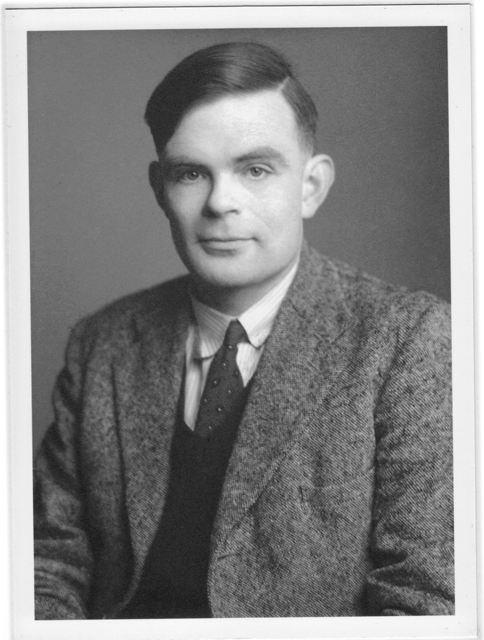
Industrial Automation

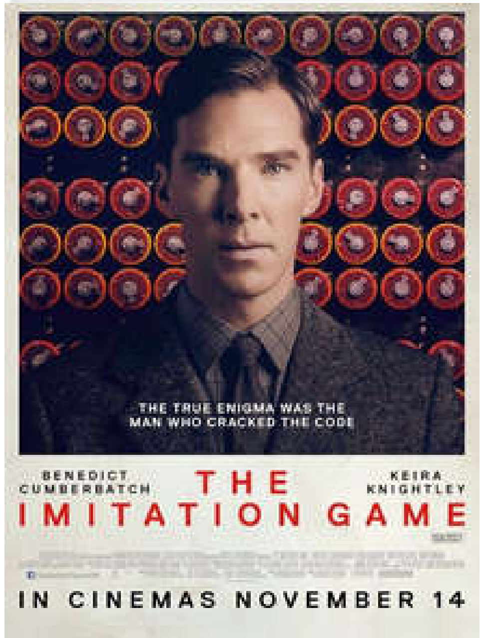
(Automação de Processos Industriais)

