

Read all questions of the exam carefully before starting to answer.

- Provide detailed justifications to all answers.
- The use of bibliographic material, either in paper or in digital format is allowed.
- Exchange of information is forbidden (e.g. voice, WiFi, Bluetooth, GPRS, WAP,...).
- Exam duration: 3 hours.

Q1. Implementation of a latch logic function with a PLC: (i) Implement in Ladder the logic function described by $X = (A \text{ OR } X) \text{ AND NOT}(B)$, where **A** and **B** are input signals and **X** is a PLC memory. (ii) Draw the time response $X(t)$ for $t \in [0, 5] \text{sec}$ considering that **X** at $t=0 \text{sec}$ is True (logic value 1), the scan period is negligible (approx. zero) and the inputs are False except during $t \in [1, 2] \text{sec}$ where **B** is True (logic 1), and during $t \in [3, 4] \text{sec}$ where **A** is True. (iii) Implement in Ladder an alternative latch function $X = A \text{ OR } (\text{NOT}(B) \text{ AND } X)$, and draw a time diagram with inputs at your own will showing that the logic functions are similar but not exactly equivalent.

Q2. Scan cycle and output description. Consider that the structured text program shown bellow is the single code run by a PLC, in a MAST section configured to be cyclic. The PLC input and output take **1msec+1msec**. TON_2 is a timer on delay with a preset value of **2000msec**.

- a) Consider that assignments shown in the program, denoted by symbols := and =>, are the single instructions consuming time, and that each one takes about **0.1msec**. Describe the output signals %q0.4.1 and %q0.4.2. In case there are constant oscillations indicate their frequencies.

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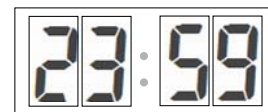
%q0.4.1 := NOT(%q0.4.1);
TON_2 (IN := NOT(%M102),
      PT:=t#2000ms, Q => %M102);
IF %M102
THEN %q0.4.2 :=NOT(%q0.4.2);
ELSE %q0.4.2 := %q0.4.2;
END_IF;

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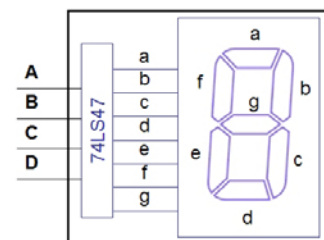
- b) What changes in (a) if MAST is configured to be periodic with period (i) **2msec** or (ii) **10msec**.

Q3. Ladder and ST. In this problem is implemented a 24 hours clock with a digital display, HH:MM, where HH indicates hours and MM are minutes (see figure bellow). Each displayed digit is encoded as four bits (A,B,C,D). The most significant bit is A. For example, (A,B,C,D)=(1,0,0,0) denotes digit 8 and (0,0,0,1) denotes digit 1. The clock can be reset to 00:00 using a pressure button.

- a) Show the electric diagram. Indicate clearly the number of bits necessary to display HH:MM.



- b) Implement the clock in Ladder. Use variables %MW1 and %MW2 to represent HH and MM, respectively. Use %i0.2.0 to set the clock to 00:00. Suggestion: use one timer and a number of counters.



- c) Write structured text to convert %MW1 and %MW2 to the output bits. Note in the figure that each digit is set though the binary inputs (A,B,C,D).

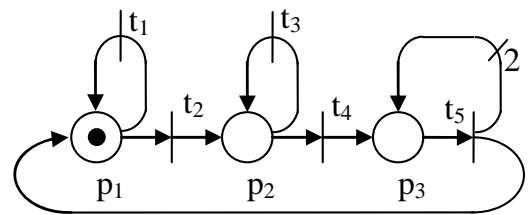
Q4. Petri net modeling. In this problem the 24 hours clock system, implemented in Q3, is modeled using Petri nets. The clock starts to be modeled by three sub-systems, as detailed in the next sentences. **Minutes** are represented by the marks in place p_1 : firing transition t_1 increments p_1 , firing t_2 decrements N marks from p_1 , where N is a constant to be chosen by you. **Hours** have a representation similar to the one used for minutes. One **timer** is represented by one marked place and one transition, which fires every 1 minute. Firing the timer transition decrements the timer marked place and then increments it back.

