Industrial Automation

(Automação de Processos Industriais)

Supervised Control of Discrete Event Systems

Supervision Controllers (Part 2/2)

http://www.isr.tecnico.ulisboa.pt/~jag/courses/api20b/api2021.html

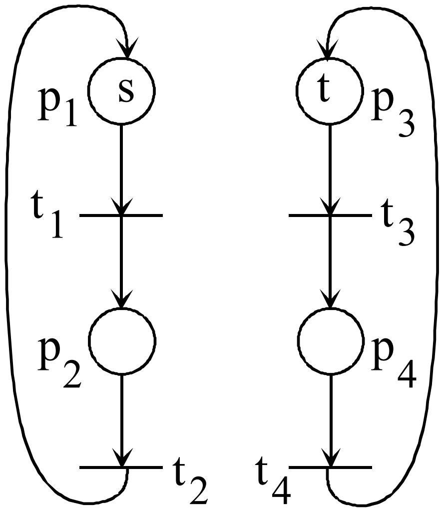
Prof. Paulo Jorge Oliveira, original slides
Prof. José Gaspar, rev. 2020/2021

Some pointers on Supervised Control of DES

Analysers & simulators	http://www.nd.edu/~isis/techreports/isis-2002-003.pdf (Users Manual) http://www.nd.edu/~isis/techreports/spnbox/ (Software)
Bibliography:	**Supervisory Control of Discrete Event Systems using Petri Nets**, J. Moody J. and P. Antsaklis, Kluwer Academic Publishers, 1998. **Supervised Control of Concurrent Systems: A Petri Net Structural Approach**, M. Iordache and P. Antsaklis, Birkhauser 2006. **Discrete Event Systems - Modeling and Performance Analysis**, Christos G. Cassandras, Aksen Associates, 1993. **Feedback Control of Petri Nets Based on Place Invariants**, K. Yamalidou, J. Moody, M. Lemmon and P. Antsaklis, http://www.nd.edu/~lemmon/isis-94-002.pdf

Methods of Synthesis

Example of controller synthesis: s Producers / t Consumers



Incidence matrix

Initial marking

$$D_P = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \qquad \mu_{P_0} = \begin{bmatrix} s \\ 0 \\ t \\ 0 \end{bmatrix}.$$

Let p2= #machines working, t2= product produced
p3= #consumers, t3= request to consume (e.g. transport product)
Q: How to write *consume only when produced* ? What is the linear constraint?

Not possible to write it as a linear constraint on places $L\mu_p \leq b$.
Is it impossible to solve this problem with the supervised control ?

Methods of Synthesis Generalized linear constraint

Let the generalized linear constraint be

$$\boxed{\begin{gathered} L\mu_P + Fq_P + Cv_P \le b, \\ \mu_P \in N_0{}^n, v_P \in N_0{}^m, q_P \in N_0{}^m, \\ L \in Z^{n_C \times n}, F \in Z^{n_C \times m}, C \in Z^{n_C \times m}, e \quad b \in Z^{n_C}, \end{gathered}}$$

where

- μ_P is the marking vector for system P
- q_P is the firing vector since t_0
- v_P is the number of transitions (firing) that can occur, also designated as Parikh vector

