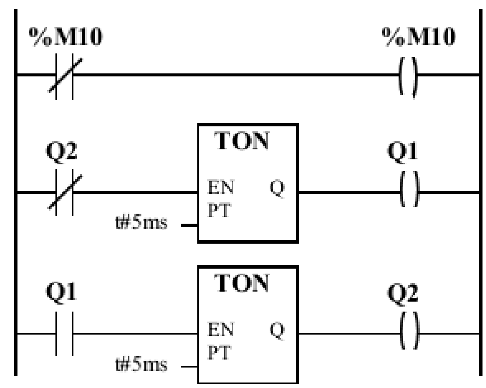
Sol! Ex2 2012/13 30.1.2013

Q1. *Majority circuit:* Implement a majority circuit with nine inputs using a standard PLC programming language. Name the inputs %i0.2.0 till %i0.2.8, and name the output %q0.4.0.

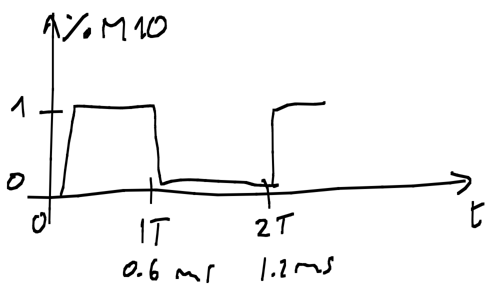
```
ACC:=0;
IF %i0.3.0 THEN ACC:=ACC+1; END_IF;
IF %i0.3.1 THEN ACC:=ACC+1; END_IF;
IF %i0.3.2 THEN ACC:=ACC+1; END_IF;
IF %i0.3.3 THEN ACC:=ACC+1; END_IF;
IF %i0.3.4 THEN ACC:=ACC+1; END_IF;
IF %i0.3.5 THEN ACC:=ACC+1; END_IF;
IF %i0.3.6 THEN ACC:=ACC+1; END_IF;
IF %i0.3.7 THEN ACC:=ACC+1; END_IF;
IF %i0.3.8 THEN ACC:=ACC+1; END_IF;
IF ACC>=5 THEN %q0.4.0:=True; ELSE %q0.4.0:=False; END_IF;
```

Q2. *Scan cycle:* Consider that the ladder diagram in the next figure is the single code run by a PLC, in a MAST section configured to be cyclic. The PLC input and output take **0.1msec+0.1msec** and each ladder instruction (contact read, coil write, timer) takes about **0.05msec**. At **t=0** the memory cells **%M10**, **Q1** and **Q2** have the logic value False. The timers have preset values of **5msec**. (i) Indicate the scan period of the PLC. (ii) Sketch the time response of **%M10** indicating clearly the time scale. (iii) Sketch the time response of **Q1**. (iv) Discuss whether **%M10** and **Q1** can or cannot be accurately replicated by an output **%q0.4.1**.

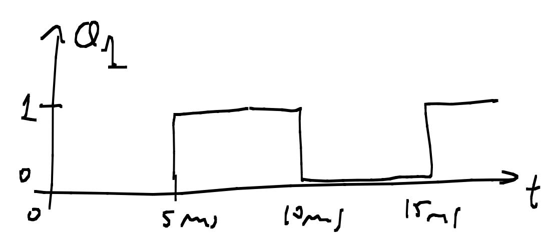


(i) $T = 0.1 + 8 \times 0.05 + 0.1 = 0.1 + 0.4 + 0.1 = 0.6 \text{ ms}$

(ii)