Discrete Event Systems: Turing Machines, *Busy Beaver*

http://www.isr.tecnico.ulisboa.pt/~jag/courses/api20b/api2021.html

Prof. José Gaspar, 2020/2021





Simple ways to learn more about
Alan Turing

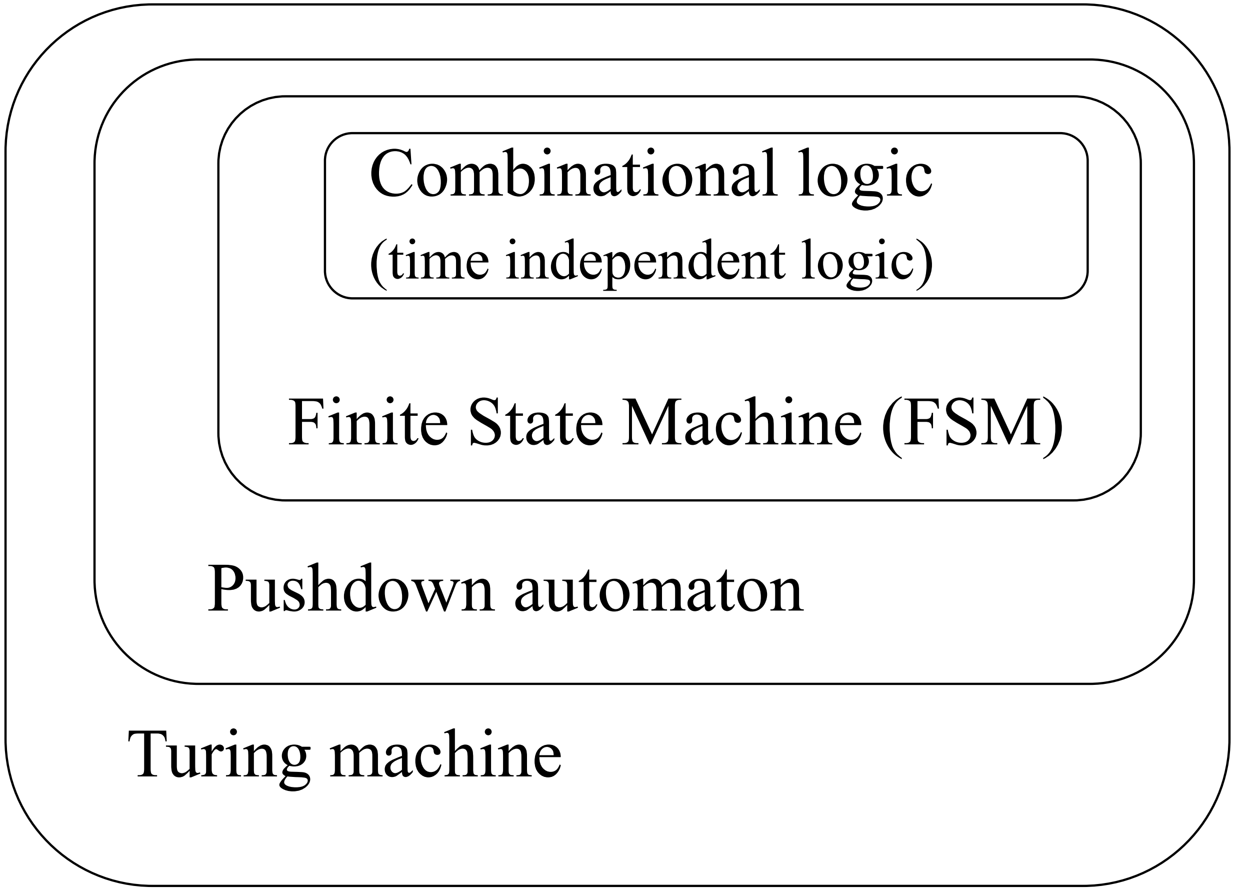
Check 2nd world war history:

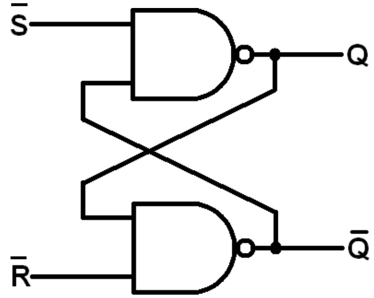
- Cryptanalysis of the ***Enigma*** Machine
- The British ***Bombe*** (Turing)

See the movie (2014):

- The Imitation Game
- *Trailer to see in the weekend ;)*

Automata theory





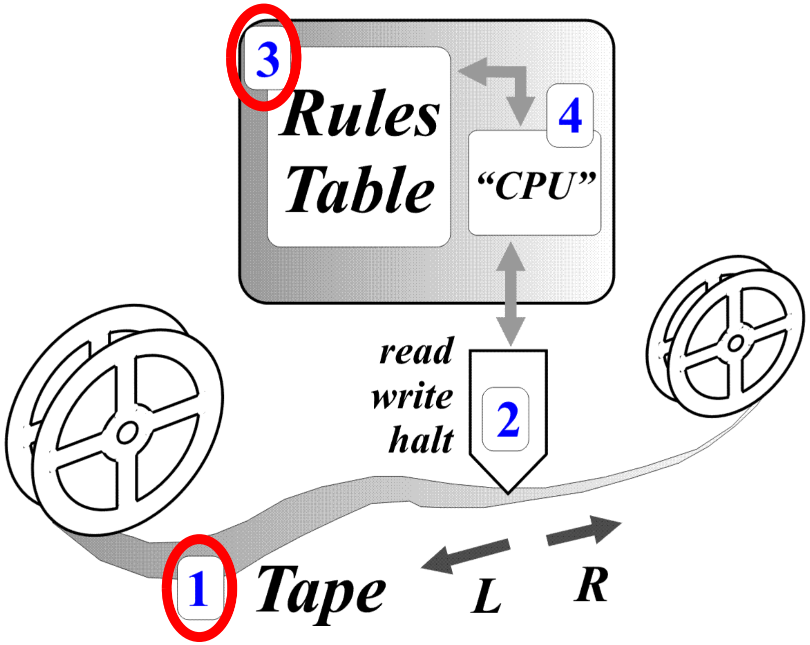
Current state	Input SR	Next state
xx	11	11
xx	10	10
xx	01	01
xx	00	xx