Example: detail elements forming the generalized linear constraint

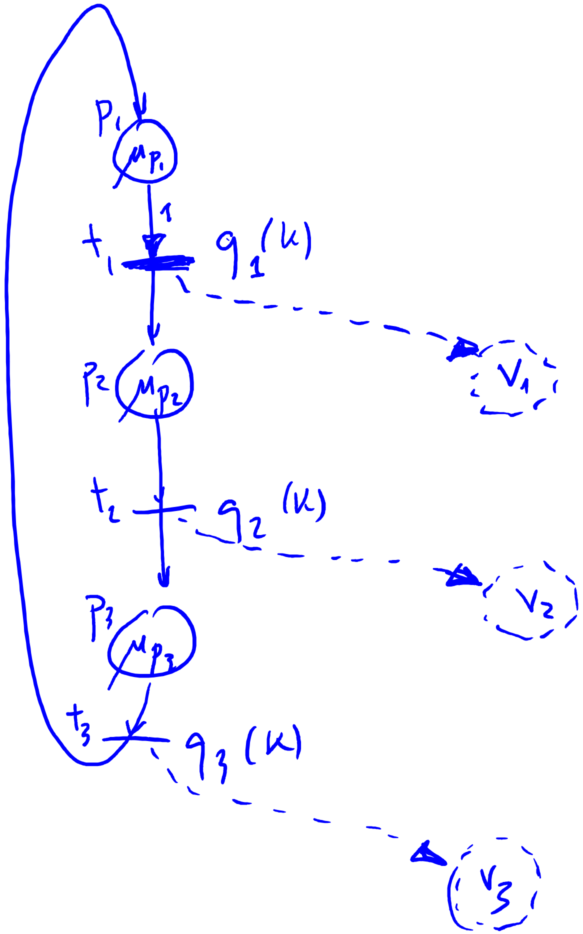
$$L\mu_P + Fq_P + Cv_P \leq b,$$

$$\mu_P \in N_0{}^n, v_P \in N_0{}^m, q_P \in N_0{}^m,$$
$$L \in Z^{n_C \times n}, F \in Z^{n_C \times m}, C \in Z^{n_C \times m},$$
$$b \in Z^{n_C},$$

State: $\mu_p(k) = \begin{bmatrix} \mu_{p1} \\ \mu_{p2} \\ \mu_{p3} \end{bmatrix}_k$

Firing vector: $q_p(k) = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}_k$

Parikh vector: $v_p(k) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_k$



Methods of Synthesis

Function LINENF of SPNBOX

Theorem*: Synthesis of Controllers based on Place Invariants, for **Generalized Linear Constraints**

Given the generalized linear constraint $\boxed{L\mu_P + Fq_P + Cv_P \leq b,}$ if $b - L\mu_{P_0} \geq 0,$ then the controller with incidence matrix and initial marking, respectively

$$\boxed{\begin{aligned} D_C^- &= \max\left(0,\ LD_P + C,\ F\right) \\ D_C^+ &= \max\left(0,\ F - \max\left(0, LD_P + C\right)\right) - \min\left(0,\ LD_P + C\right), \end{aligned}}$$

$$\boxed{\mu_{C_0} = b - L\mu_{P_0} - Cv_{P0},}$$

guarantees that constraints are verified for the states resulting from the initial marking.

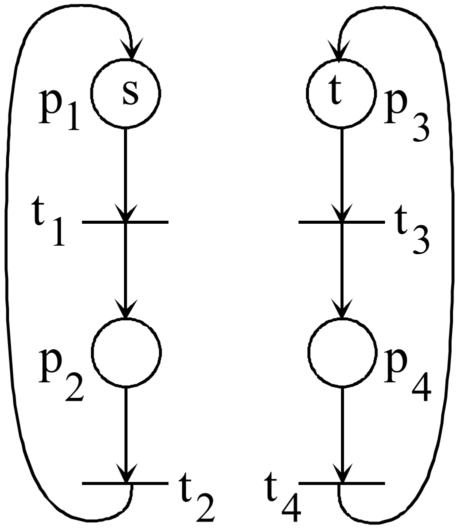
* *In the next slides this will be called the* ***LINENF theorem.***

Methods of Synthesis

Example of controller synthesis

$\forall s \in N_0, \forall t \in N_0, \forall n \in N_0$

Producer / Consumer



Linear constraint: $v_3 \le v_2$

that can be written as:

$$Cv_P \le b$$

$$L = 0, F = 0$$

$$\begin{bmatrix} 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \le 0.$$

Incidence matrix

$$D_P = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Initial marking

$$\mu_{P_0} = \begin{bmatrix} s \\ 0 \\ t \\ 0 \end{bmatrix}.$$

Methods of Synthesis

Example of controller synthesis

Producer / Consumer

1) Test $$b - L\mu_{P_0} = 0 - 0 \geq 0.$$

OK.

2) Compute

$$\boxed{\begin{aligned} D_C^- &= \max\left(0,\, LD_P + C,\, F\right) \\ D_C^+ &= \max\left(0,\, F - \max\left(0, LD_P + C\right)\right) - \min\left(0,\, LD_P + C\right), \end{aligned}}$$

$$D_C^- = \max\left(0,\ [0\ \ -1\ \ 1\ \ 0],\ 0\right) = [0\ \ 0\ \ 1\ \ 0]$$

$$D_C^+ = \max\left(0, -[0\ \ 0\ \ 1\ \ 0]\right) - \min\left(0, [0\ \ -1\ \ 1\ \ 0]\right)$$

