Modeling and Automation of Industrial Processes

Modelação e Automação de Processos Industriais / MAPI

Supervised Control of Discrete Event Systems
Supervision Controllers (Part 2/2)

http://www.isr.tecnico.ulisboa.pt/~jag/courses/mapi2122

Prof. Paulo Jorge Oliveira, original slides
Prof. José Gaspar, rev. 2021/2022

Some pointers on Supervised Control of DES

Analysers & simulators

http://www.nd.edu/~isis/techreports/isis-2002-003.pdf (Users Manual)
http://www.nd.edu/~isis/techreports/spnbox/ (Software)

Bibliography:

Supervisory Control of Discrete Event Systems using Petri Nets, J. Moody J. and P. Antsaklis, Kluwer Academic Publishers, 1998.

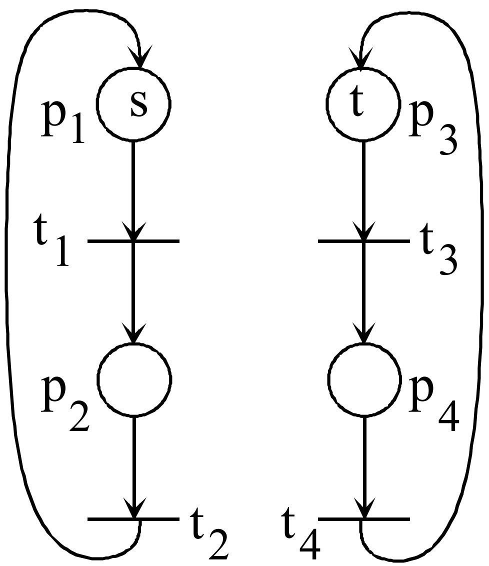
Supervised Control of Concurrent Systems: A Petri Net Structural Approach, M. Iordache and P. Antsaklis, Birkhauser 2006.

Discrete Event Systems - Modeling and Performance Analysis, Christos G. Cassandras, Aksen Associates, 1993.

Feedback Control of Petri Nets Based on Place Invariants, K. Yamalidou, J. Moody, M. Lemmon and P. Antsaklis, http://www.nd.edu/~lemmon/isis-94-002.pdf

Methods of Synthesis

Example of controller synthesis: s Producers / t Consumers



Incidence matrix

$$D_P = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Initial marking

$$\mu_{P_0} = \begin{bmatrix} s \\ 0 \\ t \\ 0 \end{bmatrix}.$$

Let p2= #machines working, t2= product produced
p3= #consumers, t3= request to consume (e.g. transport product)

Q: How to write *consume only when produced* ? What is the linear constraint?

Not possible to write it as a linear constraint on places $L\mu_p \leq b$.
Is it impossible to solve this problem with the supervised control ?

Methods of Synthesis Generalized linear constraint

Let the generalized linear constraint be

$$L\mu_P + Fq_P + Cv_P \leq b,$$
$$\mu_P \in {N_0}^n, v_P \in {N_0}^m, q_P \in {N_0}^m,$$
$$L \in Z^{nc\times n}, F \in Z^{nc\times m}, C \in Z^{nc\times m}, e \quad b \in Z^{nc},$$

where

* μ_P is the marking vector for system P
* q_P is the firing vector since t_0
* v_P is the number of transitions (firing) that can occur, also designated as Parikh vector