SR latch is an FSM example. The input *(S,R)=(0,0)* keeps the **memorized** value

$$(Q,\overline{Q}) = (x,x)$$

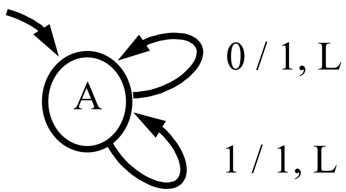
How many different states can the SR latch show?

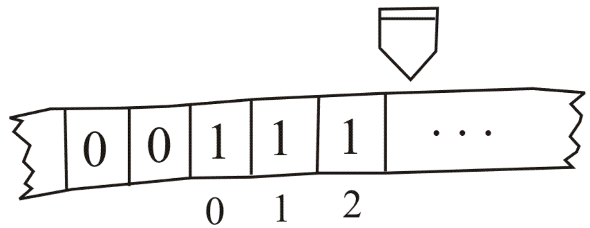
Turing machine (TM)



Components:
(1) Infinite length magnetic **Tape**
(2) Read/Write head
(3) Rules table, e.g. an FSM
(4) State register

Example of a simple Rules table, namely an **FSM**. Using this FSM the TM writes forever ones into the tape. Read the FSM as "if 0 or 1 is read from the tape, then write 1 to the tape, move tape to the left and continue in state A".



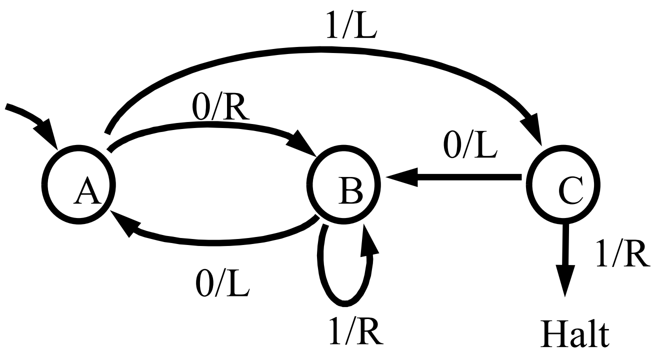


Note: a TM is not just an FSM; for example, it contains also an **infinite memory**.

Turing machine example: *Busy Beaver*

The objective is to fill the TM tape with ones, as many as possible, using a rules table (FSM) with a minimum number of states. By definition of *Busy Beaver*, the TM must halt (stop) some time after starting.

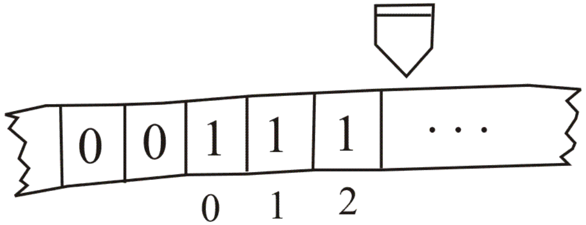
One implementation of the 3-states 2-symbols Busy Beaver is:



Current state	Input	Action R/W	Action L/R/N	Next state
A	0	write1	right	B
A	1	write1	left	C
B	0	write1	left	A
B	1	write1	right	B
C	0	write1	left	B
C	1	write1	null	halt

Turing machine in Matlab:

(1) **tape** and
(2) read/write head

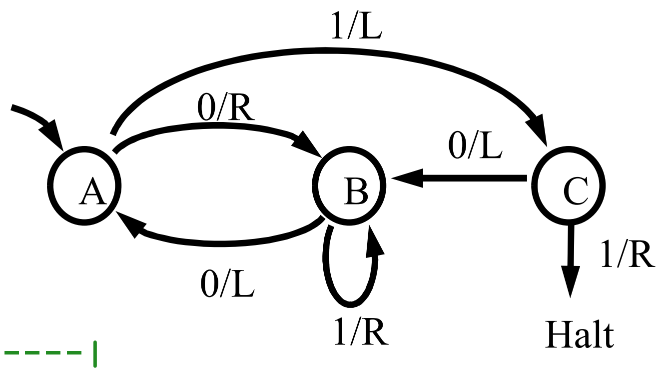



```
function TM_reset
global TMT
TMT= struct('pos',0,
            'val',[],
            'valNeg',[]);
```

```
function ret= TM_tape( op, arg1 )
% Tape for a Turing machine. Basic operations:
%   read/write and move Left/Right/None
global TMT; if isempty(TMT), TM_reset; end
switch op
    case 'reset', TM_reset;
    case 'left',  TMT.pos= TMT.pos+1;
    case 'right', TMT.pos= TMT.pos-1;
    case 'null_move' % do nothing
    case 'read', % 1st call may need tape
        realloc_if_needed( TMT.pos );
        if TMT.pos>=0, ret= TMT.val( TMT.pos+1 );
        else           ret= TMT.valNeg( -TMT.pos );
        end
    case 'write', % 1st call may need tape
        realloc_if_needed( TMT.pos );
        if TMT.pos>=0, TMT.val( TMT.pos+1 )= arg1;
        else           TMT.valNeg( -TMT.pos )= arg1;
        end
    otherwise, error('inv op')
end
```

Turing machine in Matlab:

(3) rules table, FSM of a *3-state Busy Beaver*




```
function FSM= def_BusyBeaver3
% FSM has four columns:
%   curr_state, true_false_condition, actions, next_state
%           |- T/F cond -----|     |- write action and move action ----|
FSM= {
    'A',  'TM_tape("read")==0',  'TM_tape("write",1); TM_tape("right");',  'B';
    'A',  'TM_tape("read")==1',  'TM_tape("write",1); TM_tape("left");' ,  'C';
    'B',  'TM_tape("read")==0',  'TM_tape("write",1); TM_tape("left");' ,  'A';
    'B',  'TM_tape("read")==1',  'TM_tape("write",1); TM_tape("right");',  'B';
    'C',  'TM_tape("read")==0',  'TM_tape("write",1); TM_tape("left");' ,  'B';
    'C',  'TM_tape("read")==1',  'TM_tape("write",1); TM_tape("null_move");', 'halt';
    };
```

Current state	Input	Action R/W	Action L/R/N	Next state
A	0	write1	right	B
A	1	write1	left	C
B	0	write1	left	A
B	1	write1	right	B
C	0	write1	left	B
C	1	write1	null	halt

Alternative, more compact, representation:

```
function FSM= def_BusyBeaver3

tbl= {'A01RB', 'A11LC', ...
      'B01LA', 'B11RB', ...
      'C01LB', 'C11NH'};

FSM= convert_table_to_list( tbl );
```

Turing machine in Matlab:

(4) state register, `curr_state` for **running** the machine

Recall the first line of the table:

```
FSM{1,:} =
'A'
'TM_tape("read")==0'
'TM_tape("write",1); TM_tape("right");'
'B'
```

and read it as "if current state is A and tape read is zero, then write 1 to the tape, move tape right, and the next state is B".

