

# **Industrial Automation**

## **(Automação de Processos Industriais)**

### **DES and Industrial Automation**

<http://users.isr.ist.utl.pt/~jag/courses/api1213/api1213.html>

Slides 2010/2011 Prof. Paulo Jorge Oliveira  
Rev. 2011-2013 Prof. José Gaspar

# Syllabus:

**Chap. 7 – Analysis of Discrete Event Systems [2 weeks]**

...

**Chap. 8 - DESs and Industrial Automation [1 week]**

GRAFCET / Petri Nets Relation

Model modification

Tools adaptation

Analysis of industrial automation solutions by analogy with  
Discrete Event Systems

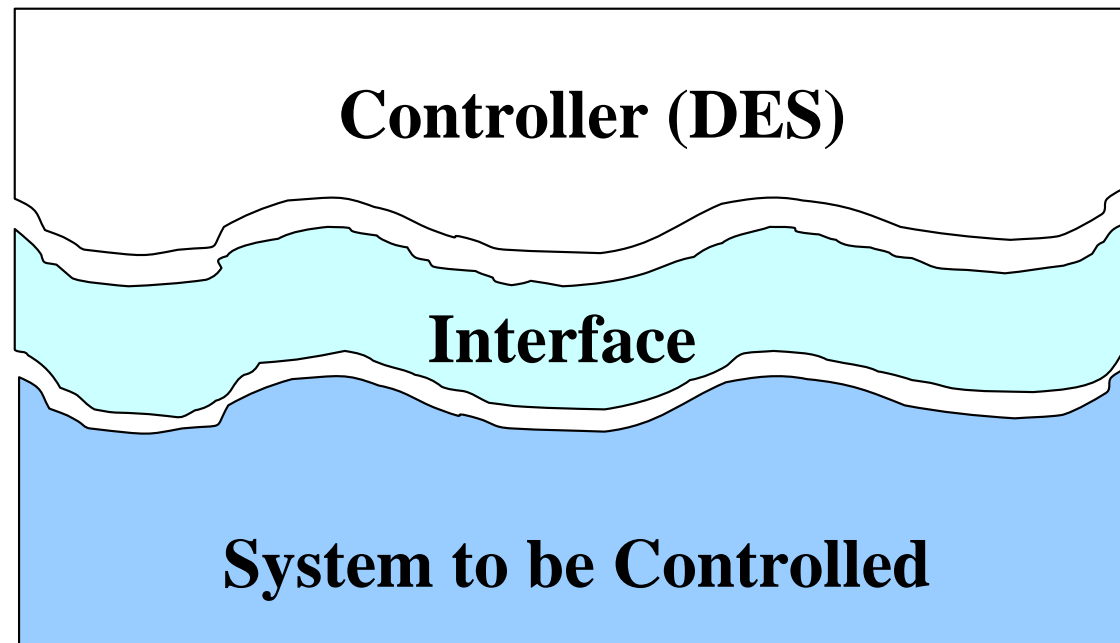
...

**Chap. 9 – Supervision of DESs [1 week]**

## Some pointers to Discrete Event Systems

- History: <http://prosys.changwon.ac.kr/docs/petrinet/1.htm>
- Tutorial: <http://www.eit.uni-kl.de/litz/ENGLISH/members/frey/VnVSurvey.htm>  
<http://vita.bu.edu/cgc/MIDEDS/>  
<http://www.daimi.au.dk/PetriNets/>
- Analysers,  
and  
Simulators: <http://www.ppgia.pucpr.br/~maziero/petri/arp.html> (in Portuguese)  
<http://wiki.daimi.au.dk:8000/cpntools/cpntools.wiki>  
<http://www.informatik.hu-berlin.de/top/pnk/download.html>
- Bibliography: \* **Petri Nets and GRAFCET: Tools for Modelling Discrete Event Systems**  
R. DAVID, H. ALLA, New York : PRENTICE HALL Editions, 1992

## DES Implementation: Models



Besides modelling the **DES** it is required to design **models** of the **System to be controlled** and of the **Interface** to be used