Example: detail elements forming the generalized linear constraint

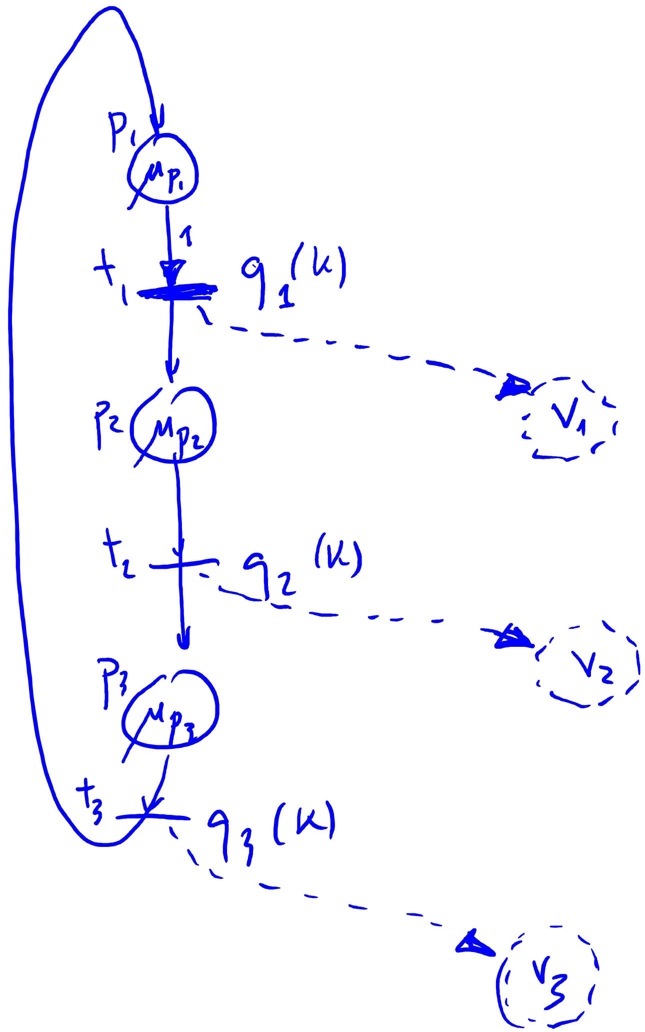
$$L\mu_P + Fq_P + Cv_P \leq b,$$

$$\mu_P \in N_0{}^n, v_P \in N_0{}^m, q_P \in N_0{}^m,$$
$$L \in Z^{n_C \times n}, F \in Z^{n_C \times m}, C \in Z^{n_C \times m},$$
$$b \in Z^{n_C},$$

State: $\mu_p(k) = \begin{bmatrix} \mu_{p1} \\ \mu_{p2} \\ \mu_{p3} \end{bmatrix}_k$

Firing vector: $q_p(k) = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}_k$

Parikh vector: $v_p(k) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_k$



Methods of Synthesis

Function LINENF of SPNBOX

Theorem*: Synthesis of Controllers based on Place Invariants, for **Generalized Linear Constraints**

Given the generalized linear constraint $L\mu_P + Fq_P + Cv_P \le b$,
if $b - L\mu_{P_0} \ge 0$, then the controller with incidence matrix and initial marking, respectively

$$D_C^- = \max\left(0,\ LD_P + C,\ F\right)$$
$$D_C^+ = \max\left(0,\ F - \max\left(0, LD_P + C\right)\right) - \min\left(0,\ LD_P + C\right),$$

$$\mu_{C_0} = b - L\mu_{P_0} - Cv_{P0},$$

guarantees that constraints are verified for the states resulting from the initial marking.

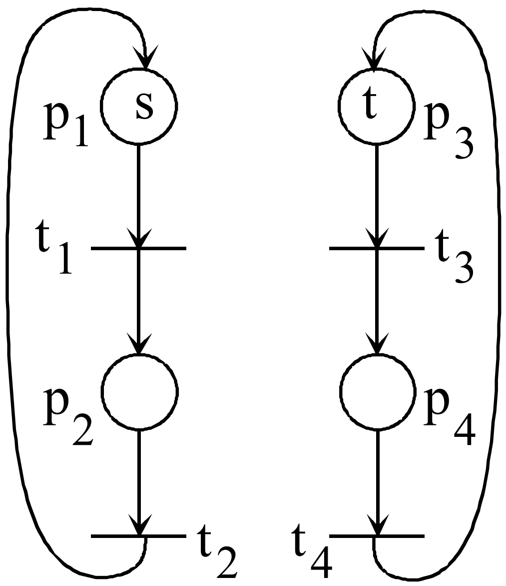
* *In the next slides this will be called the* ***LINENF theorem.***

Methods of Synthesis

Example of controller synthesis

$$\forall s \in N_0, \forall t \in N_0, \forall n \in N_0$$

Producer / Consumer



Linear constraint: $v_3 \leq v_2$

that can be written as:

$$Cv_P \leq b$$

$$L = 0, F = 0$$

$$\begin{bmatrix} 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \leq 0.$$

Incidence matrix

$$D_P = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Initial marking

$$\mu_{P_0} = \begin{bmatrix} s \\ 0 \\ t \\ 0 \end{bmatrix}.$$

Methods of Synthesis

Example of controller synthesis

Producer / Consumer

1) Test $b - L\mu_{P_0} = 0 - 0 \geq 0.$

OK.

