Problem

Consider a discrete event system describing an interface between two manufacturing stages in an industrial environment. This interface comprises robots and a transport conveyor. The system can be described by the following 5-tuple:

(P,T,A,w,μ_0)

$P=\{p_1, p_2, p_3, p_4\}$

$T=\{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8\}$

$A=\{(t_1,p_1), (p_1,t_2), (t_2,p_2), (p_2,t_3), (p_2,t_4), (t_5,p_3), (p_3,t_6), (t_7,p_4), (p_4,t_8)\}$

$w(t_1,p_1)=1$, $w(p_1,t_2)=1$, $w(t_2,p_2)=1$, $w(p_2,t_3)=1$, $w(p_2,t_4)=1$,

$w(t_5,p_3)=1$, $w(p_3,t_6)=1$, $w(t_7,p_4)=1$, $w(p_4,t_8)=1$

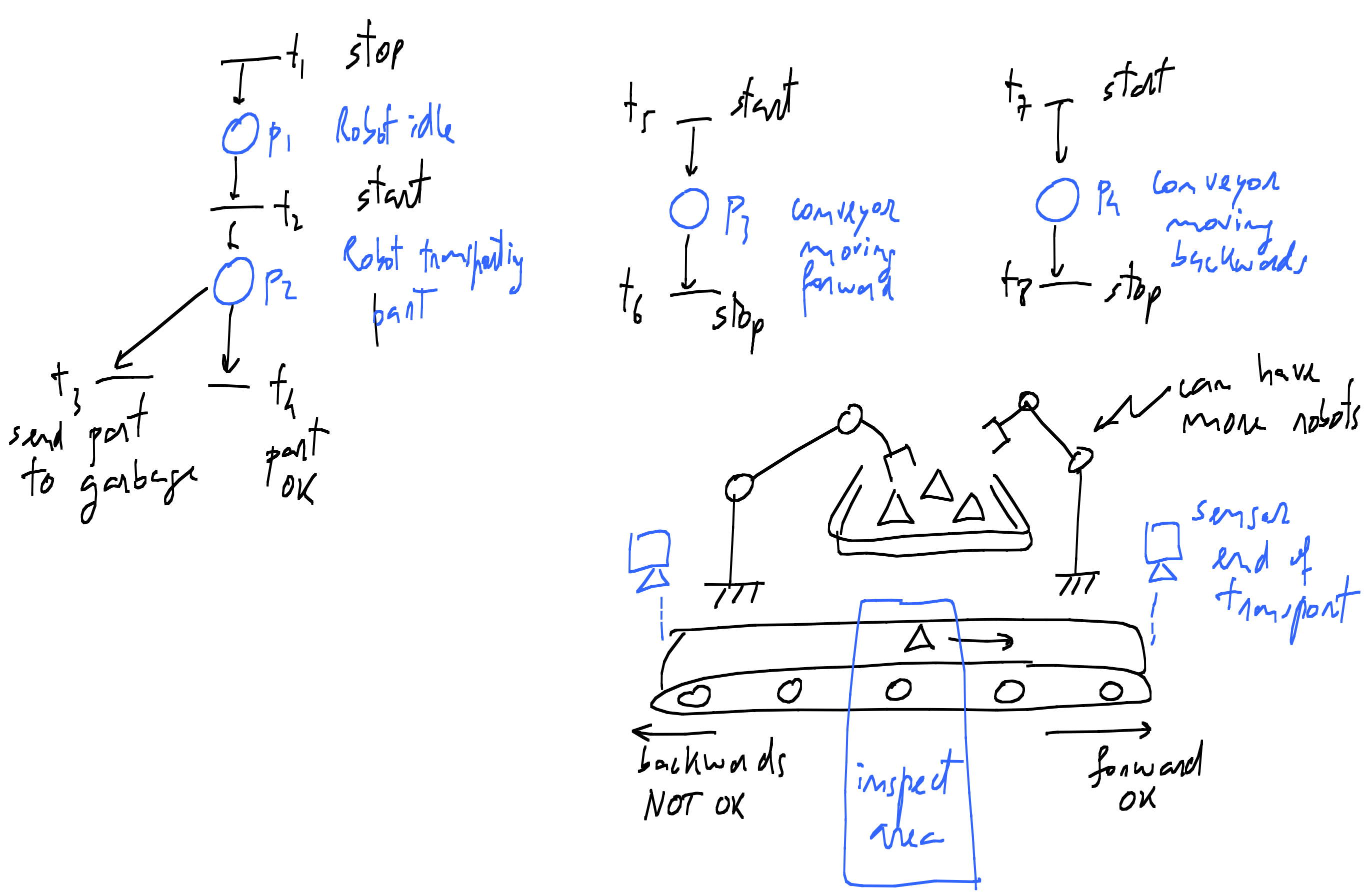
$\mu 0=[0\ 0\ 0\ 0]^T$

with the following interpretation of conditions and events:

p_1 – Robot idle
p_2– Robot transporting a part
p_3- Conveyor moving forward
p_4- Conveyor moving backwards

t_1 – Robot ends operation
t_2 - Robot starts operation
t_3 – Defective part
t_4 – Valid part
t_5 – Conveyor starts moving forward
t_6 - Conveyor end moving forward
t_7- Conveyor starts moving backwards
t_8 - Conveyor ends moving backwards

a) [1v] Draw the graph of the Petri net.



b) [1v] What is the incidence matrix, D_p, of the Petri net?

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
p_1	+1	−1						
p_2		+1	−1	−1				
p_3					+1	−1		
p_4							+1	−1

Note 1: $(t_i, p_j) \Rightarrow +1$, $(p_i, t_j) \Rightarrow -1$

Note 2: 3 blocks matrix, why?

c) [2v] Design one supervisor based on invariant places, specifying that there are at most 2 robots (robotic manipulators) and 1 transport conveyor. The transport conveyor cannot be moving forward and backwards simultaneously. Superimpose the supervisor on the drawing of the Petri net drawn in a).

At most 2 robots & 1 conveyor.

Conveyor cannot move forward backwards simultaneously.

$$\begin{cases} \mu_1 + \mu_2 \leq 2 \\ \mu_3 + \mu_4 \leq 1 \end{cases} \qquad L = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

(i) $b - L\mu_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} - L \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \geq 0$ cqd

(ii) $D_c = -LD_p = -\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} D_p = \begin{bmatrix} -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{bmatrix}$ //

(iii) $\mu_{c0} = b - L\mu_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} - L \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ /

d) [1v] Using the Method of the Matrix Equations, determine whether the Petri net is strictly conservative and, if so, compute the (non trivial) marking invariants that verify the minimum firings.

$$w^T D = 0 \quad , \quad w = [w_1 \ w_2 \ w_3 \ w_4 \ w_5]^T$$

$$[w_1 \ w_2 \ w_3 \ w_4 \ w_5] \begin{array}{c} \\ \\ \\ \\ p_5 \\ p_6 \end{array} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{bmatrix} = 0$$

$$\begin{array}{cccccccc} & & \multicolumn{2}{c}{=} & \multicolumn{2}{c}{x-1} & \multicolumn{2}{c}{x-1} \end{array}$$

Two blocks:
Two independent PNs

$$\begin{cases} w_1 - w_5 = 0 \\ -w_1 + w_2 = 0 \\ -w_2 + w_5 = 0 \\ w_3 - w_6 = 0 \\ w_4 - w_6 = 0 \end{cases} \qquad \begin{cases} w_1 = w_5 \\ w_1 = w_2 \\ w_2 = w_5 \\ w_3 = w_6 \\ w_4 = w_6 \end{cases}$$

$w_1 = w_2 = w_5$ e.g. $\begin{cases} w_1 = 1 \\ w_2 = 1 \\ w_5 = 1 \end{cases}$ ∴ strictly conservative

$w_3 = w_4 = w_6$ e.g. $\begin{cases} w_3 = 1 \\ w_4 = 1 \\ w_6 = 1 \end{cases}$ ∴ strictly conservative

Transition invariants, firing vectors:

$$Dq = 0$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix} = 0$$

Summary:
$q_1 - q_2 = 0$
$q_2 - q_3 - q_4 = 0$

$$\begin{cases} q_1 - q_2 = 0 \\ q_2 - q_3 - q_4 = 0 \\ q_5 - q_6 = 0 \\ q_7 - q_8 = 0 \\ -q_1 + q_3 + q_4 = 0 \\ -q_5 + q_6 - q_7 + q_8 = 0 \end{cases} \qquad \begin{cases} q_1 = q_2 \\ q_2 = q_3 + q_4 \\ (q_2 = q_3 + q_4) \\ \hline q_5 = q_6 \\ q_7 = q_8 \\ 0 = 0 \end{cases}$$