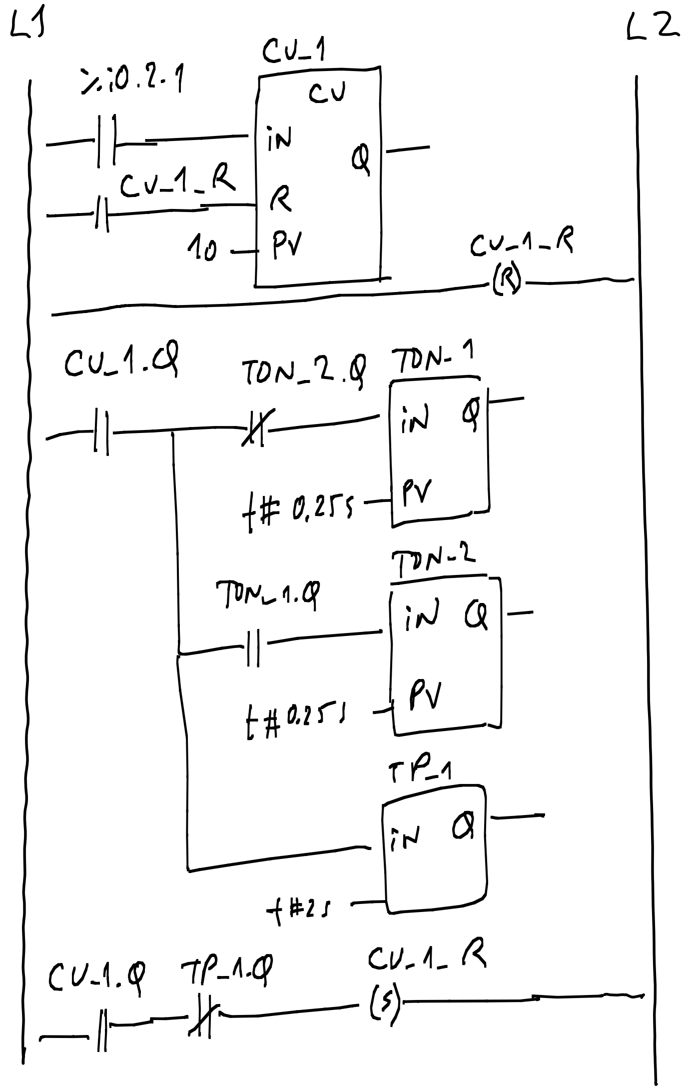
(iii)



(iv) usually outputs are stable for $\Delta t \geq 4$ms, hence %M10 won't appear in the output

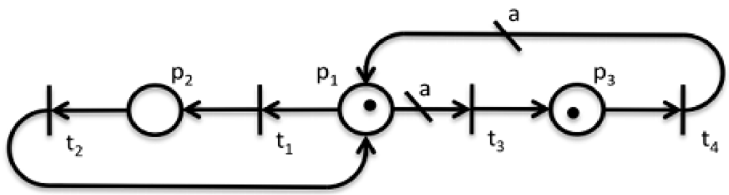
Q1 can be seen if output is driven by transistors (if it is driven by relays, then Q1 cannot be seen and/or can break after some short term the relays)

Q3. *Program:* Considering the PLC programming languages learnt in the course, implement the following logic function: After a counter reaches the counting of ten rising edge triggers in the PLC input %i0.2.1, a light connected to the output %q0.4.1 flashes at 2Hz, 50% duty cycle, during two seconds. The counter restarts only after ending the flashing of the light.



Q4. *PN properties:* Consider the Petri net graph shown in the next figure. Note that there are two arcs with generic non-negative weights 'a'.

See ex1 2010

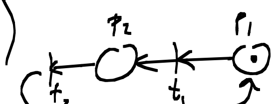


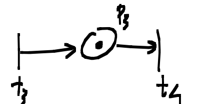
Let **a=0**.

a) Discuss the conservativeness and the boundness of the aforementioned Petri net, resorting to a reachability (sub)tree.

b) Discuss the liveness of each transition and the overall level of liveness for the Petri net.

a)





unbounded

non-conservative

$(1,0) \xrightarrow{t_1} (0,1) \xrightarrow{t_2} (1,0)$ dup

safe, i.e. 1-bounded

conservative

(1)

$\xrightarrow{t_4}$ (0) $\xrightarrow{t_3}$ (1) dup

$\xrightarrow{t_3}$ ($\cancel{2}$) (ω)

∴ the complete net is:
unbounded &
non-conservative

b)

t_1, t_2 are live L_4

t_3 is live L_4

t_4 is live L_4 (can fire ∞ times, provided t_3 fires ∞ times)

Let **a=1**.