2) Compute

$$D_C^- = \max\left(0,\, LD_P + C,\, F\right)$$
$$D_C^+ = \max\left(0,\, F - \max\left(0, LD_P + C\right)\right) - \min\left(0,\, LD_P + C\right),$$

$$D_C^- = \max\left(0,\ [0\ \ -1\ \ 1\ \ 0],\ 0\right) = [0\ \ 0\ \ 1\ \ 0]$$
$$D_C^+ = \max\left(0, -[0\ \ 0\ \ 1\ \ 0]\right) - \min\left(0, [0\ \ -1\ \ 1\ \ 0]\right)$$
$$= [0\ \ 0\ \ 0\ \ 0] - [0\ \ -1\ \ 0\ \ 0] = [0\ \ 1\ \ 0\ \ 0]$$

and

$$\mu_{C_0} = b - L\mu_{P_0} - Cv_{P0},$$
$$\mu_{C_0} = b - L\mu_{P_0} = 0 - 0 = 0.$$

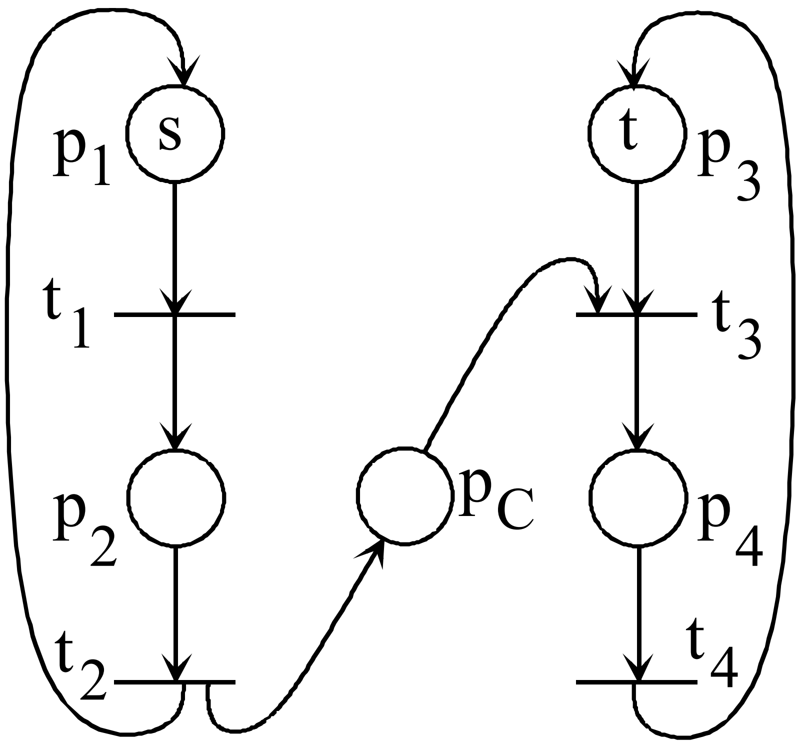
OK.

Methods of Synthesis

Example of controller synthesis

Producer / Consumer

3) Resulting in



$$D = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

$$\mu_0 = \begin{bmatrix} s \\ 0 \\ t \\ 0 \\ 0 \end{bmatrix}$$

OK.
UAU!!!.