Sol. 1: e.g. $q_3 = 1$, $q_4 = 1$; $q_1 = q_2 = 2$

or Sol. 2: hip. $q_4 = 0$, $q_3 = 1$: $\begin{cases} q_1 = 1 \\ q_2 = 1 \\ q_3 = 1 \\ q_4 = 0 \end{cases}$

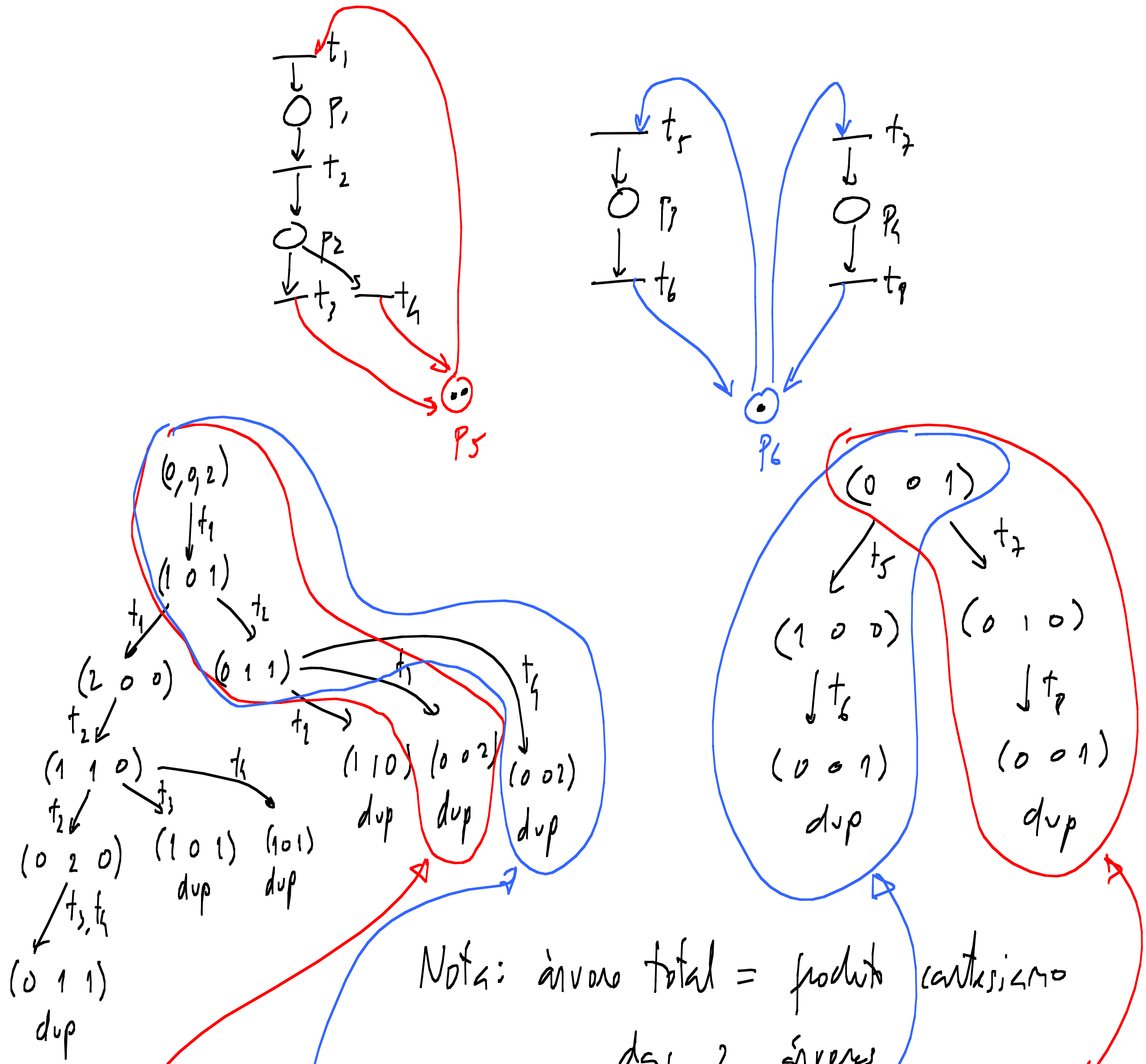
or Sol. 3: hip. $q_4 = 1$, $q_3 = 0$: $\begin{cases} q_1 = 1 \\ q_2 = 1 \\ q_3 = 0 \\ q_4 = 1 \end{cases}$