## Given a Discrete Event System how to implement it?

### 1. Use a GRAFCET

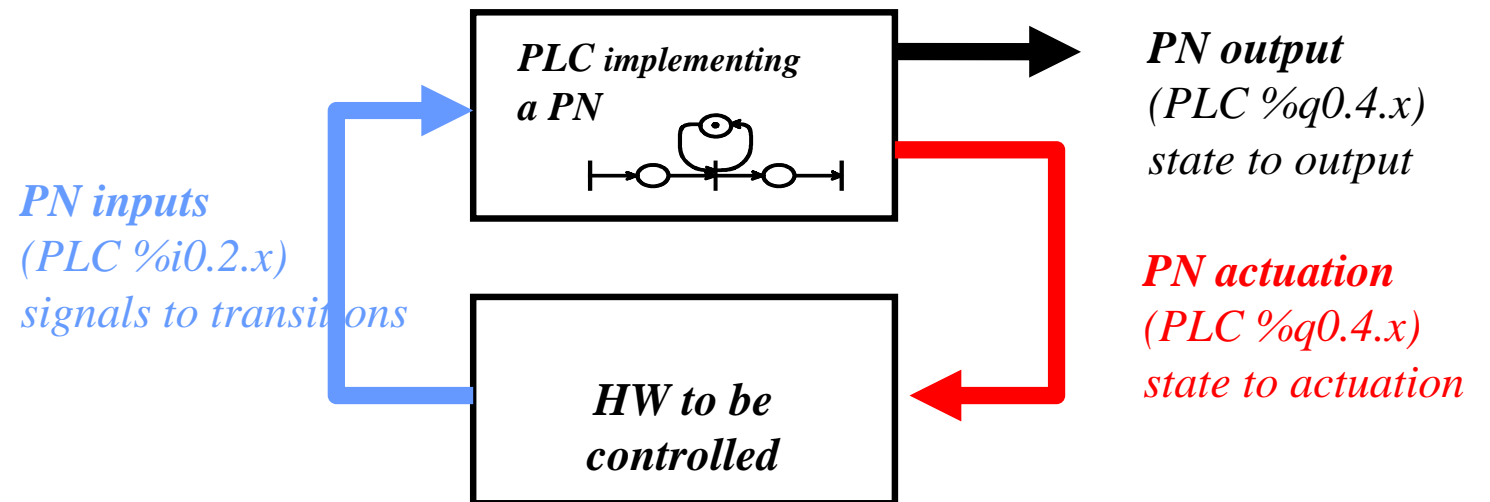
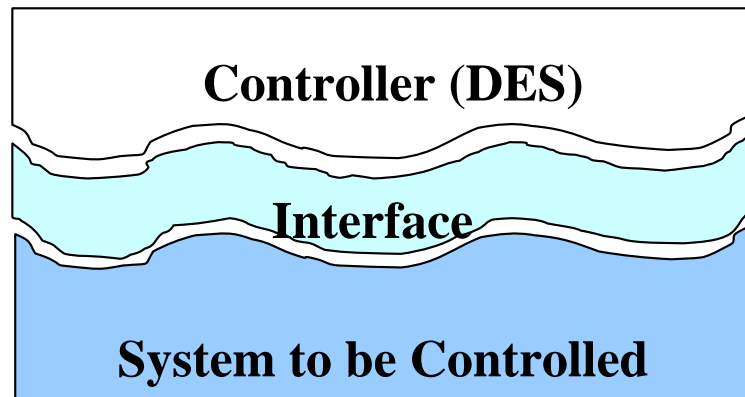
- a) Less modeling ability
- b) Implementation in PLCs is straightforward
- c) **No analysis (or very scarce) methods available**

### 2. Use a Petri Net

- a) More modeling capacity
- b) **No direct implementation in PLCs** (therefore indirect or special software solutions required)
- c) Classical analysis methods available