Methods of Synthesis

Example of controller synthesis: Producer Consumer

```
% The Petri net D=Dp-Dm, and m0
%  (Dplus-Dminus= Post-Pre)

Dm= [1 0 0 0;
     0 1 0 0;
     0 0 1 0;
     0 0 0 1];

 Dp= [0 1 0 0;
     1 0 0 0;
     0 0 0 1;
     0 0 1 0];

m0= [1 0 1 0]';

% Supervisor constraint
%
L= []; F= [];  C= [0 -1 1 0];
b= 0;

% Computing the supervisor
%
[Dfm, Dfp, ms0] = linenf(Dm, Dp, L, b, m0, F, C)
Df= Dfp-Dfm
ms0
```

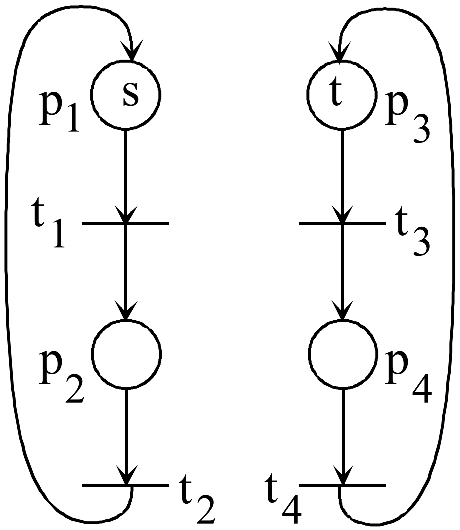
Result using the function LINENF.m of the toolbox SPNBOX:

```
Df =

    -1     1     0     0
     1    -1     0     0
     0     0    -1     1
     0     0     1    -1
     0     1    -1     0


ms0 =

     1
     0
     1
     0
     0
```

Methods of Synthesis

Example of controller synthesis

Bounded Producer / Consumer



Incidence matrix

$$D_P = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Initial marking

$$\mu_{P_0} = \begin{bmatrix} s \\ 0 \\ t \\ 0 \end{bmatrix}.$$

TWO linear constraints:

$$\begin{cases} v_3 \le v_2 \\ v_2 \le v_3 + n \end{cases} \Leftrightarrow \begin{cases} v_3 - v_2 \le 0 \\ v_2 - v_3 \le n \end{cases}$$

$$\forall s \in N_0, \forall t \in N_0, \forall n \in N_0$$

The two linear constraints can be written as:

$$Cv_P \le b$$
$$i.e.\ L = 0, F = 0$$
$$\Leftrightarrow \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \le \begin{bmatrix} 0 \\ n \end{bmatrix}$$

Methods of Synthesis

Example of controller synthesis

Bounded Producer / Consumer

1) Test

$$b - L\mu_{P_0} = b = \begin{bmatrix} 0 \\ n \end{bmatrix} \geq 0.$$

OK.

2) Compute

$$D_C^- = \max\left(0, \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}, 0\right) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$D_C^+ = \max\left(0, 0 - \max\left(0, \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}\right)\right) - \min\left(0, \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

and

$$\mu_{C_0} = b - L\mu_{P_0} = \begin{bmatrix} 0 \\ n \end{bmatrix}.$$

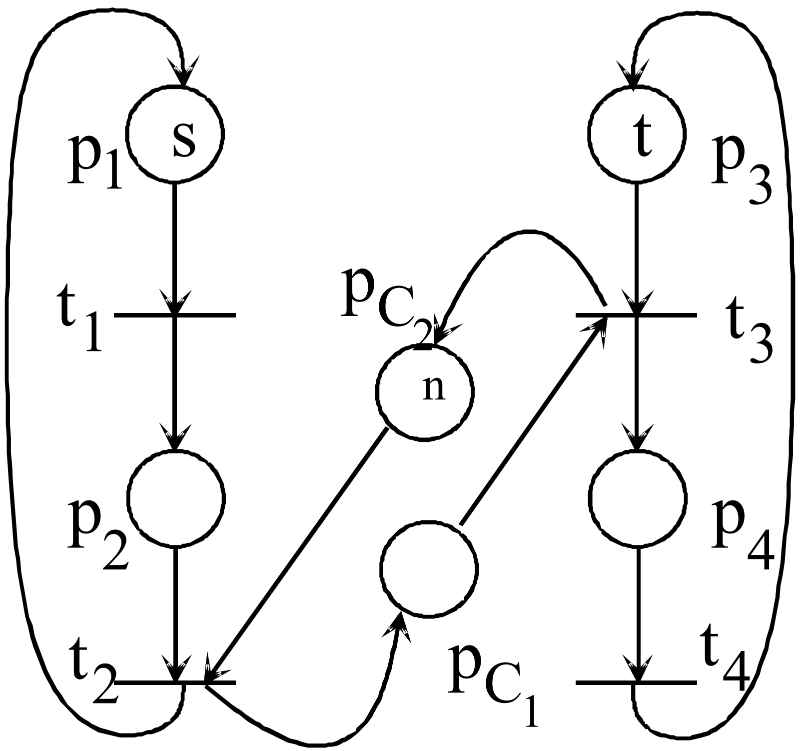
OK.

