$\therefore$ PN is not strictly L3

h)

$a=0$ (seen before) $\rightarrow$ PN is L_4

$a \geq 2$ $(=2$ see in g$)$ $\rightarrow$ $\begin{cases} t_3 \text{ firing} \Rightarrow \mu_3 \mathrel{+}= 1,\ \mu_1 \mathrel{-}= a \\ t_4 \text{ firing} \Rightarrow \mu_1 \mathrel{+}= a,\ \mu_3 \mathrel{-}= 1 \end{cases}$

Q5. *Supervision:* Consider a discrete event system describing the state of a fleet of Automatic Guided Vehicles (AGVs) transporting parts in a factory and the state of a set of energy-charging stations. The state of the fleet and the charging stations is described by the 5-tuple **{P,T,A,w, μ_0}** where

$P=\{p_1, p_2, p_3, p_4, p_5\}$, **$T=\{t_1, t_2, t_3, t_4, t_5, t_6, t_7\}$**
$A=\{(t_1,p_1), (p_1,t_2), (t_2,p_2), (p_2,t_3), (t_3,p_3), (p_3,t_4), (t_5,p_4), (p_4,t_6), (t_6,p_5), (p_5,t_7)\}$
$\forall_{i,j}\, w(t_i,p_j)=1, \forall_{k,l}\, w(p_k,t_l)=1$, and **$\mu0=[0\ 1\ 0\ 0\ 0]^T$**

The meaning of the conditions and the events is the following:

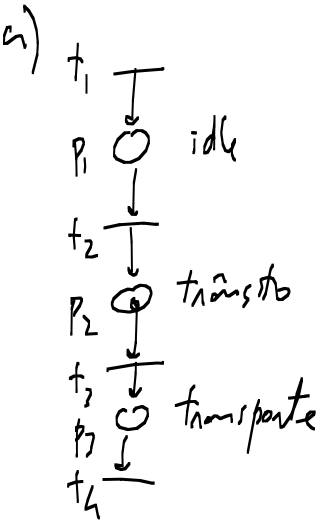
- **p_1** - AGV(s) idle
- **p_2** - AGV(s) moving
- **p_3** - AGV(s) transporting loads
- **p_4** - Charger(s) idle
- **p_5** - Charger(s) charging AGV(s)

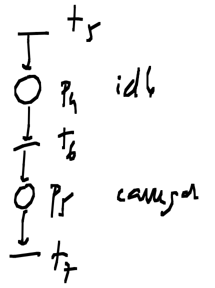
- **t_1** - AGV(s) end operation
- **t_2** - AGV(s) start moving
- **t_3** - AGV(s) start transporting loads
- **t_4** - AGV(s) stop transporting loads
- **t_5** - Charger(s) changing to idle
- **t_6** - Charger(s) start charging
- **t_7** - Charger(s) stop charging

a) Draw the graph and write the incidence matrix D_p of the Petri net.

b) Design a supervisor based on place invariants stating that there are at most **10** AGVs and **4** charging stations. Draw the supervisor in the Petri net shown in a).

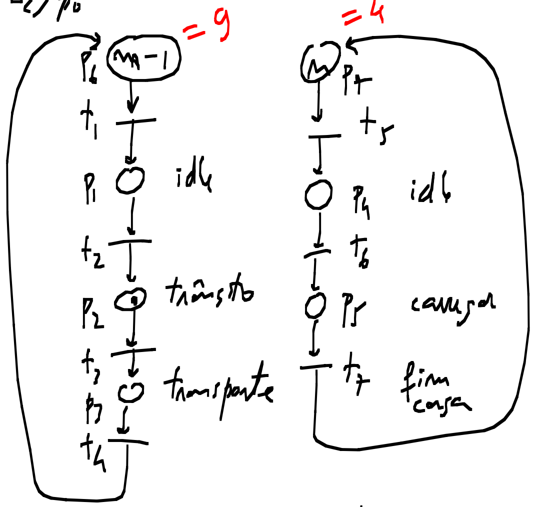
a)





D=	t_1	t_2	t_3	t_4	t_5	t_6	t_7
p_1	1	-1					
p_2		1	-1				
p_3			1	-1			
p_4					1	-1	
p_5						1	-1

$$\mu_{c_1} = b_1 - L_1 \mu_{p_0} = 10 - 1 = 9$$
$$\mu_{c_2} = b_2 - L_2 \mu_{p_0} = 4 - 0 = 4$$



D=	t_1	t_2	t_3	t_4	t_5	t_6	t_7
p_1	1	-1					
p_2		1	-1				
p_3			1	-1			
p_4					1	-1	
p_5						1	-1
p_6	-1			1			
p_7					-1		1

b)

$$\begin{cases} \mu_1+\mu_2+\mu_3 \le m & (m=10) \\ \mu_4+\mu_5 \le m & (m=4) \end{cases}$$

$$L_1 = [1\ 1\ 1\ 0\ 0],\quad L_2 = [0\ 0\ 0\ 1\ 1]$$

$$D_{c_1} = -L_1 D_p = [1\ 0\ 0\ -1\ 0\ 0\ 0]$$
$$D_{c_2} = -L_2 D_p = [0\ 0\ 0\ 0\ 1\ 0\ -1]$$

c) Use the incidence matrix to verify if the Petri net containing the supervisor is conservative. Compute the places weighting vector if the net is conservative.

$$w^T D = 0$$

$$\begin{cases} w_1 - w_6 = 0 \\ -w_1 + w_2 = 0 \\ -w_2 + w_3 = 0 \\ -w_3 + w_6 = 0 \\ w_4 - w_7 = 0 \\ -w_4 + w_5 = 0 \\ -w_5 + w_7 = 0 \end{cases} \quad \begin{cases} w_1 = w_6 = w_2 \\ w_2 = w_3 = w_6 \\ w_4 = w_7 = w_5 \end{cases} \quad \begin{cases} w_1 = w_2 = w_3 = w_6 \\ w_4 = w_5 = w_7 \end{cases}$$

e.g.

$$\begin{cases} w_1 = w_2 = w_3 = w_6 = 1 \\ w_4 = w_5 = w_7 = 1 \end{cases} \quad \therefore \text{ is conservative}$$

$$w^T = [1\ 1\ 1\ 1\ 1\ 1\ 1] \quad // \text{ cqd}$$

d) Change the Petri net by adding two transitions t_8 and t_9, and two arcs (p_2, t_8) and (t_9, p_2) both with unitary weights. The new transitions have the meanings t_8 - AGV battery discharged, t_9 - AGV battery recharged. Design a supervisor based on place invariants, considering generalized linear constraints, such that one moving AGV whenever it detects it is discharged it can go to a charging station (if available) and come back to the moving condition. Draw the supervisor in the global Petri net.

1) $v_6 \leq v_8$ $\quad D_{c_1} = -C_1 = -[\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ 1\ \cdot\ -1\ \cdot]$ $\quad \mu_{c_1} = b_1 = 0$

2) $v_8 \leq v_7 + m$ $\quad D_{c_2} = -C_2 = -[\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ -1\ 1\ \cdot]$ $\quad \mu_{c_2} = b_2 = m = 4$

3) $v_9 \leq v_7$ $\quad D_{c_3} = -C_3 = -[\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ -1\ \cdot\ 1]$ $\quad \mu_{c_3} = b_3 = 0$