Sol. 1: $\begin{cases} q_5 = 1 \\ q_6 = 1 \\ q_7 = 1 \\ q_8 = 1 \end{cases}$

or Sol. 2: $q_7 = 0$, $q_6 = 1$: $\begin{cases} q_5 = 1 \\ q_6 = 1 \\ q_7 = 0 \\ q_8 = 0 \end{cases}$

or Sol. 3: $q_6 = 0$, $q_7 = 1$: $\begin{cases} q_5 = 0 \\ q_6 = 0 \\ q_7 = 1 \\ q_8 = 1 \end{cases}$

e) [1v] Build the reachability tree for the Petri net obtained in c). Discuss the conservation of the net. Show in the tree the invariants determined in d).



Nota: árvore total = produto cartesiano das 2 árvores

$$\begin{cases} q_1 = 1 \\ q_2 = 1 \\ q_3 = 1 \\ q_4 = 0 \end{cases} \quad \text{ou} \quad \begin{cases} q_1 = 1 \\ q_2 = 1 \\ q_3 = 0 \\ q_4 = 1 \end{cases}$$

$$\begin{cases} q_5 = 1 \\ q_6 = 1 \\ q_7 = 0 \\ q_8 = 0 \end{cases} \quad \text{ou} \quad \begin{cases} q_5 = 0 \\ q_6 = 0 \\ q_7 = 1 \\ q_8 = 1 \end{cases}$$

f) [2v] Draw a supervisor based on marking invariants, using generalized linear constraints, such that after detecting a defective part the conveyor is actuated to move backwards. In the case of a valid part, the conveyor is actuated forward. One part is classified as valid or defective at the end of its transport done by the robot.

$$\begin{cases} v_3 \geq v_7 \\ v_4 \geq v_5 \end{cases} \qquad \begin{cases} v_7 - v_3 \leq 0 \\ v_5 - v_4 \leq 0 \end{cases} \qquad C = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} D_c^- = \max(0,\ LD_p + C,\ F) \\ D_c^+ = \max(0,\ F - \max(0, LD_p + C)) - \min(0,\ LD_p + C) \\ \mu_{c_0} = b - L\mu_{p_0} - C v_{p_0} \end{cases}$$

$$D_p = \left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{array}\right]$$

$LD_p + C = 0 + C = C$

$D_c^- = \max(0, C, 0) =$

$= \left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$

$D_c^+ = \max(0, -\max(0, C)) - \min(0, C)$

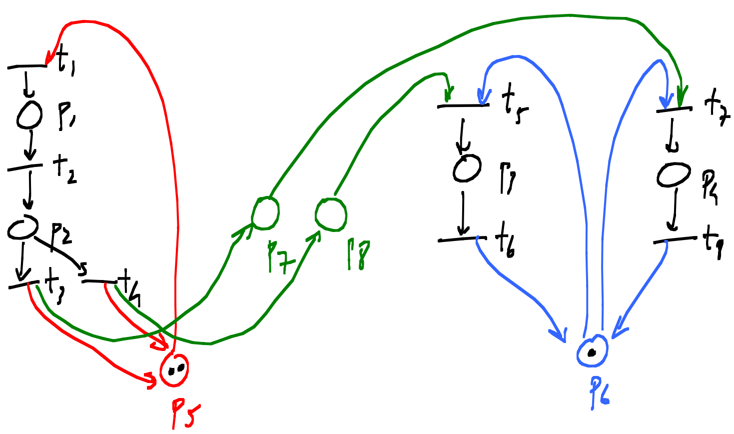
$= \max(0, -C^+) + C^- = C^-$

$= \left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right]$

$$D_c = D_c^+ - D_c^- = \left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{array}\right] /\!/ \qquad (\text{Note: } D_c = -C)$$

$$\mu_{c_0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - 0 \cdot \mu_{p_0} - C \cdot 0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} /\!/$$

g) [1v] Superimpose the supervisor just determined on the Petri net drawn in a) and c).



h) [1v] Compute the weights associated to the conservation of the net just obtained. Is it the same as determined in e)?