Methods of Synthesis

Example of controller synthesis

Bounded Producer / Consumer

3) Resulting in



$$D = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

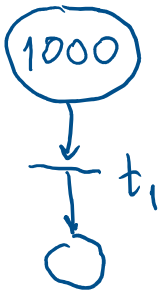
$$\mu_0 = \begin{bmatrix} s \\ 0 \\ t \\ 0 \\ 0 \\ n \end{bmatrix}$$

OK.
UAU!!!.

Methods of Synthesis

Example of controller synthesis – *Flow regulation*

Consider a Petri net with a large initial marking



Objective: do ***NOT allow consuming too many tokens*** in a single step.

For example, one wants to enforce max q_1 to be 2, i.e. accepting only $q_1 = 0$ or $q_1 = 1$ or $q_1 = 2$.

Constraint

$1 \cdot q_1 \leq 2$

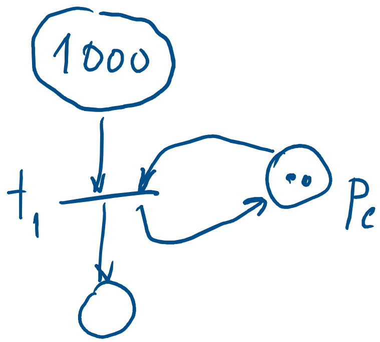
(F = 1, b = 2)

Solution

$D_c^+ = 1$

$D_c^- = 1$

$\mu_{c_0} = 2$



Methods of Synthesis

Function LINENF of SPNBOX

LINENF Lemma 1: From General Constraints to Theorem T1

Given the generalized linear constraint $L\mu_P + Fq_P + Cv_P \leq b$ and the conditions of the LINENF theorem:

If $L \neq 0,$ $F = 0,$ $C = 0$

then $D_C^+ = (LD_P)^-,$ $D_C^- = (LD_P)^+$ and $D_C = -LD_P$

$$\mu_{C0} = b - L\mu_{P0}$$

(see proof in the next page)

Notation:

$$D^+ = \max(0, D)$$
$$D^- = -\min(0, D)$$
$$D = D^+ - D^-$$

$D^+, D^- \in N_0^{n\times m}$ and $D \in Z_0^{n\times m}$

$$D_C^- = \max\left(0, LD_P + C, F\right)$$
$$D_C^+ = \max\left(0, F - \max\left(0, LD_P + C\right)\right) - \min\left(0, LD_P + C\right),$$

$$\mu_{C_0} = b - L\mu_{P_0} - C\nu_{P0},$$

$$L \neq 0, \quad F = 0, \quad C = 0 \quad \Rightarrow L\mu_P \leq b$$

$$D_c^- = \max(0, LD_p + \overset{=0}{C}, \overset{=0}{F}) = \max(0, LD_p) = (LD_p)^+$$

$$D_c^+ = \max(0, \overset{=0}{F} - \max(0, LD_p + \overset{=0}{C})) - \min(0, LD_p + \overset{=0}{C}) = \max(0, \underbrace{-(LD_p)^+}_{\leq 0}) + (LD_p)^- = +(LD_p)^-$$

$D^- = -\min(0, D)$

$D^- \in \mathbb{N}_0^{m \times m}$

$$D_c = D_c^+ - D_c^- = (LD_p)^- - (LD_p)^+ = -\left((LD_p)^+ - (LD_p)^-\right) = -LD_p$$

$$\mu_{c_0} = b - L\mu_{p_0} - \overset{=0}{C}\nu_{p_0} = b - L\mu_{p_0}$$

Methods of Synthesis

Function LINENF of SPNBOX

LINENF Lemma 2: Firing Regulation

Given the generalized linear constraint $L\mu_P + Fq_P + Cv_P \leq b$ and the conditions of the LINENF theorem:

If $L = 0,$ $F \neq 0,$ $C = 0$

then $D_C^+ = F^+,$ $D_C^- = F^+$ and $D_C = 0$

$\mu_{C0} = b$

(homework, prove this lemma)

Methods of Synthesis

Function LINENF of SPNBOX

LINENF Lemma 3: Constraints on Counters

Given the generalized linear constraint $L\mu_P + Fq_P + Cv_P \leq b$ and the conditions of the LINENF theorem:

If $L = 0$, $F = 0$, $C \neq 0$

then $D_C^+ = C^-$, $D_C^- = C^+$ and $D_C = -C$

$$\mu_{C0} = b - Cv_{P0}$$

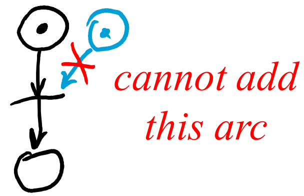
(homework, prove this lemma)

(empty page, do yourself the proof of the last two lemmas)

Methods of Synthesis: intro to **Uncontrollable** and **Unobservable** transitions

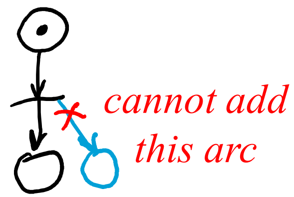
Definition of **Uncontrollable Transition:**

A transition is uncontrollable if its firing **cannot be inhibited** by an external action (e.g. a supervisory controller).



Definition of **Unobservable Transition:**

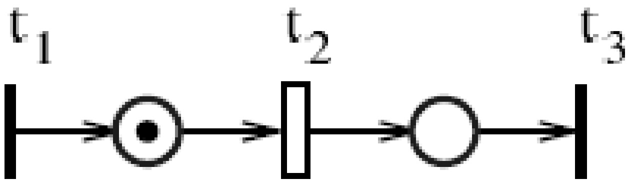
A transition is unobservable if its firing **cannot be detected or measured** (therefore the study of any supervisory controller can not depend from that firing).



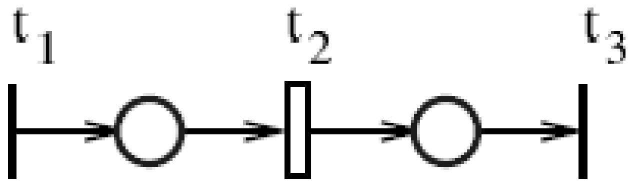
Proposition:

In a Petri net based controller, both input and output arcs to/from plant transitions are used to trigger state changes in the controller. *Since a controller cannot have arcs connecting to unobservable transitions, then all* ***unobservable transitions are also implicitly uncontrollable***.

Methods of Synthesis: intro to **Uncontrollable** and **Unobservable** transitions



(a)



(b)

If **t1 is controllable** and **t2 is uncontrollable**:
- case (a), then t2 cannot be directly inhibited; it will eventually fire
- case (b), then **t2 can be indirectly prevented** from firing by inhibiting t1.

i.e. may exist indirect solution despite t2 being uncontrollable.

If **t2 is unobservable** and **t3 is observable**, then we cannot detect when t2 fires. The state of a supervisor is not changed by firing t2. However we can **indirectly detect that t2 has fired**, by detecting the firing of t3.

i.e. may exist indirect solution despite t2 being unobservable.

∴ *may exist indirect solution despite t2 uncontrollable and/or unobservable.*

Methods of Synthesis

Definition: A marking μ_P is admissible if

i) $L\mu_P \leq b$ and **ii)** $\forall \mu' \in R(C, \mu_P)$ verifies $L\mu' \leq b$

Definition: A Linear Constraint (L, b) is admissible if

i) $L\mu_{Po} \leq b$ and

ii) $\forall \mu' \in R(C, \mu_{Po})$ such that $L\mu' \leq b$

μ' is an admissible marking.

Note: ii) indicates that the firing of uncontrollable transitions can never lead from a state that satisfies the constraint to a new state that does not satisfy the constraint.

