Petri Nets Properties

Examples & Solutions

2021/2022 J. Gaspar

Example1: simple Petri net, properties?

(P, T, A, w, x_0)

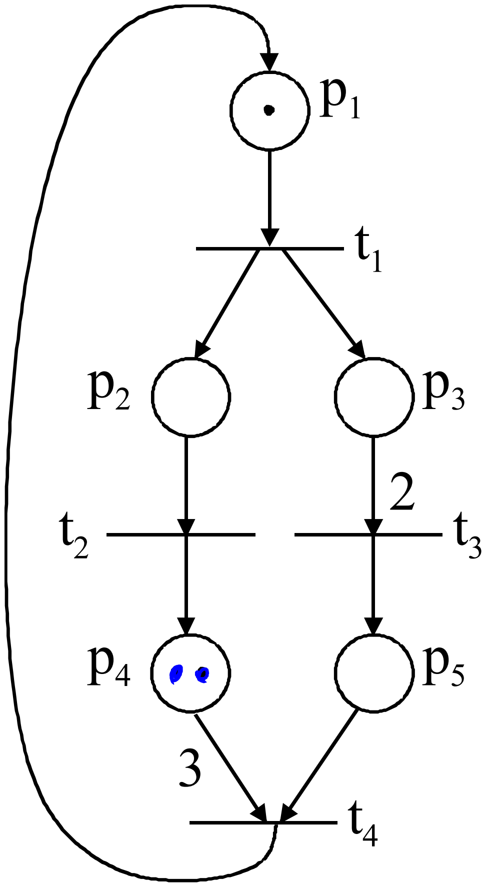
$P=\{p_1, p_2, p_3, p_4, p_5\}$

$T=\{t_1, t_2, t_3, t_4\}$

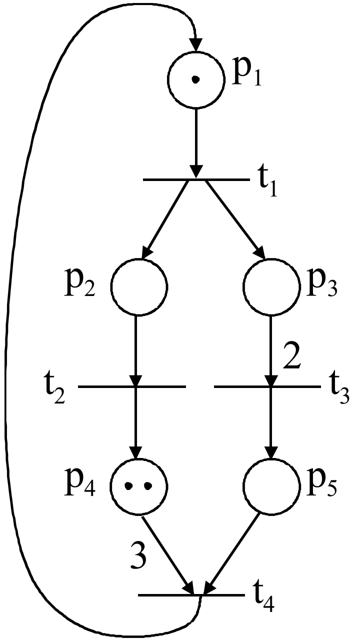
$A=\{(p_1, t_1), (t_1, p_2), (t_1, p_3), (p_2, t_2),$
$(p_3, t_3), (t_2, p_4), (t_3, p_5), (p_4, t_4),$
$(p_5, t_4), (t_4, p_1)\}$

$w(p_1, t_1)=1,$
$w(t_1, p_2)=1, w(t_1, p_3)=1,$
$w(p_2, t_2)=1, w(p_3, t_3)=2,$
$w(t_2, p_4)=1, w(t_3, p_5)=1,$
$w(p_4, t_4)=3, w(p_5, t_4)=1,$
$w(t_4, p_1)=1$

$x_0= \{1, 0, 0, 2, 0\}$

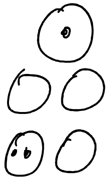


1. *Reachability?*
2. *Coverability?*
3. *Safeness?*
4. *Boundness?*
5. *Conservation?*
6. *Liveness?*

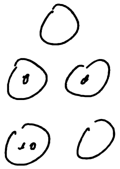


study of properties based on the Reachability tree

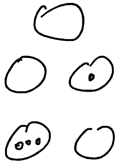
1)



2)



3)



$(1, 0, 0, 2, 0)$

$\downarrow t_1$

$(0, 1, 1, 2, 0)$

$\downarrow t_2$

$(0, 0, 1, 3, 0)$

Term

1. Finite reachable set
2. No state covers / is covered by another one
3. Is not safe (p_4 reaches 3 marks)
4. Net is 3-bounded
5. Net not strictly conservative
6. t_3 and t_4 are level ϕ, other ones are L_1

Example2: simple automation system modeled using PNs, properties?