$$= [0\ \ 0\ \ 0\ \ 0] - [0\ \ -1\ \ 0\ \ 0] = [0\ \ 1\ \ 0\ \ 0]$$

and

$$\boxed{\mu_{C_0} = b - L\mu_{P_0} - Cv_{P0},}$$

$$\mu_{C_0} = b - L\mu_{P_0} = 0 - 0 = 0.$$

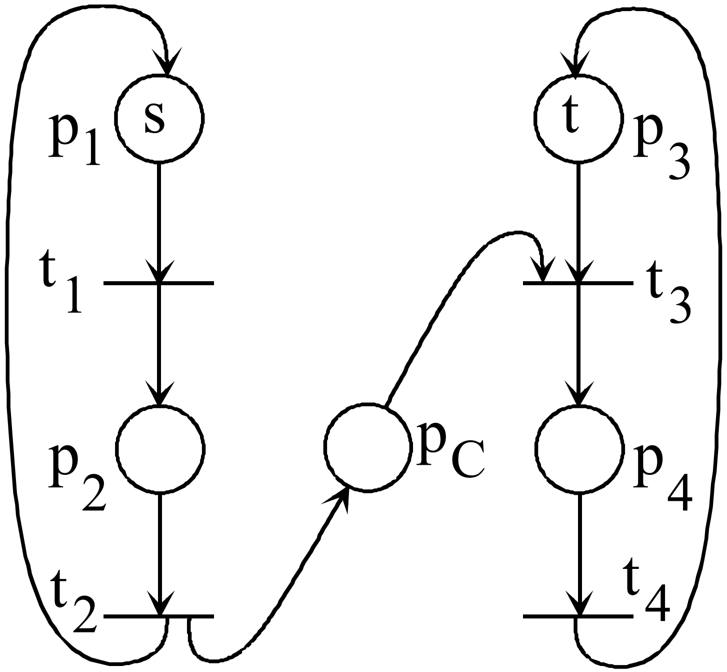
OK.

Methods of Synthesis

Example of controller synthesis

Producer / Consumer

3) Resulting in



$$D = \left[\begin{array}{cccc} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ \multicolumn{4}{c}{\boxed{0 \quad 1 \quad -1 \quad 0}} \end{array}\right]$$

$$\mu_0 = \begin{bmatrix} s \\ 0 \\ t \\ 0 \\ \boxed{0} \end{bmatrix}$$

OK.
UAU!!!.

Methods of Synthesis

Example of controller synthesis: Producer Consumer

```
% The Petri net D=Dp-Dm, and m0
%  (Dplus-Dminus= Post-Pre)

Dm= [1 0 0 0;
     0 1 0 0;
     0 0 1 0;
     0 0 0 1];

 Dp= [0 1 0 0;
     1 0 0 0;
     0 0 0 1;
     0 0 1 0];

m0= [1 0 1 0]';

% Supervisor constraint
%
L= []; F= [];  C= [0 -1 1 0];
b= 0;

% Computing the supervisor
%
[Dfm, Dfp, ms0] = linenf(Dm, Dp, L, b, m0, F, C)
Df= Dfp-Dfm
ms0
```

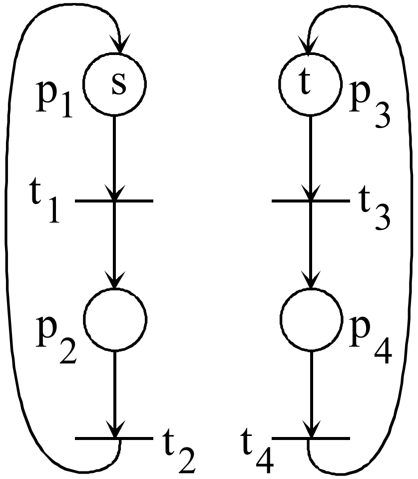
Result using the function LINENF.m of the toolbox SPNBOX:

```
Df =

    -1     1     0     0
     1    -1     0     0
     0     0    -1     1
     0     0     1    -1
     0     1    -1     0


ms0 =

     1
     0
     1
     0
     0
```

Methods of Synthesis

Example of controller synthesis

Bounded Producer / Consumer



Incidence matrix

$$D_P = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Initial marking

$$\mu_{P_0} = \begin{bmatrix} s \\ 0 \\ t \\ 0 \end{bmatrix}.$$

TWO linear constraints:

$$\begin{cases} v_3 \le v_2 \\ v_2 \le v_3 + n \end{cases} \Leftrightarrow \begin{cases} v_3 - v_2 \le 0 \\ v_2 - v_3 \le n \end{cases}$$

$$\forall s \in N_0, \forall t \in N_0, \forall n \in N_0$$

The two linear constraints can be written as:

$$Cv_P \le b$$
$$i.e.\ L = 0, F = 0$$

$$\Leftrightarrow \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \le \begin{bmatrix} 0 \\ n \end{bmatrix}$$

Methods of Synthesis

Example of controller synthesis

Bounded Producer / Consumer

1) Test

$$b - L\mu_{P_0} = b = \begin{bmatrix} 0 \\ n \end{bmatrix} \geq 0.$$

OK.