Methods of Synthesis

Proposition: Admissibility of a constraint

A linear constraint is admissible ***iff***

- The initial markings satisfy the constraint.
- There **exists a controller** with maximal permissivity that forces the constraint and **does not inhibit any uncontrollable transition**.

Two sufficient (not necessary) conditions:

Corollary: given a system with uncontrollable transitions,

$\boxed{l^T D_{uc} \leq 0}$ implies admissibility.

Corollary: given a system with unobservable transitions,

$\boxed{l^T D_{uo} = 0}$ implies admissibility.

Methods of Synthesis

Function MRO_ADM of SPNBOX

Lemma *: Structure of Constraint transformation

If $L'\mu_p \leq b'$ where

$L' = R_1 + R_2 L$ and $b' = R_2(b+1) - 1$

$R_1 \in Z^{n_C \times n}$ and $R_1 \mu_p \geq 0$

$R_2 \in Z^{n_C \times n_C}$ is a matrix with positive elements in the diagonal

then $L\mu_p \leq b$ is also verified.

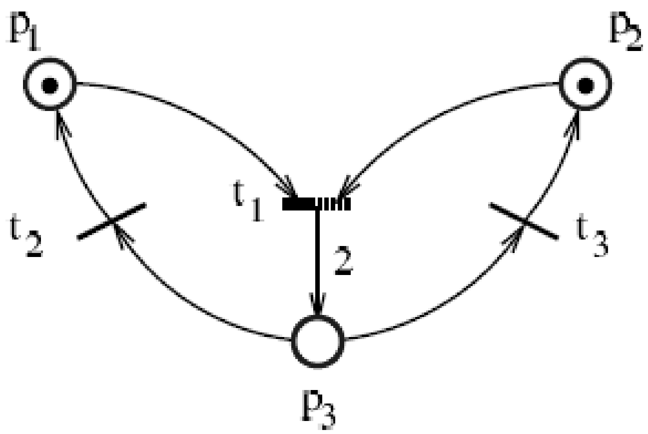
Typical usage:

- constraint (L, b), and **extra constraints** → $(R_1, R_2) \rightarrow (L', b') \rightarrow (D_C^+, D_C^-, \mu_{C0})$
- see unobservable / uncontrollable extra constraints example in the next slides

* Lemma 4.10 in [Moody98] pg46

Methods of Synthesis

Example: design controller with t1 unobservable (1/4)



$$D = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{bmatrix}, \quad D_{uo} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

Objectives: $\mu_1 + \mu_3 \geq 1$ and $\mu_2 + \mu_3 \geq 1$ which can be written in matrix form as

$$L\mu \leq b, \qquad L = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}, \qquad b = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Example extracted from "Supervised Control of Concurrent Systems: A Petri Net Structural Approach", M. Iordache and P. Antsaklis, Birkhauser 2006.

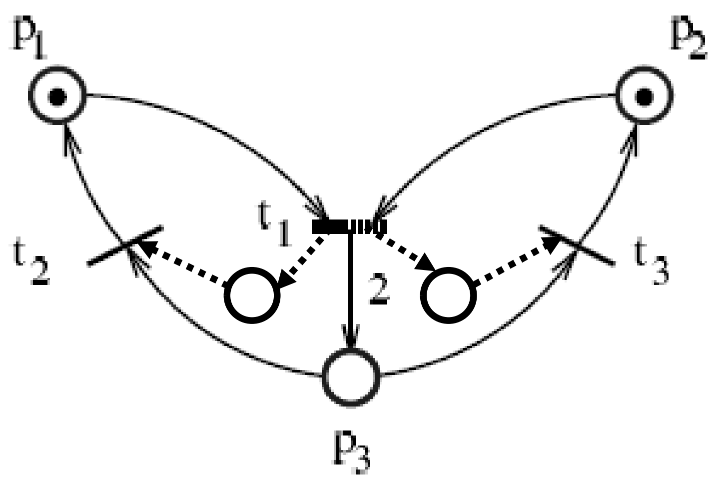
Methods of Synthesis

Example: design controller with t1 unobservable (2/4)

```
% System and constraints

D= [-1  1  0;
    -1  0  1;
    +2 -1 -1];

Dm= -D.*(D<0);
Dp=  D.*(D>0);

m0= [1 1 0]';

L= [-1 0 -1; 0 -1 -1];
b= [-1; -1];


% Supervisor computation

[Dfp, Dfm, mf0] =
    linenf( Dp, Dm, L, b, m0 );
```