- a) Draw the initial Petri net. Identify clearly all conditions, all events and the initial marking.
b) Design supervisors based on marking invariants to upper bound the numbers of minutes and hours.

- c) Design supervisors, based in marking invariants, using generalized constraints, such that the one minute timeouts make advance the minutes counter, and accumulated minutes increment the number of hours when necessary.
- d) Discuss the properties of safeness, boundedness and coverability of the supervised Petri net. If needed, add additional constraints such that the supervised Petri net is bounded.
- e) One may conjecture that the constraints associated to the sequence or quantity of firings can be rewritten with simple linear constraints by adding an arc and an auxiliary place at the output of the transitions under consideration. Try to repeat (c) using this conjecture.
- f) In this Petri net most of the transitions are supposed to fire automatically as soon as their preconditions (markings) are met. Consider now that transitions associated to counting hours depend on hardware which is malfunctioning and thus do not fire. In your solution, does the counting of minutes stop after a while? If so, how long does it take the stop of counting hours to cause a stop of counting minutes? Propose changes to the previous questions such that stopping the hours counting also stops, as soon as possible, counting minutes.

Q5. Petri net properties. This problem focus on the discrete event systems analysis tools studied in the course. Consider the Petri net shown in the next figure.

- a) Discuss the boundedness, safeness and coverability properties resorting to a reachability tree. Discuss the liveness of each transition.
- b) Using the method of matrix equations, classify the Petri net about its conservation.
- c) Using the same method, determine if the state (3, 4, 12) is reachable. If it is reachable then indicate a sequence of firings to reach that state.
- d) Determine the shortest operation cycles for the Petri net.



- e) Change two arc weights, namely $w(t_4, p_3)=a$ and $w(p_3, t_5)=b$. Discuss boundedness and liveness when (i) $(a,b)=(1,2)$ or (ii) $(a,b)=(2,2)$. Suggest another arc to change its weight such that when $(a,b)=(1,1)$ the net is bounded and has no deadlocks. Indicate the chosen arc and its weight.

Q6. Petri net simulation. In the Petri net simulator used in the laboratory, conflicting transitions are prioritized by their identification numbers: the transition with the lowest number is the one selected to be fired. Let D^- and D^+ denote the preconditions and pos-conditions matrices, and let q denote a firing vector. Using matrix operations, propose $f_1(\cdot), f_2(\cdot)$ and $f_3(\cdot)$, to obtain a new Petri net characterized by $D_2^- = f_1(D^-)$, $D_2^+ = f_2(D^+)$ and $q_2 = f_3(q)$ such that the original effects of transitions are kept but q_2 has the desired priorities.

Q7. Petri net supervision: Let \mathbf{M} denote a Petri net which is described by its incidence matrix $\mathbf{D}_p = \mathbf{D}_p^+ - \mathbf{D}_p^-$ and initial marking μ_{p0} .

- a) What is the expression of a supervisor, \mathbf{D}_c based on place invariants with linear constraints $\mathbf{L}\mu + \mathbf{F}q + \mathbf{C}v \leq \mathbf{b}$, when \mathbf{L} and \mathbf{C} are null? Show also $\mathbf{D}_c^+, \mathbf{D}_c^-$, and μ_{c0} . Use the notation $\mathbf{F}^+ = \max(\mathbf{0}, \mathbf{F})$ and $\mathbf{F}^- = \text{abs}(\min(\mathbf{0}, \mathbf{F}))$.
- b) Describe the graphical representation of the supervisor, more precisely, describe how one introduces the supervisor obtained in (a), i.e. $\mathbf{D}_c^+, \mathbf{D}_c^-, \mu_{c0}$, into the graphical representation of \mathbf{M} . Give one example for a net having a single place, a single arc, a single transition and a larger than 1 initial marking.
- c) Consider one single $\mathbf{F}q \leq \mathbf{b}$ constraint on a single transition j . In other words, \mathbf{b} is a scalar and only the entry j of \mathbf{F} is not zero. (i) Write expressions indicating upper bounds to the maximum consumed and created marks by firing the transition j . Do not include the supervisor contribution in the expressions. (ii) Indicate how to upper bound the number of marks consumed by firing j . (iii) Discuss whether nonzero lower bounds can also be found.

PS: Do not forget to identify all sheets of paper.