2) Compute

$$D_C^- = \max\left(0, \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}, 0\right) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$D_C^+ = \max\left(0, 0 - \max\left(0, \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}\right)\right) - \min\left(0, \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

and

$$\mu_{C_0} = b - L\mu_{P_0} = \begin{bmatrix} 0 \\ n \end{bmatrix}.$$

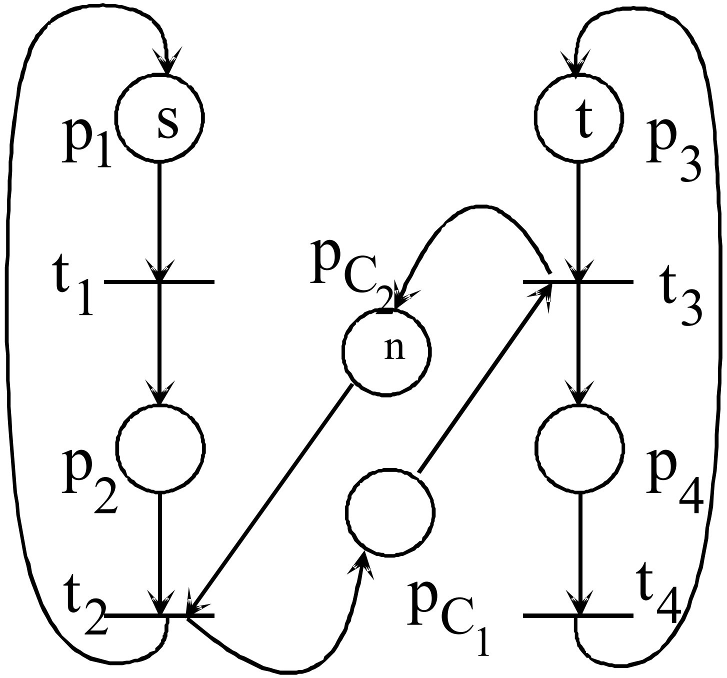
OK.

Methods of Synthesis

Example of controller synthesis

Bounded Producer / Consumer

3) Resulting in



$$D = \begin{bmatrix} \begin{matrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{matrix} \\ \boxed{\begin{matrix} 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{matrix}} \end{bmatrix}$$

$$\mu_0 = \begin{bmatrix} s \\ 0 \\ t \\ 0 \\ \boxed{\begin{matrix} 0 \\ n \end{matrix}} \end{bmatrix}$$

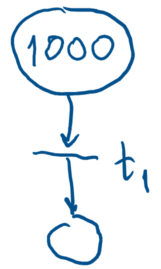
OK.
UAU!!!.

Methods of Synthesis

Example of controller synthesis

Flow regulation

Given



enforce max q_1 to be 2

i.e. q_1 can be 0, 1 or 2

Constraint

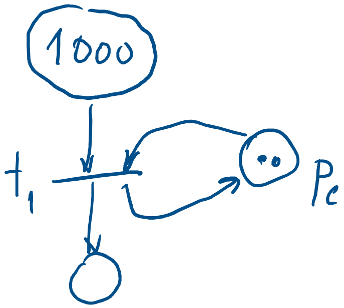
$$\underset{\overset{\uparrow}{F}}{1} \cdot q_1 \leq \underset{\overset{\uparrow}{b}}{2}$$

Solution

$D_c^+ = 1$

$D_c^- = 1$

$\mu_{c_0} = 2$



Methods of Synthesis

Function LINENF of SPNBOX

LINENF Lemma 1: From General Constraints to Theorem T1

Given the generalized linear constraint $L\mu_P + Fq_P + Cv_P \leq b$ and the conditions of the LINENF theorem:

If $\boxed{L \neq 0,}$ $F = 0,$ $C = 0$

then $D_C^+ = (LD_P)^-,$ $D_C^- = (LD_P)^+$ and $\boxed{D_C = -LD_P}$

$$\boxed{\mu_{C0} = b - L\mu_{P0}}$$

(see proof in the next page)

Notation:

$D^+ = \max(0, D)$
$D^- = -\min(0, D)$
$D = D^+ - D^-$

$D^+, D^- \in N_0^{n\times m}$ and $D \in Z_0^{n\times m}$

$$D_C^- = \max(0,\, LD_P + C,\, F)$$
$$D_C^+ = \max(0,\, F - \max(0, LD_P + C)) - \min(0,\, LD_P + C),$$

$$\mu_{C_0} = b - L\mu_{P_0} - Cv_{P0},$$

$$\boxed{L \neq 0, \quad F = 0, \quad C = 0 \quad \Rightarrow L\mu_P \leq b}$$

$$D_c^- = \max(0,\, LD_p + \overset{=0}{\cancel{C}},\, \overset{=0}{\cancel{F}})$$
$$= \max(0,\, LD_p)$$
$$= (LD_p)^+ \; /\!/$$

$$D_c^+ = \max(0,\, \overset{=0}{\cancel{F}} - \max(0, LD_p + \overset{=0}{\cancel{C}})) \ominus \min(0,\, LD_p + \overset{=0}{\cancel{C}})$$
$$= \underbrace{\max(0,\, \underbrace{-(LD_p)^+}_{\leq 0})}_{=0} \oplus (LD_p)^-$$
$$= +(LD_p)^- \; /\!/$$

$D^- = -\min(0, D)$

$D^- \in \mathbb{N}_0^{m \times m}$

$$D_c = D_c^+ - D_c^- = (LD_p)^- - (LD_p)^+ = -\left((LD_p)^+ - (LD_p)^-\right) = -LD_p \; /\!/$$

$$\mu_{c_0} = b - L\mu_{p_0} - \overset{=0}{\cancel{C}}\, v_{p_0} = b - L\mu_{p_0} \; /\!/$$

Methods of Synthesis

Function LINENF of SPNBOX

LINENF Lemma 2: Firing Regulation

Given the generalized linear constraint $L\mu_P + Fq_P + Cv_P \leq b$ and the conditions of the LINENF theorem:

If $L = 0$, $\boxed{F \neq 0,}$ $C = 0$

then $D_C^+ = F^+$, $D_C^- = F^+$ and $\boxed{D_C = 0}$

$$\boxed{\mu_{C0} = b}$$

(homework, prove this lemma)

Methods of Synthesis

Function LINENF of SPNBOX

LINENF Lemma 3: Constraints on Counters

Given the generalized linear constraint $L\mu_P + Fq_P + Cv_P \leq b$ and the conditions of the LINENF theorem:

If $L = 0, \quad F = 0, \quad \boxed{C \neq 0}$

then $D_C^+ = C^-, \qquad D_C^- = C^+$ and $\boxed{D_C = -C}$

$$\boxed{\mu_{C0} = b - Cv_{P0}}$$

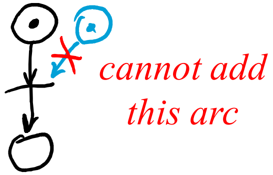
(homework, prove this lemma)

(empty page, do yourself the proof of the last two lemmas)

Methods of Synthesis: intro to **Uncontrollable** and **Unobservable** transitions

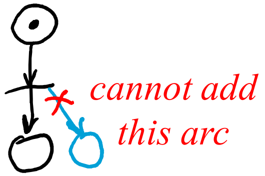
Definition of **Uncontrollable Transition:**

A transition is uncontrollable if its firing **cannot be inhibited** by an external action (e.g. a supervisory controller).



Definition of **Unobservable Transition:**

A transition is unobservable if its firing **cannot be detected or measured** (therefore the study of any supervisory controller can not depend from that firing).



Proposition:

In a Petri net based controller, both input and output arcs to/from plant transitions are used to trigger state changes in the controller. *Since a controller cannot have arcs connecting to unobservable transitions, then all* ***unobservable transitions are also implicitly uncontrollable****.*

Methods of Synthesis: intro to **Uncontrollable** and **Unobservable** transitions

(a) (b)

If **t1 is controllable** and **t2 is uncontrollable**:

- case (a), then t2 cannot be directly inhibited; it will eventually fire
- case (b), then **t2 can be indirectly prevented** from firing by inhibiting t1.

i.e. may exist indirect solution despite t2 being uncontrollable.

If **t2 is unobservable** and **t3 is observable**, then we cannot detect when t2 fires. The state of a supervisor is not changed by firing t2. However we can **indirectly detect that t2 has fired**, by detecting the firing of t3.

i.e. may exist indirect solution despite t2 being unobservable.

