

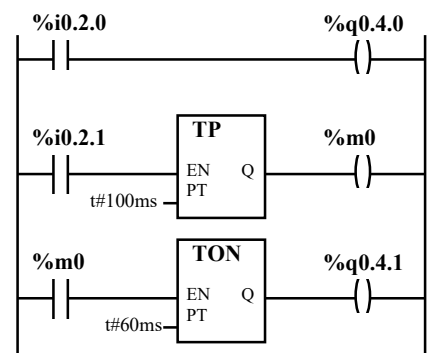
Read all questions of the exam carefully before starting to answer.

- Provide detailed justifications to all answers.
- The use of bibliographic material, either in paper or in digital format is allowed.
- Exchange of information is forbidden (e.g. voice, WiFi, Bluetooth, GPRS, WAP, ...).
- Exam duration: 3 hours.

Q1. Logic function. Implement in ladder the logic function $Z = (I_1 \text{ xor } I_2) \text{ xor } I_3$ where "xor" denotes the "exclusive or" logic function. Implement in ladder a "xor" function generalized to N inputs, $Z = f(I_1, I_2, \dots, I_N)$. When an odd number of inputs are 1 (True) what is the value of the output Z ? What happens to Z when a single input changes its state (0→1 or 1→0)?

Q2. Scan cycle: Consider that the ladder diagram in the next figure is the single code run by a PLC, in a MAST section configured to be cyclic. The PLC input and output take **0.1msec+0.1msec** and each ladder instruction (contact read, coil write, timer) takes about **0.1msec**. The timers have preset values of **100msec** and **60msec**.

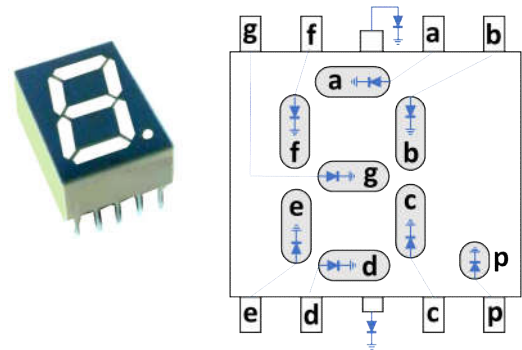
- Indicate the scan period of the PLC.
- Indicate the smallest time intervals, with probabilities larger than zero, for rising edges in the inputs, **%i0.2.0** and **%i0.2.1**, making changes in the outputs, **%q0.4.0** and **%q0.4.1**, respectively.
- What changes in (b) if **t#100ms** changes to **t#40ms**?



Q3. Display one digit. Consider the 7-segments display, common cathode, of the next figure.

a) Draw an electric diagram to plug the 7-segments display to a PLC which has digital inputs and outputs. How many resistors are needed for the correct operation of the 7-segments display?

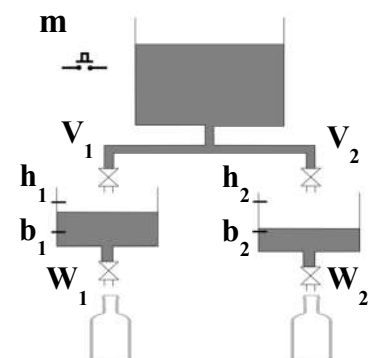
b) `BYTE_TO_BIT(X, q0, q1, q2, q3, q4, q5, q6, q7)` is a ST function that converts one byte X into eight BOOL values here denoted as q_0 till q_7 . Use the referred function to display the digit 0. More in general, and considering also the dot on the side of the 7-segments display, how many different images, including non-digits, can one show?



- Consider a byte variable named MYCNT which can have values between zero and nine. Write a table converting each possible value of MYCNT to the seven BOOL values, 'a' till 'g', of the 7-segments display.
- Write a complete ST program to increment MYCNT on rising edges of **%i0.2.0** and to decrement MYCNT based on rising edges of **%i0.2.1**. Avoid that MYCNT gets negative values or values larger than nine. Display MYCNT in the 7-segments display.

Q4. PN model. Consider the two-bottles filling system shown in the figure, formed by 1 start pushbutton, 4 valves and 4 height sensors.

- Propose one Petri net to represent the process of starting with a pressure of a button, filling in parallel two auxiliary reservoirs and then filling two bottles. Ensure the Petri net can repeat the process.
- Draw the reachability tree. Study the properties reachability, coverability, safeness, boundedness, conservation and liveness.
- Using the Method of Matrix Equations (MME), prove (i) that the system cannot be idle (stopped) and at the same time filling an auxiliary reservoir, and (ii) there are operation cycles.



Q5. Supervision: This problem focus on Discrete Event Systems analysis tools studied on the course, for the Petri net defined as

$$C = (P, T, A, w, \mu_0) \text{ with } P = \{p_1, p_2, p_3, p_4\}, T = \{t_1, t_2, t_3, t_4\}, \mu_0 = [20 \ 0 \ 0 \ 0]^T,$$

$$A = \{(p_1, t_1), (t_1, p_2), (t_1, p_3), (p_2, t_2), (p_3, t_2), (t_3, p_4), (p_4, t_4)\}.$$

The arc weights are $w(p_i, t_j)=1, w(t_j, p_k)=1$ for all possible i, j, k .

- Draw the petri net and discuss its conservativeness.
- Design one or more supervisors to impose that each of the places p_2 and p_4 cannot have more than 10 marks. Discuss the boundedness of the marking of the place p_3 .
- Design a supervisor based on marking invariants, using generalized linear constraints, such that firing t_2 represents producing parts and firing t_3 represents consuming the produced parts. The transition t_2 is allowed to fire up-to 3 times before being put on hold, waiting for some consumption done by t_3 . Draw the supervisor just determined on the Petri net. Suggestion: start by proving that in this question the supervisor can be computed with simplified expressions.
- One may conjecture that the constraints associated to the sequence or quantity of firings can be rewritten with simple linear constraints by adding an arc and an auxiliary place at the output of the transitions under consideration. Try to repeat (c) using this conjecture.
- Design one supervisor to impose that each firing of transition t_1 allows at most consuming one mark from the place p_1 .

Q6. Combining Petri nets: Let μ_a, μ_b, D_a and D_b denote the state vectors and incidence matrices of two Petri nets, a and b . (i) Show that an enlarged net with state $\mu = [\mu_a^T \ \mu_b^T]^T$, running the two nets simultaneously, has an incidence matrix which is a function of the other two matrices, i.e. $D = f(D_a, D_b)$. (ii) Consider now the addition of some extra arcs (t_{ai}, p_{bj}) and (t_{bk}, p_{ai}) linking transitions of a to places of b , and transitions of b to places of a . What changes in D ? (iii) Apply the results to the case where $D_a = [-1 \ 0; 0 \ -1]$ and $D_b = [-1]$ and one wants to add arcs (t_{a1}, p_{b1}) and (t_{b1}, p_{a2}) . Draw the Petri nets before and after adding the two additional arcs.

Q7. Turing Machine vs Petri net. List the main components constituting a Turing Machine. Compare a Turing Machine with a Petri net in terms of memory and parallelism representation. Use the terminology $\#(p_i, I(t_j))$ or $\#(p_i, O(t_j))$ to characterize the cardinality of transition input or output arcs, respectively. Use the terminology $|\bullet t| = n$ or $|t \bullet| = m$ to indicate the number of input or output arcs to a transition.

PS: Do not forget to identify all sheets of paper.

Good Luck,
José Gaspar