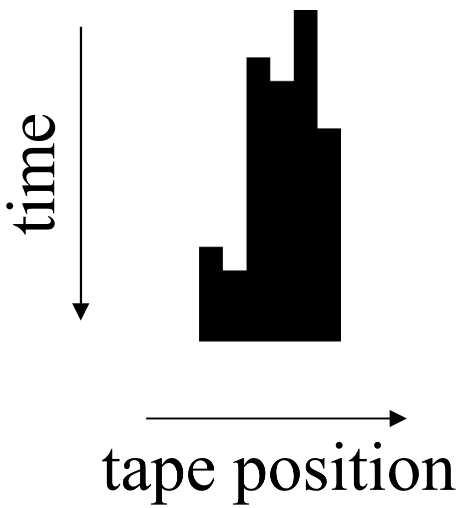
```
function TM_run
TM_tape( 'reset' );
FSM= TM_ini( 'BusyBeaver3' );
curr_state= FSM{1,1};
while ~strcmpi(curr_state, 'halt')
    for i=1:size(FSM,1)
        if strcmpi(FSM{i,1}, curr_state) ...
                && eval( FSM{i,2} )
            % found state and true condition
            eval( FSM{i,3} );
            % curr_state <- next state
            curr_state= FSM{i,4};
            break;
        end
    end
end
```

Download the complete implementation from:
http://isr.tecnico.ulisboa.pt/~jag/course_utils/Turing_Machines_sim/Turing_Machines_sim.html

Turing Machine *Busy Beaver*: simulation results

3-state Busy Beaver:
a0 -> b1r a1 -> h1r
b0 -> c0r b1 -> b1r
c0 -> c1l c1 -> a1l

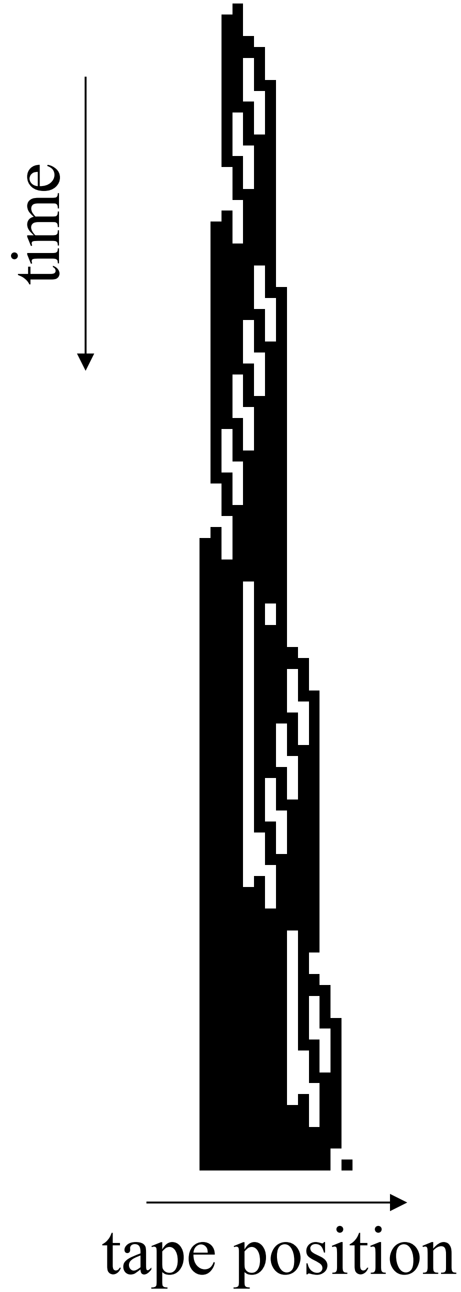
halts after **21 time steps**
fills **6 ones**



4-state Busy Beaver:
a0 -> b1r a1 -> b1l
b0 -> a1l b1 -> c0l
c0 -> h1r c1 -> d1l
d0 -> d1r d1 -> a0r