∴ *may exist indirect solution despite t2 uncontrollable and/or unobservable.*

Methods of Synthesis

Definition: A **marking** $\boldsymbol{\mu_P}$ is **admissible** if

i) $L\mu_P \leq b$ and **ii)** $\forall \mu' \in R(C, \mu_P)$ verifies $L\mu' \leq b$

Definition: A **Linear Constraint** $(\boldsymbol{L}, \boldsymbol{b})$ is **admissible** if

i) $L\mu_{Po} \leq b$ and

ii) $\forall \mu' \in R(C, \mu_{Po})$ such that $L\mu' \leq b$

μ' is an admissible marking.

Note: ii) indicates that the firing of uncontrollable transitions can never lead from a state that satisfies the constraint to a new state that does not satisfy the constraint.

Methods of Synthesis

Proposition: Admissibility of a constraint

A linear constraint is admissible ***iff***

- The initial markings satisfy the constraint.
- There **exists a controller** with maximal permissivity that forces the constraint and **does not inhibit any uncontrollable transition**.

Two sufficient (not necessary) conditions:

Corollary: given a system with uncontrollable transitions,

$\boxed{l^T D_{uc} \leq 0}$ implies admissibility.

Corollary: given a system with unobservable transitions,

$\boxed{l^T D_{uo} = 0}$ implies admissibility.

Methods of Synthesis

Function MRO_ADM of SPNBOX

Lemma *: Structure of Constraint transformation

If $\boxed{L'\mu_p \leq b'}$ where

$$L' = R_1 + R_2 L \qquad \text{and} \quad b' = R_2(b+1) - 1$$

$\boxed{R_1} \in Z^{nc \times n}$ and $R_1 \mu_p \geq 0$

$\boxed{R_2} \in Z^{nc \times nc}$ is a matrix with positive elements in the diagonal

then $\boxed{L\mu_p \leq b}$ is also verified.

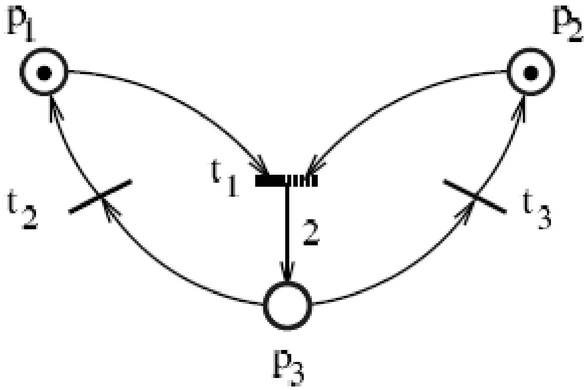
Typical usage:

- constraint (L, b), and **extra constraints** → $(R_1, R_2) \rightarrow (L', b') \rightarrow (D_C^+, D_C^-, \mu_{C0})$
- see unobservable / uncontrollable extra constraints example in the next slides

* Lemma 4.10 in [Moody98] pg46

Methods of Synthesis

Example: design controller with t1 unobservable (1/4)



$$D = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{bmatrix}, \quad D_{uo} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

Objectives: $\mu_1 + \mu_3 \geq 1$ and $\mu_2 + \mu_3 \geq 1$ which can be written in matrix form as

$$L\mu \leq b, \qquad L = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}, \qquad b = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Example extracted from "Supervised Control of Concurrent Systems: A Petri Net Structural Approach", M. Iordache and P. Antsaklis, Birkhauser 2006.

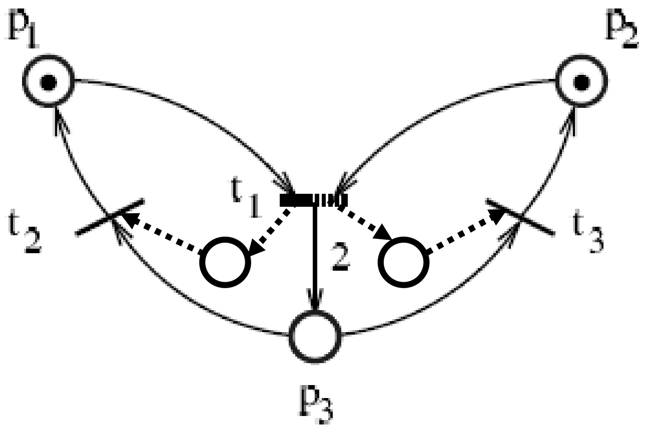
Methods of Synthesis

Example: design controller with t1 unobservable (2/4)

```
% System and constraints

D= [-1  1  0;
    -1  0  1;
    +2 -1 -1];

Dm= -D.*(D<0);
Dp=  D.*(D>0);

m0= [1 1 0]';

L= [-1 0 -1; 0 -1 -1];
b= [-1; -1];


% Supervisor computation

[Dfp, Dfm, mf0] =
    linenf( Dp, Dm, L, b, m0 );
```