### (3. Use an Automaton)

# DES Implementation: Petri Net and HW



## Analysis of solutions

### GRAFCET and Petri Nets

#### Similarities to exploit:

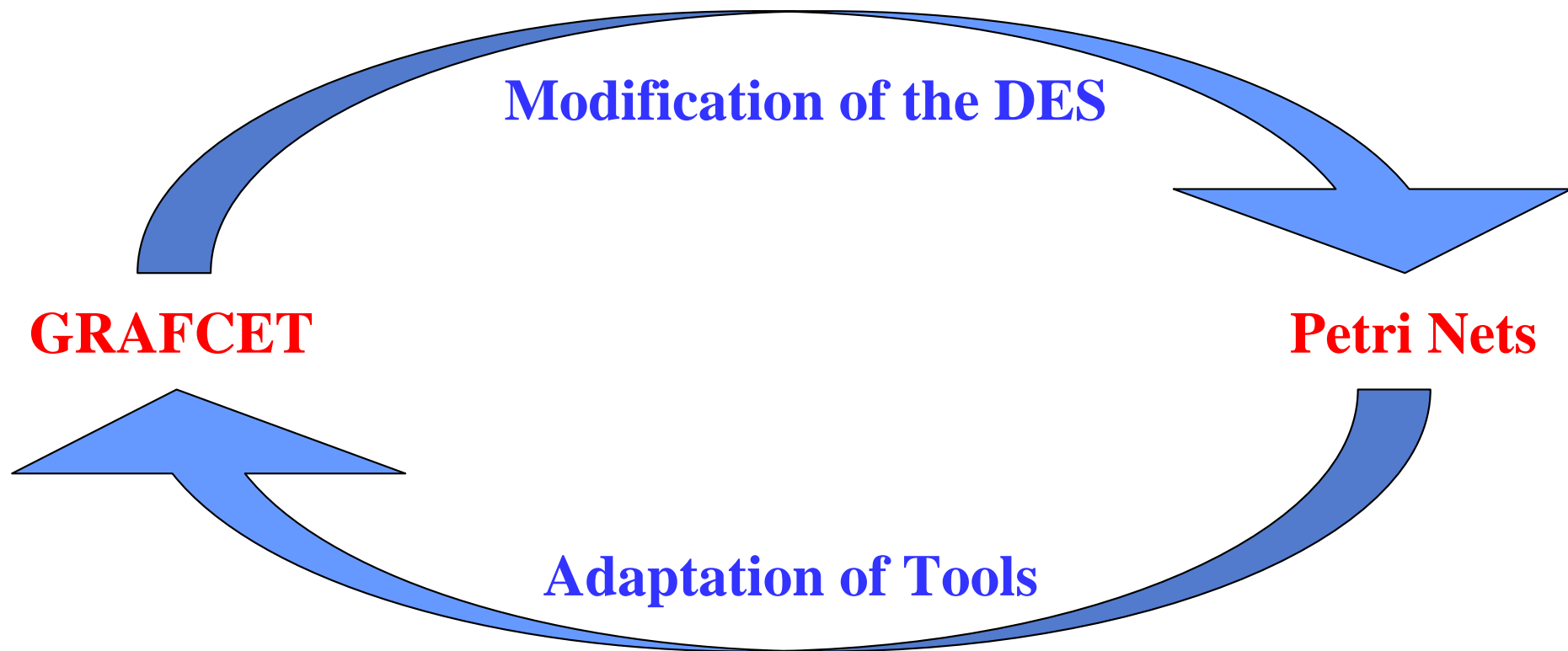
- a) Steps and Places are similar
- b) Transitions compose both tools
- c) Places can be used to implement **counters** (on marking changes) and **binary variables** (marked vs unmarked place)
- d) **Logic** functions can be rewritten resorting to the firing of transitions

#### Differences to be taken into account:

- a) Firing rules (mutual exclusion)
- b) Conflicts
- c) **Binary** activation of stages
- d) Interface with the system to be controlled
- e) Activation functions