$$D_{P_2} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 7 \end{array} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$w^T D_{P_2} = 0$$

$$\begin{cases} w_1 - w_5 = 0 \\ -w_1 + w_2 = 0 \\ -w_2 + w_5 + w_7 = 0 \\ -w_2 + w_5 + w_8 = 0 \\ w_3 - w_6 - w_8 = 0 \quad \} \; w_8 = 0 \\ -w_3 + w_6 = 0 \\ +w_4 - w_6 - w_7 = 0 \quad \} \; w_7 = 0 \\ -w_4 + w_6 = 0 \end{cases}$$

$\therefore$ not conservative ($\exists\, w_i = 0$)

Could be seen by inspection that $t_1\ t_2\ t_3$ imply $\mu_7 \leftarrow \mu_7 + 1$

or $t_1\ t_2\ t_4$ imply $\mu_8 \leftarrow \mu_8 + 1$

i) [2v] Which is the liveness level of the complete Petri net? Justify.

Level 4 : P_1, P_2, P_5 execute an ∞ loop

that ∞ loop feeds the loops P_3, P_6 and P_4, P_6

j) [2v] One may conjecture that the constraints associated to the sequence or quantity of firings can be obtained with ~~simple linear constraints by~~ adding an arc and an auxiliar place at the output of the transitions under consideration. Try to repeat f) using this conjecture.

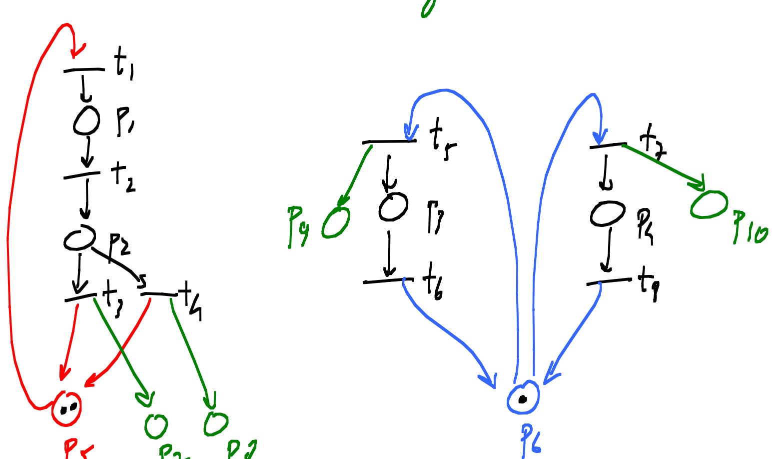
F, C

qK

✓

see green lines

Objective $\begin{cases} v_3 \geq v_7 \\ v_4 \geq v_5 \end{cases}$



Now one can write $\begin{cases} v_3 \geq v_7 \\ v_4 \geq v_5 \end{cases} \Rightarrow \begin{cases} P_7 \geq P_{10} \\ P_8 \geq P_9 \end{cases} \Leftrightarrow \begin{cases} -P_7 + P_{10} \leq 0 \\ -P_8 + P_9 \leq 0 \end{cases}$

in other words

$$L_3 = \left[\begin{array}{cccccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{array}\right], \quad b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

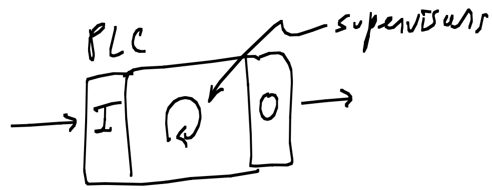
and finally

$$D = -L_3 D_{p_3} = \dots = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \quad \text{cqd}$$

k) [1v] Discuss the implementation in industrial automata (PLC) of the control and supervision derived in the preceeding questions, using the GRAFCET language. Write the GRAFCET section implementing the controller / supervisor.

Separate the supervisons

Define system/devices inputs and outputs



l) [2v] Discuss the veracity of the next statement: "If a Petri net has liveness level n then is has also liveness level n+1, for n={0, 1, 2, 3}".

False Counter-examples $L_0 \not\Rightarrow L_1$

$L_3 \not\Rightarrow L_4$

m) [1v] Suppose that during the implementation of the controller / supervision system you need to guarantee that a GRAFCET step has to actuate an output each time a set of parts (e.g. with m pieces) is completed. How would you implement it?

n) [2v] It is common using PLCs to implement the system studied in the previous questions in an industrial environment. Describe how do the PLCs work. Number some key aspects important in the choice of a PLC. Describe typical PLC failures and the ageing process.