An automatic soda selling machine accepts

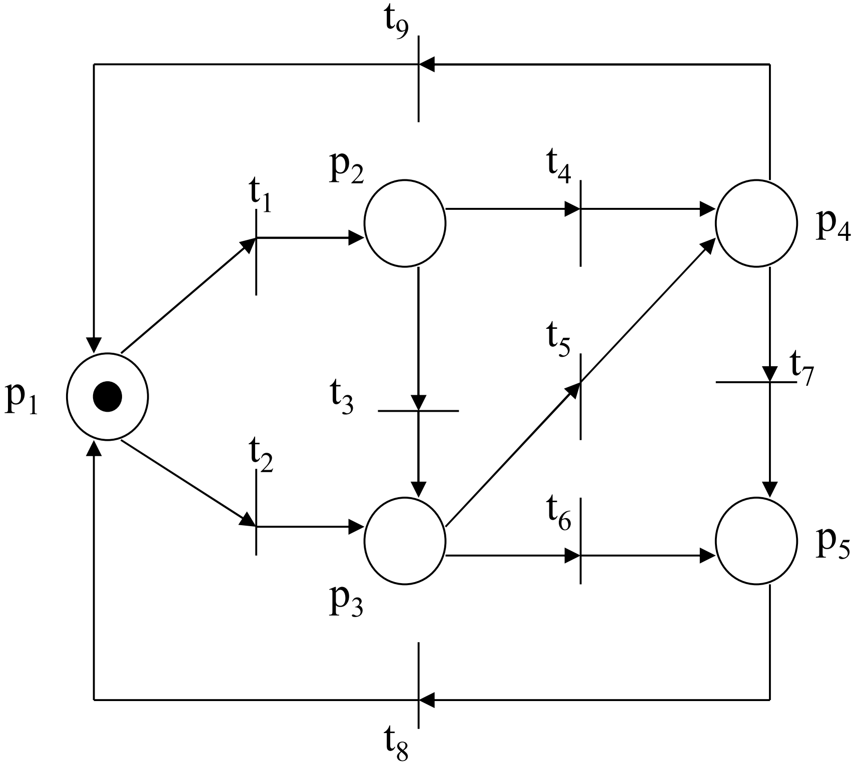
50c and \$1 coins and

sells 2 types of products:

SODA A, that costs \$1.50 and

SODA B, that costs \$2.00.

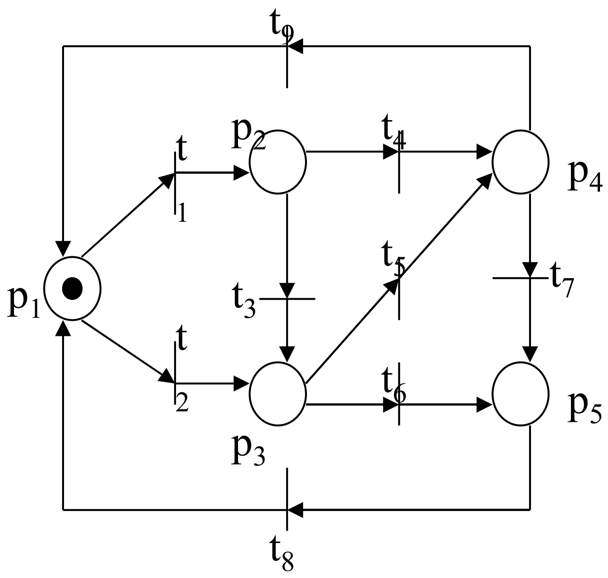
Assume that the money return operation is omitted.