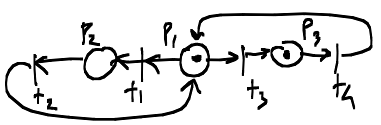
c) Discuss the conservativeness of the Petri net, for this case, and provide the weight vector.

d) Resorting to the Method of the Matrix Equations, study if and how the marking u=[1 1 1]' can be reached.

e) Build the reachability tree. Is the marking u=[0 2 0]' reachable?

f) Find the cycles of operation or place invariants, for this Petri net.

c)



$$D = \begin{array}{c} \begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array} \\ \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \end{array}$$

$wD=0 \Leftrightarrow [w_1\ w_2\ w_3]D=0$

$$\begin{cases} -w_1+w_2=0 \\ w_1-w_2=0 \\ -w_1+w_3=0 \\ w_1-w_3=0 \end{cases} \quad \begin{cases} w_1=w_2 \\ w_1=w_3 \end{cases}$$

∴ is strictly conservative

e.g. $w_1=w_2=w_3=1$

d)

$\mu' = \mu + Dq$

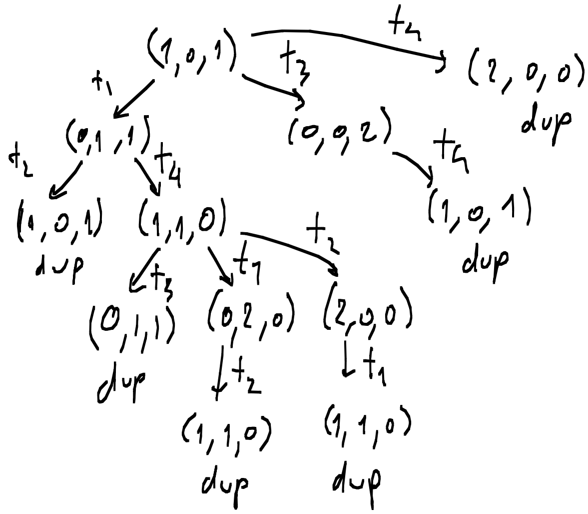
$$\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix} + Dq$$

$$\begin{cases} -q_1+q_2-q_3+q_4=0 \\ +q_1-q_2=1 \\ q_3-q_4=0 \end{cases} \quad \begin{cases} q_1-q_2=0 \\ q_1-q_2=1 \end{cases}$$

impossible

∴ u is not reachable

e)



Yes $\mu = [0\ 2\ 0]'$ is reachable

e.g. (1, 0, 1)

↓

(0, 1, 1)

↓

(1, 1, 0)

↓

(0, 2, 0)

f)

$x^T D = 0$ (done before) $\quad x = [1\ 1\ 1]^T$ all places can participate in the cycles

cycles $Dq = 0$

$$\begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = 0$$

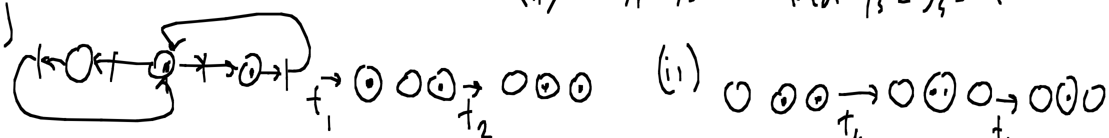
only 2 eqs relevant $(D(1,:) = -D(2,:) - D(3,:))$

$$\begin{cases} q_1 - q_2 = 0 \\ q_3 - q_4 = 0 \end{cases} \quad \begin{cases} q_1 = q_2 \\ q_3 = q_4 \end{cases}$$

eg. (i) $q_1 = q_2 = 1$ and $q_3 = q_4 = 0$ or

(ii) $q_1 = q_2 = 0$ and $q_3 = q_4 = 1$

(i)



