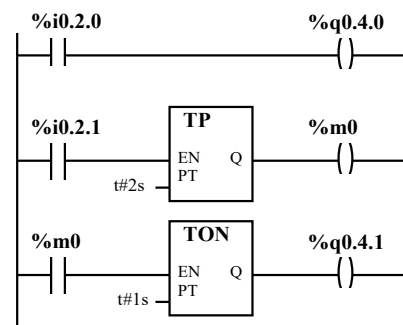


Read all questions of the exam carefully before starting to answer.

- Provide detailed justifications to all answers.
- The use of bibliographic material, either in paper or in digital format is allowed.
- Exchange of information is forbidden (e.g. voice, WiFi, Bluetooth, GPRS, WAP, ...).
- Exam duration: 2 hours and 50 min.

**Q1. [Scan cycle]:** Consider that the ladder diagram in the next figure is the single code run by a PLC, in a MAST section configured to be cyclic. The PLC input and output take **0.5msec+0.5msec** and each ladder instruction (contact read, coil write, timer) takes about **0.1msec**. Timers preset values are **2sec** and **1sec**.

- Indicate the scan period of the PLC.
- Indicate the smallest time intervals, with probabilities larger than zero, for rising edges in the inputs, **%i0.2.0** and **%i0.2.1**, making changes in the outputs, **%q0.4.0** and **%q0.4.1**, respectively.
- What changes in (b) if **t#2s** changes to **t#0.5s**?



**Q2. [PLC IO]** Consider a square wave generator with **10Hz** and **50% duty cycle** is plugged to **%i0.2.0**. The PLC scan cycle period is **1msec**. The structure text instruction **%m0 := RE(%i0.2.0)** creates a square wave within the PLC memory. In this question, answer approximate values by ignoring small synchronization mismatches between the input square wave and the timings of the PLC input.

- Characterize the square wave at **%m0**, in terms of frequency and duty cycle (ON time / period time).
- Let **%m1 := %i0.2.0 AND RE(%i0.2.0)**. Use a truth table to show **%m1 = %m0**.
- What would be the signal in the case **%m2 := %i0.2.0 AND FE(%i0.2.0)**? Justify also with a truth table.

**Q3. [Square wave]** The code provided in this question allows generating a square wave based on a sequence of pulses. Various wave forms are available depending on **your student number** written in variable **%MD0**. Consider the code shown below is the single code run by a PLC in a MAST section configured as cyclic.

- Assume that the assignments denoted by the symbol **":="** are the single instructions consuming time, and that each one takes about **0.2msec**. The PLC input and output take **1msec + 1msec**. Note some scan cycles are faster than others. Compute the minimum scan cycle period and the maximum scan cycle period.
- Write additional Structured Text code so that at the first scan cycle, and just in that scan cycle, the memory variable **%MD0** is set with **your student number** (5 or 6 digits).

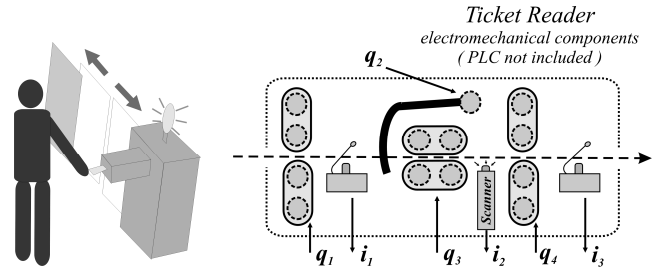
```
(* sequence of 1sec pulses *)
TP_0(IN:= NOT(%m0),
      PT:= t#1s,
      Q => %m0 );

(* output signal on %m10 *)
if RE(%m0) then
  %MD10:= MOD(%MD0/10, 10);
  %MD12:= MOD(%MD0, 10);
  %MD14:= ROL(1,8) +ROL(%MD10,4) +%MD12;
  %m10 := DINT_TO_BOOL(
    ROR(%MD14, INT_TO_UINT(8-%MW2)) );
  %MW2 := MOD(%MW2+1,9);
end_if;
```

- Consider that **%MD0** contains your student number. Noting the PLC overlaps in memory the words and double words, indicate the values of **%MW0** and **%MW1**. Given the program, indicate the values of **%MD10** and **%MD12**, and the possible values of **%MW2**.
- Assume **%MW2** is zero at **t=0** (as usual in PLCs after boot). Plot the values of the memory **%m10** from **t=0** to **t=10** seconds, according to the program running having **%MD0** set with your student number.