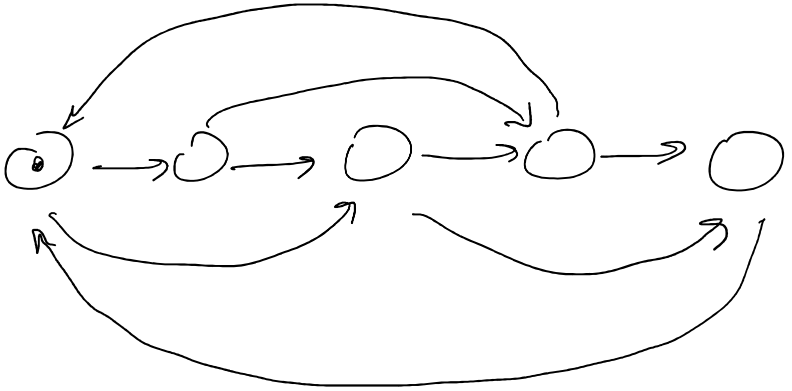


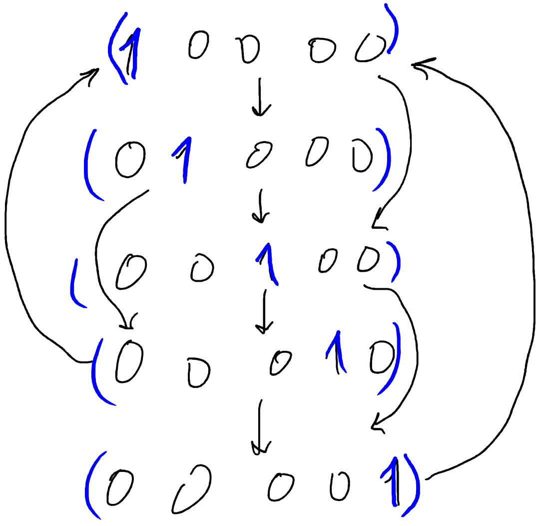
p_1: machine with \$0.00;

t_1: coin of 50 c introduced;

t_8: SODA B sold.







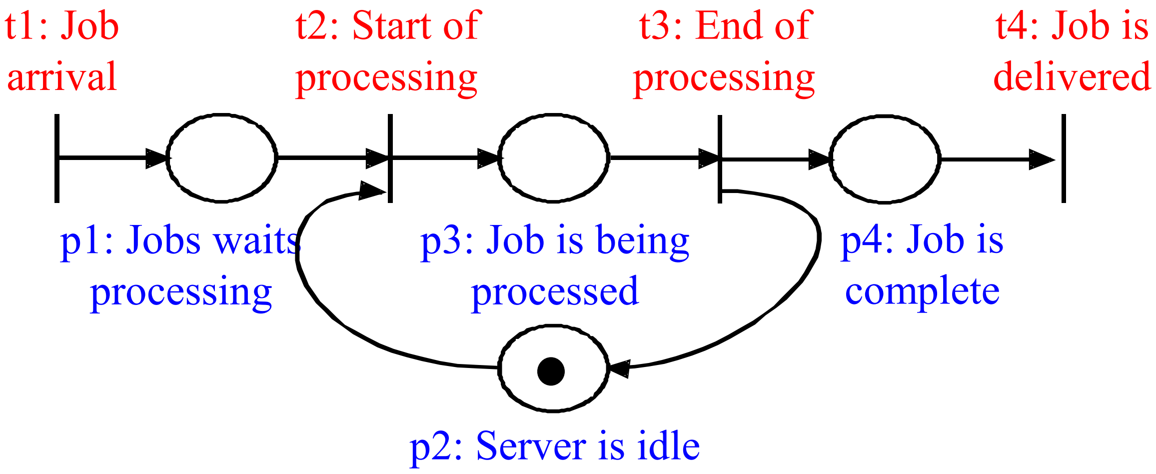
1. Finite reachable set (5 states)
2. no state covered by another one
3. Is safe
4. Is 1-bounded
5. Is conservative
6. Liveness, all transitions are L4

$\cancel{w^T\mu_{k+1}} = \cancel{w^T\mu_k} + w^T D q_k$

$w^T D = 0$

possible?

Example for the analysis of properties:



t1: Job arrival

t2: Start of processing

t3: End of processing

t4: Job is delivered

p1: Jobs waits processing

p3: Job is being processed

p4: Job is complete

p2: Server is idle

Event	Pre-conditions	Pos-conditions
t1	-	p1
t2	p1, p2	p3
t3	p3	p4, p2
t4	p4	-

Q: Exists ***conservation*** *?*

A: Find w such that $\mathbf{w^T.D=0}$

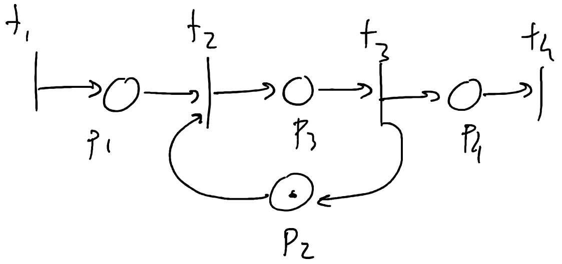
if $\exists w>0$ *then net is conservative*

else it is not conservative

$$D = \begin{bmatrix} 1 & -1 & & \\ & -1 & 1 & \\ & 1 & -1 & \\ & & 1 & -1 \end{bmatrix}$$

$w^T = [w_1\ w_2\ w_3\ w_4] = ?$

Q2: What changes if initial marking in p2 is zero?