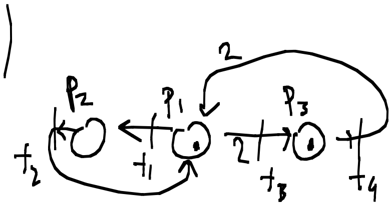
$$\mathrm{null}(D^T) = \mathrm{null}\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \qquad \begin{cases} x_1 = x_2 \\ x_2 = x_3 \end{cases}$$

Let **a=2**.

g) Discuss the following statement "This Petri net is strictly of level 3".

h) Discuss the liveness levels for a=0 and a greater or equal to 2.

g)



All transitions are Level 4 ($\forall \mu\ \exists q$ such that t_i fires)

$\therefore$ PN is not strictly L3

h)

a=0 (seen before) → PN is L4

$a \geq 2$ (=2 see in g) → $\begin{cases} t_3 \text{ firing} \Rightarrow \mu_3 \mathrel{+}= 1,\ \mu_1 \mathrel{-}= a \\ t_4 \text{ firing} \Rightarrow \mu_1 \mathrel{+}= a,\ \mu_3 \mathrel{-}= 1 \end{cases}$

Q5. *Supervision:* Consider a discrete event system describing the state of a fleet of Automatic Guided Vehicles (AGVs) transporting parts in a factory and the state of a set of energy-charging stations. The state of the fleet and the charging stations is described by the 5-tuple **{P,T,A,w, μ_0}** where

P={p_1, p_2, p_3, p_4, p_5}, **T={t_1, t_2, t_3, t_4, t_5, t_6, t_7}**
A={(t_1,p_1), (p_1,t_2), (t_2,p_2), (p_2,t_3), (t_3,p_3), (p_3,t_4), (t_5,p_4), (p_4,t_6), (t_6,p_5), (p_5,t_7)}
$\forall_{i,j}$ w(t_i,p_j)=1, $\forall_{k,l}$ w(p_k,t_l)=1, and **μ0=[0 1 0 0 0]^T**

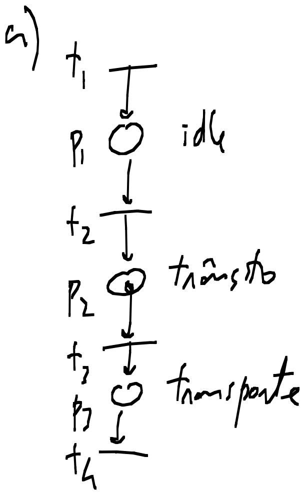
The meaning of the conditions and the events is the following:

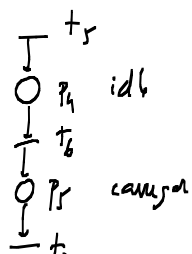
p_1 - AGV(s) idle
p_2 - AGV(s) moving
p_3 - AGV(s) transporting loads
p_4 - Charger(s) idle
p_5 - Charger(s) charging AGV(s)

t_1 - AGV(s) end operation
t_2 - AGV(s) start moving
t_3 - AGV(s) start transporting loads
t_4 - AGV(s) stop transporting loads
t_5 - Charger(s) changing to idle
t_6 - Charger(s) start charging
t_7 - Charger(s) stop charging

a) Draw the graph and write the incidence matrix **D_p** of the Petri net.
b) Design a supervisor based on place invariants stating that there are at most **10** AGVs and **4** charging stations. Draw the supervisor in the Petri net shown in a).