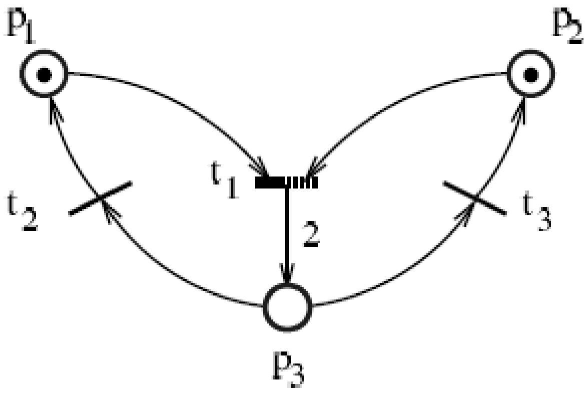
```
Dfp =                        Dfm =                        mf0 =

     0     1     0                1     0     0              1
     0     0     1                1     0     0              1
     2     0     0                0     1     1              0
     1     0     0                0     0     1              0
     1     0     0                0     1     0              0
```

^ *Bad news, supervisor touches* t_1 *.*



Methods of Synthesis

Example: design controller with t1 unobservable (3/4)



```
D= [-1  1  0;
    -1  0  1;
     2 -1 -1];

Tuo= [1]; Tuc= [];

L= [-1 0 -1; 0 -1 -1];
b= [-1 -1]';

[La, ba, R1, R2] = mro_adm( L, b, D, Tuc, Tuo );
```

Solution obtained with the function MRO_ADM.m of the SPNBOX toolbox:

$$R1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{La} = \begin{bmatrix} -2 & 0 & -1 \\ 0 & -2 & -1 \end{bmatrix} \qquad \mathbf{ba} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$R2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Note: verify that $L_a \mu \leq b_a$ implies $L \mu \leq b$

Methods of Synthesis

Example: design controller with t1 unobservable (4/4)

Finally the supervised controller is simply obtained from L_a and b_a:

$$D_c = -L_a D_p$$

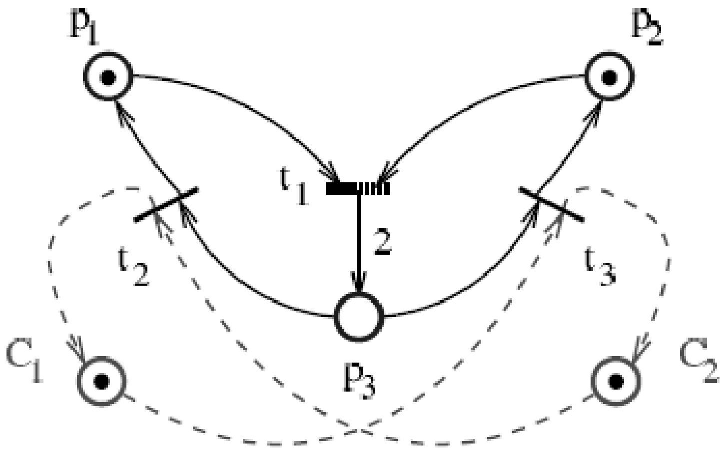
$$= \begin{bmatrix} -2 & 0 & -1 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\mu_{c0} = b_a - L_a \mu_{p0}$$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix} - \begin{bmatrix} -2 & 0 & -1 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



Obtained the desired result: supervisor does not touch t_1.

This course is ending. What is next?

This course is ending .