**Q4. [SFC and DES model]** Consider a ticket reader device (see figure). The electromechanical components are governed by a PLC that constantly monitors the sensors and actuates the motors.

a) Draw an electric diagram detailing a PLC connected to the ticket reader electromechanical components. Indicate all sensors and actuators. Include a power supply for the input and output modules. Include a normally open switch (push button) to turn the system ON and a normally closed to turn the system OFF.

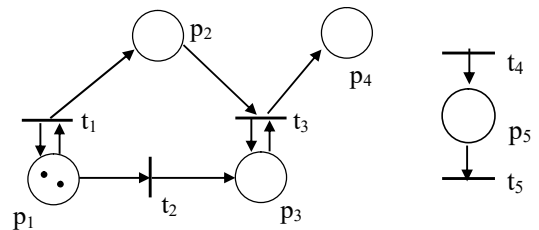


b) Propose a GRAFCET / SFC program that starts with a step named *idle waiting a ticket*. Then, when a ticket is introduced, at the entry gate, i.e. happens a rising edge on  $i_1$ , the system drives motors  $q_1$ ,  $q_2$  and  $q_3$  till the end of scanning (a rising edge on  $i_2$ ). After scanning, the ticket is placed ready to be taken out using  $q_4$ . The ticket reader returns to *idle waiting a ticket* after a falling edge on  $i_3$ . Do not include here the ON/OFF buttons, they will be used in another question.

c) Convert the GRAFCET / SFC to a Petri net, clearly indicating the meaning of places and transitions. Study the liveness level of the Petri net transitions. Study also the conservation of the net and comment whether the system has, or does not have, parallelism.

d) Consider ON/OFF is represented by an additional Petri net with incidence matrix  $D = \begin{bmatrix} -1 & +1 \\ +1 & -1 \end{bmatrix}$  where the two places are named ON and OFF. (i) Design one supervisor imposing the mutual exclusion between the OFF place and the place of the main system *idle waiting a ticket*. (ii) Ignore the previous supervisor, and design one new supervisor where a token at place OFF prevents firing transition(s) at the end of arc(s) starting at place *idle waiting a ticket*. Note: Indicate the initial states of the ON/OFF Petri net you chose to design the supervisors.

**Q5. [Properties and Supervision]** This problem focus on Discrete Event Systems analysis tools studied on the course, for the Petri net defined in the figure.



a) Discuss the liveness level of the transitions.

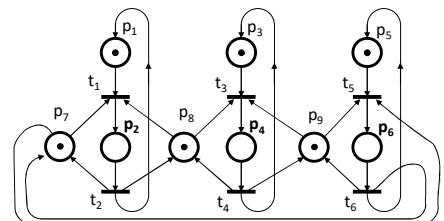
b) Using the method of matrix equations, prove whether or not are reachable the states  $(0,0,2,N,0)$ ,  $N=1$  or  $N=2$ . Can one infer coverability from these states?

c) Design one supervisor to impose that the place  $p_5$  cannot have more than 15 marks.

d) Design a supervisor based on marking invariants, using generalized linear constraints, such that firing  $t_3$  represents producing parts and firing  $t_4$  represents consuming the produced parts. The transition  $t_3$  is allowed to fire up-to 4 times before being put on hold, waiting for some consumption done by  $t_4$ . Draw the supervisor just determined on the Petri net. Comment any possible change on liveness.

e) Design one supervisor to impose that each firing of transition  $t_5$  allows at most consuming one mark from the place  $p_5$ .

**Q6. [Dinning philosophers]** Consider a 3 philosophers dinner represented by the Petri net shown. Places  $p_1$ ,  $p_3$  and  $p_5$  mean meditating. Places  $p_2$ ,  $p_4$  and  $p_6$  mean eating. Places  $p_7$ ,  $p_8$  and  $p_9$  indicate one chopstick. Each philosopher needs two chopsticks to eat the noodles from his bowl.



a) Write the incidence matrix and the initial marking. Determine the reachable set of the Petri net.

b) Consider a firing vector  $q_k = [q_1 \ 1 \ q_3 \ 1 \ q_5 \ 1]^T$ . Let  $x_i$  denote samples from a continuous uniform distribution  $x_i \sim unif(0,1)$ , with  $i \in \{1, 3, 5\}$ . Propose functions  $q_i = f_i(x_i)$  such that  $q_1, q_3, q_5$  are nonzero on average 1%, 2% and 3% (respectively) of the number of running time-steps  $k \rightarrow \infty$ .

c) Write a matrix with state transition probabilities from any state to any other one. Discuss whether there is a limit distribution indicating the percentages of number of time-steps each philosopher effectively gets eating. Discuss what happens if the values 1%, 2% and 3% are all raised to 100%, considering transitions are assessed to fire (i) by a specific order or (ii) randomly with uniform probability.