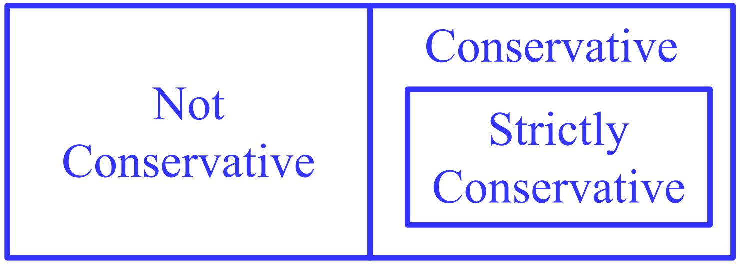
Conservation

$$w^T D = 0$$

$$\mu(K+1) = \mu(K) + D\,q(K)$$

$$\underbrace{w^T\mu(K+1) = w^T\mu(K)}_{\text{equal}} + \underbrace{w^T D}_{=0}\,\underbrace{q(K)}_{\text{any}}$$

$$D = \begin{array}{c|cccc} & t_1 & t_2 & t_3 & t_4 \\ \hline p_1 & +1 & -1 & & \\ p_2 & & -1 & +1 & \\ p_3 & & +1 & -1 & \\ p_4 & & & +1 & -1 \end{array}$$



Not Conservative

Conservative

Strictly Conservative

$$\underbrace{\begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix}}_{w^T} \underbrace{\begin{bmatrix} 1 & -1 & & \\ & -1 & 1 & \\ & 1 & -1 & \\ & & 1 & -1 \end{bmatrix}}_{D} = 0$$

$$\begin{cases} w_1 = 0 \\ -w_1 - w_2 + w_3 = 0 \\ w_2 - w_3 + w_4 = 0 \\ -w_4 = 0 \end{cases} \qquad \begin{cases} w_1 = 0 \\ w_2 = w_3 \\ - \\ w_4 = 0 \end{cases}$$

e.g. $w = [\,0 \quad 1 \quad 1 \quad 0\,]$ //

problems

NOT conservative

Discrete Event Systems

Example of a simple automation system modeled using PNs

An automatic soda selling machine accepts

50c and $1 coins and

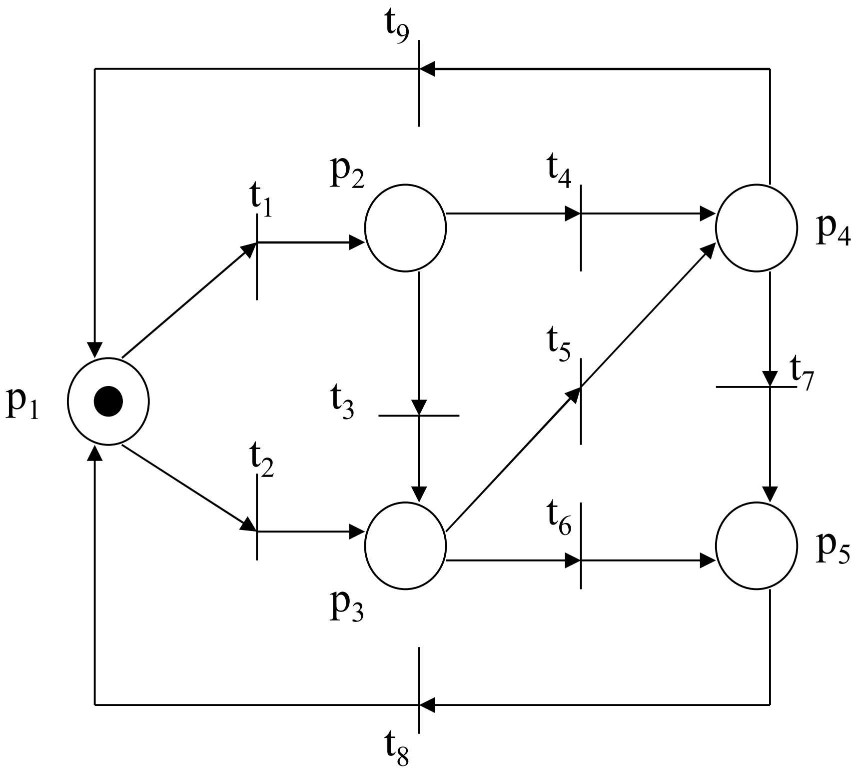
sells 2 types of products:

SODA A, that costs $1.50 and

SODA B, that costs $2.00.

Assume that the money return operation is omitted.

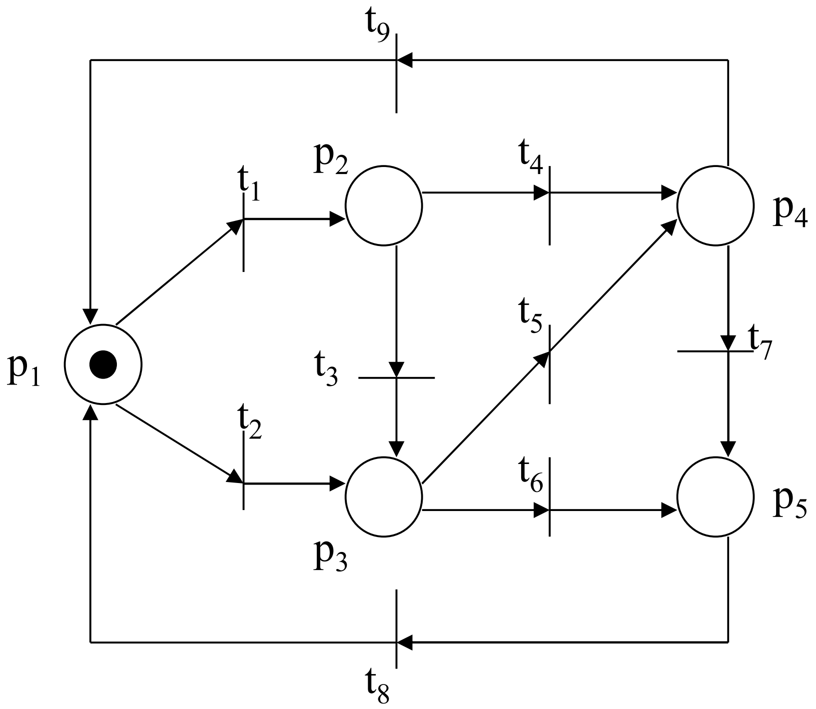
Q: Are there transition firing vectors that make the Petri net return to the same state? In other words, does the Petri net have ***cycles of operation****?*



p_1: machine with $0.00;
t_1: coin of 50 c introduced;
t_8: SODA B sold.

Discrete Event Systems

Example of a simple automation system modeled using PNs



```
D= [ -1  -1   0   0   0   0   0   1   1
      1   0  -1  -1   0   0   0   0   0
      0   1   1   0  -1  -1   0   0   0
      0   0   0   1   1   0  -1   0  -1
      0   0   0   0   0   1   1  -1   0 ]
```

Time invariance *? Find q such that. D.q=0*

```
>> q= null( D, 'r` )
q =
   1  -1   1   0   1
  -1   1  -1   1   0
   1   0   0   0   0
   0  -1   1   0   1
   0   1   0   0   0
   0   0  -1   1   0
   0   0   1   0   0
   0   0   0   1   0
   0   0   0   0   1

>> q(:,1)= q(:,1)+q(:,4);
>> q(:,2)= q(:,2)+q(:,5);
>> q(:,3)= q(:,3)+q(:,4);
q =
   1   0   1   0   1
   0   1   0   1   0
   1   0   0   0   0
   0   0   1   0   1
   0   1   0   0   0
   1   0   0   1   0
   0   0   1   0   0
   1   0   1   1   0
   0   1   0   0   1
```

Note: there are more solutions; see function invar(D) of the SPNBOX toolbox