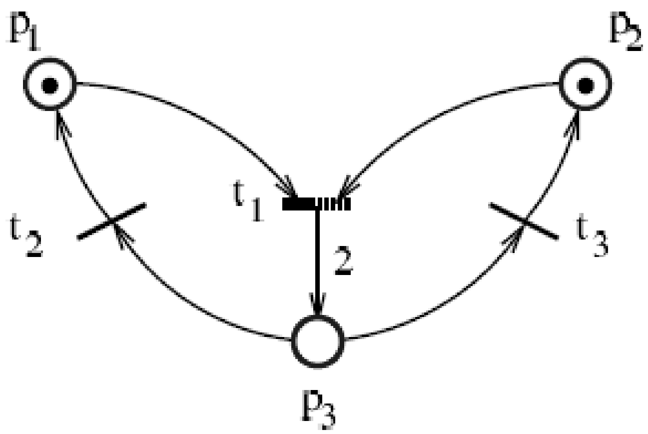
```
Dfp =

     0     1     0
     0     0     1
     2     0     0
     1     0     0
     1     0     0

Dfm =

     1     0     0
     1     0     0
     0     1     1
     0     0     1
     0     1     0

mf0 =

     1
     1
     0
     0
     0
```

^ *Bad news, supervisor touches t_1 .*



Methods of Synthesis

Example: design controller with t1 unobservable (3/4)




```
D= [-1  1  0;
     -1  0  1;
      2 -1 -1];

Tuo= [1]; Tuc= [];

L= [-1 0 -1; 0 -1 -1];
b= [-1 -1]';

[La, ba, R1, R2] = mro_adm( L, b, D, Tuc, Tuo );
```

Solution obtained with the function MRO_ADM.m of the SPNBOX toolbox:

$$R1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{La} = \begin{bmatrix} -2 & 0 & -1 \\ 0 & -2 & -1 \end{bmatrix} \qquad \mathbf{ba} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$R2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Note: verify that $L_a \mu \le b_a$ implies $L \mu \le b$

Methods of Synthesis

Example: design controller with t1 unobservable (4/4)

Finally the supervised controller is simply obtained from L_a and b_a:

$$D_c = -L_a D_p$$

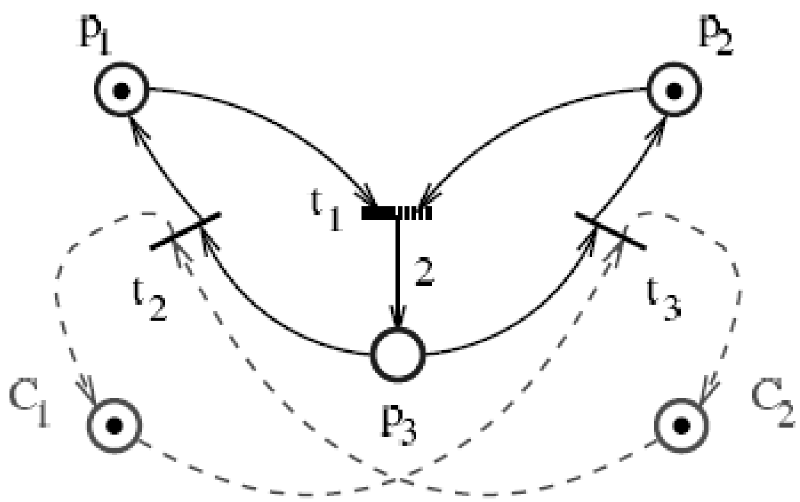
$$= \begin{bmatrix} -2 & 0 & -1 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\mu_{c0} = b_a - L_a \mu_{p0}$$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -2 & 0 & -1 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



Obtained the desired result: supervisor does not touch t_1.

End of chapter on supervision control.

What is next?