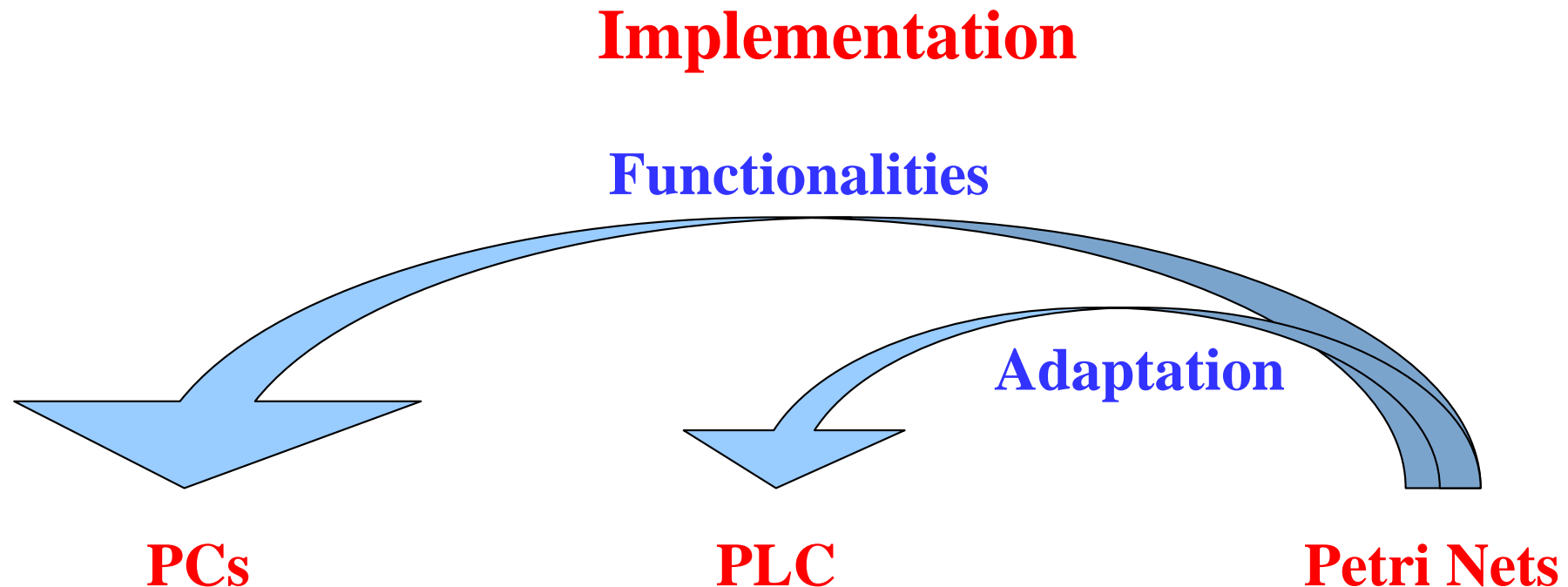
# Implementation of DES using GRAFCET

## Analysis





# Implementation of DES using Petri Nets

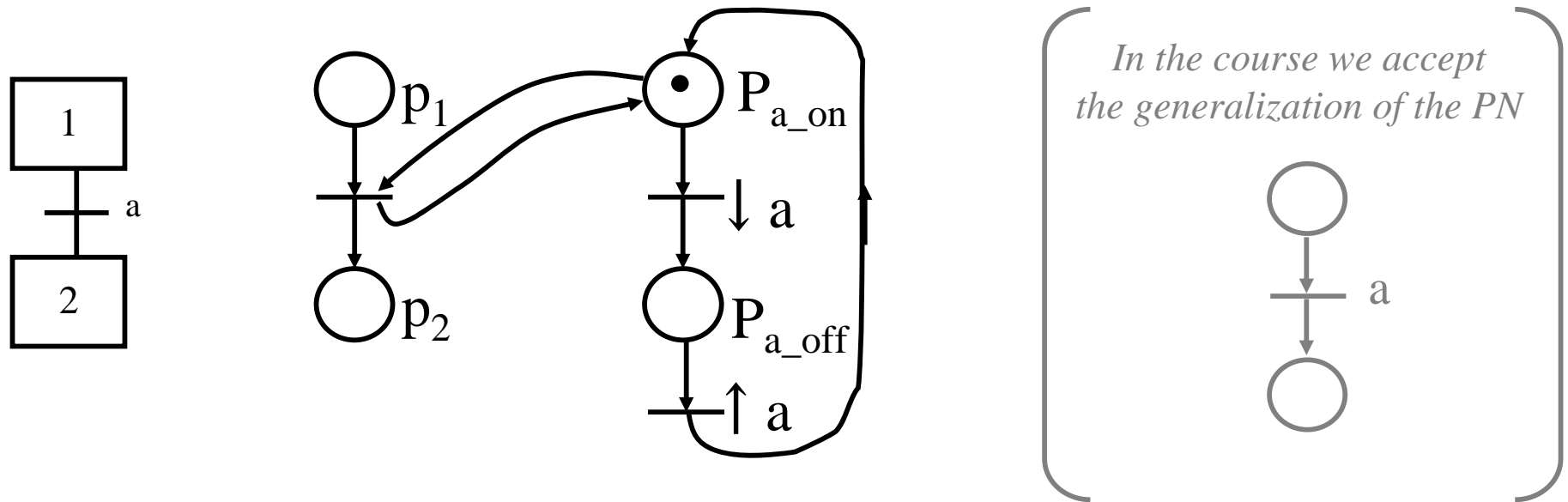


Both solutions are valid  
(but out of the scope of this course).