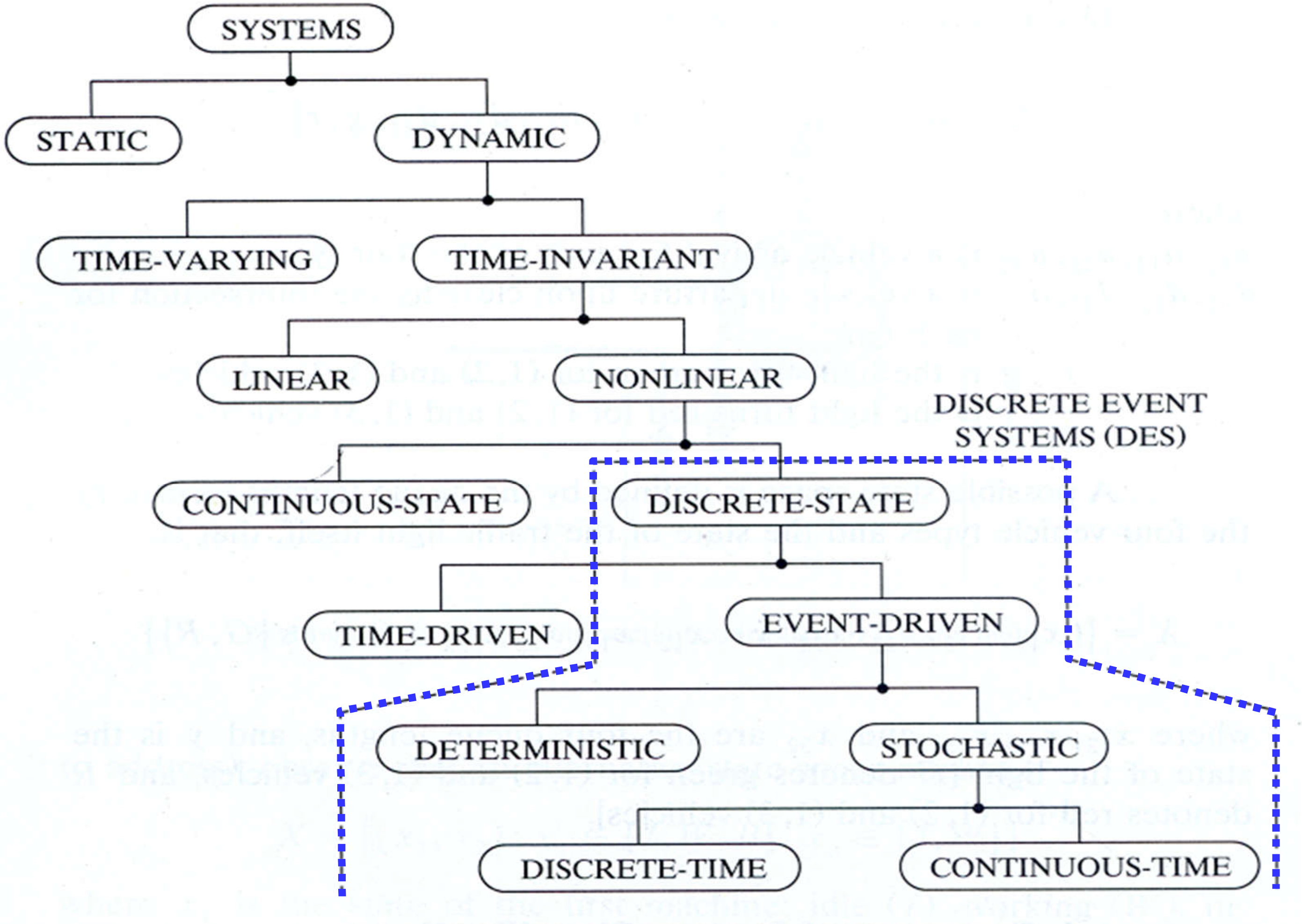


Figure 1.29. Major system classifications.

What
is
next ?

Top 10 Challenges in Logic Control for Manufacturing Systems

by Dawn Tilbury from University of Michigan

10. Distributed Control	(General management of distributed control applications, Open/distributed control -- ethernet-based control)
9. Theory	(No well-developed and accepted theory of discrete event control, in contrast to continuous control)
8. Languages	(None of the programming languages do what we need but nobody wants a new programming language)
7. Control logic synthesis	(automatically)
6. Standards	(Machine-control standards -- every machine is different, Validated standards, Standardizing different types of control logic programming language)
5. Verification	(Standards for validation, Simulation and verification of controllers)
4. Software	(Software re-usability -- cut and paste, Sophisticated software for logic control, User-unfriendly software)
3. Theory/Practice Gap	(Bridging the gap between industry and academia, Gap between commercial software and academic research)
2. Education	(Educating students for various PLCs, Education and keeping current with evolution of new control technologies, Education of engineers in logic control, Lack of curriculum in discrete-event systems)
	And the number one challenge in logic control for manufacturing systems is...
1. Diagnostics	(Integrating diagnostic tools in logic control, Standardized methodologies for design, development, and implementation of diagnostics)

The End .