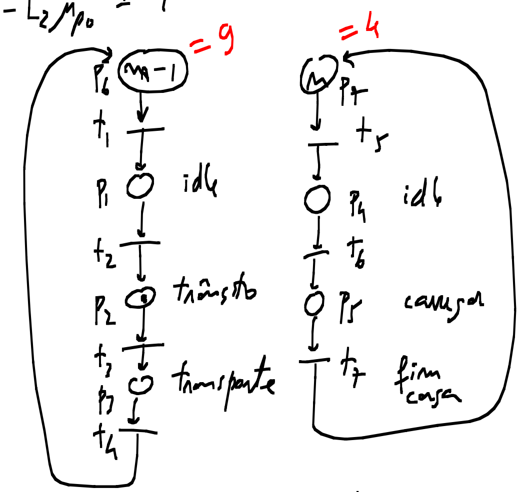
a)





D =	t_1	t_2	t_3	t_4	t_5	t_6	t_7
p_1	1	-1					
p_2		1	-1				
p_3			1	-1			
p_4					1	-1	
p_5						1	-1

$$\mu_{C_1} = b_1 - L_1\mu_{P_0} = 10 - 1 = 9$$
$$\mu_{C_2} = b_2 - L_2\mu_{P_0} = 4 - 0 = 4$$



b)

$$\begin{cases} \mu_1+\mu_2+\mu_3 \leq m & (m=10) \\ \mu_4+\mu_5 \leq m & (m=4) \end{cases}$$

$L_1 = [1\ 1\ 1\ 0\ 0]$, $L_2 = [0\ 0\ 0\ 1\ 1]$

$D_{C_1} = -L_1 D_p = [1\ 0\ 0\ -1\ 0\ 0\ 0]$

$D_{C_2} = -L_2 D_p = [0\ 0\ 0\ 0\ 1\ 0\ -1]$

D =	t_1	t_2	t_3	t_4	t_5	t_6	t_7
p_1	1	-1					
p_2		1	-1				
p_3			1	-1			
p_4					1	-1	
p_5						1	-1
p_6	-1			1			
p_7					-1		1

c) Use the incidence matrix to verify if the Petri net containing the supervisor is conservative. Compute the places weighting vector if the net is conservative.

$wD = 0$

$$\begin{cases} w_1 - w_6 = 0 \\ -w_1 + w_2 = 0 \\ -w_2 + w_3 = 0 \\ -w_3 + w_6 = 0 \\ w_4 - w_7 = 0 \\ -w_4 + w_5 = 0 \\ -w_5 + w_7 = 0 \end{cases} \quad \begin{cases} w_1 = w_6 = w_2 \\ w_2 = w_3 = w_6 \\ w_4 = w_7 = w_5 \end{cases} \quad \begin{cases} w_1 = w_2 = w_3 = w_6 \\ w_4 = w_5 = w_7 \end{cases}$$

e.g.

$$\begin{cases} w_1 = w_2 = w_3 = w_6 = 1 \\ w_4 = w_5 = w_7 = 1 \end{cases} \quad \therefore \text{ is conservative}$$

$w = [1\ 1\ 1\ 1\ 1\ 1\ 1]$ // c.q.d.

d) Change the Petri net by adding two transitions t_8 and t_9, and two arcs (p_2, t_8) and (t_9, p_2) both with unitary weights. The new transitions have the meanings t_8 - AGV battery discharged, t_9 - AGV battery recharged. Design a supervisor based on place invariants, considering generalized linear constraints, such that one moving AGV whenever it detects it is discharged it can go to a charging station (if available) and come back to the moving condition. Draw the supervisor in the global Petri net.

1) $v_6 \leq v_8$ $\quad D_{c_1} = -c_1 = -[\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ 1\ \cdot\ -1\ \cdot]$ $\quad \mu_{c_1} = b_1 = 0$

2) $v_8 \leq v_7 + m$ $\quad D_{c_2} = -c_2 = -[\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ -1\ 1\ \cdot]$ $\quad \mu_{c_2} = b_2 = m = 4$

3) $v_9 \leq v_7$ $\quad D_{c_3} = -c_3 = -[\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ -1\ \cdot\ 1]$ $\quad \mu_{c_3} = b_3 = 0$