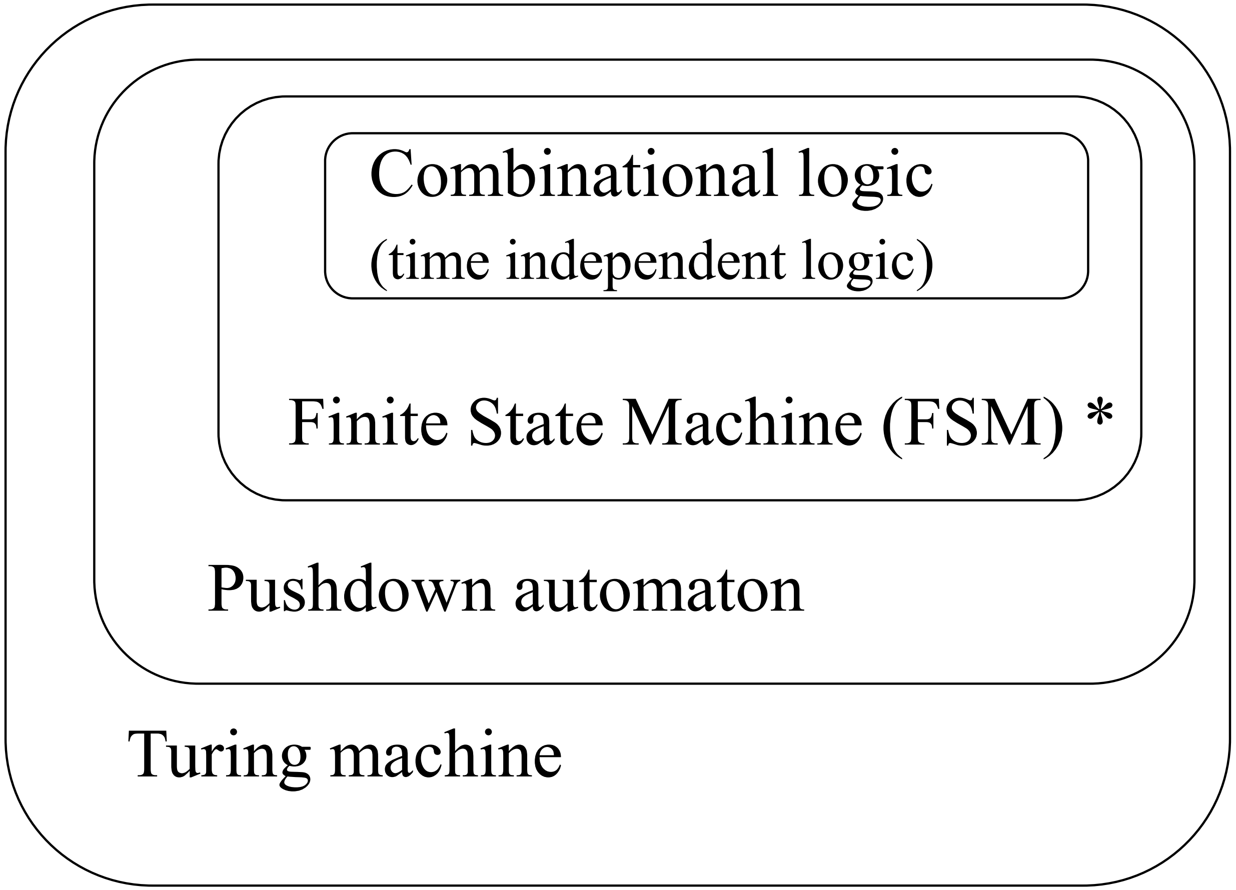
halts after **107 time steps**
fills **13 ones**

States	*Halts after n time steps*	*Fills m ones in the tape*
2	**6**	**4**
3	**21**	**6**
4	**107**	**13**
5	**47,176,870 ?**	**4098 ?**
6	**> 7.4 × 10^{36534}**	**> 3.5 × 10^{18267}**



http://www.catonmat.net/blog/busy-beaver/
http://www.logique.jussieu.fr/~michel/bbc.html
(competition results visited Nov 2016)

Automata theory



* *Time dependency, **memory**, is an essential component for automata.*
*Petri nets will introduce another essential component: **parallelization**.*