# Analysis of solutions

## GRAFCET → Petri Nets

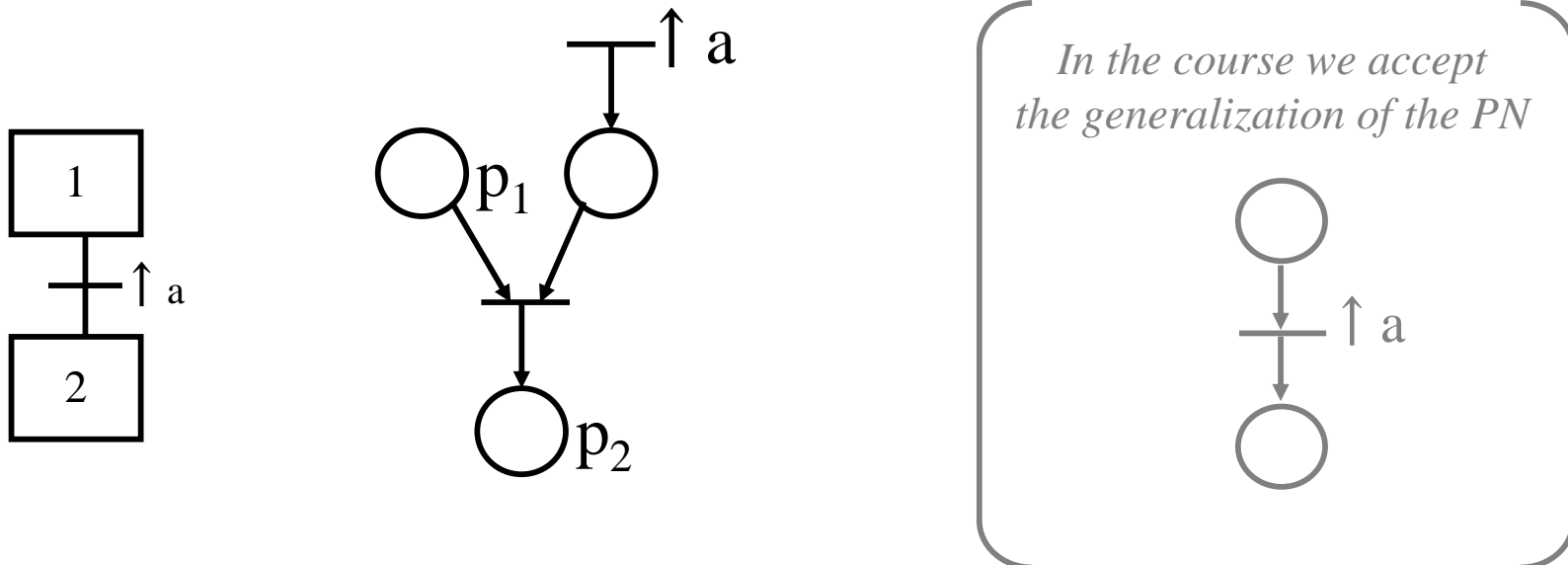
### Representation of variables active on level



## Analysis of solutions

### GRAFCET → Petri Nets

### Representation of variables active at **edge**



## Analysis of solutions

Petri Nets → GRAFCET

### Adaptation of Tools:

- 1) Reachability Tree → Reachability Graph
- 2) Method of the Matrix Equations  
to describe the state evolution

## Petri Nets → GRAFCET

### Reachability Graph

Is a graph containing the **reachable makings**.

Is composed by two types of nodes:

- **terminal**
- **interior**

The **duplicated** nodes are not represented. *They become connected to the respective copies.*

The **symbol infinity ( $\omega$ )** is not used in GRAFCET. *While in Petri nets  $\omega$  is necessary to obtain finite trees, when a marking covers other(s), in GRAFCET the finite number of states renders  $\omega$  unnecessary.*

## Petri Nets → GRAFCET

### Reachability Graph

**Theorem** - If a reachability graph has **terminal nodes** then the corresponding GRAFCET has **deadlocks**.

*This reachability graph will be used to study the properties introduced in Chapter 6.*

## Petri Nets → GRAFCET

### Reachable Set

Given the GRAFCET  $G=(S, T, I, O, \mu_0)$  with initial marking  $\mu_0$ , the set of all markings that are reachable is the **reachable set**  $\mu' \in R(C, \mu)$ .

*Remark: the Reachable Set **is not infinite!***

*Given a GRAFCET with  $m$  steps it has at most  $2^m$  nodes.*

## Petri Nets → GRAFCET

### Safeness, Boundedness and Limitation

The GRAFCET  $G=(S, T, I, O, \mu_0)$  is always safe

*The GRAFCET may become not safe if one uses some auxiliary elements such as counters or buffers.*

*If one uses these auxiliary elements, the analysis methods studied for Petri Nets can be used to assert specific safeness, boundedness and limitation properties.*



## Petri Nets → GRAFCET

### Conservation

A GRAFCET  $G=(S, T, I, O, \mu_0)$  is **strictly conservative** if for all  $\mu' \in R(C, \mu)$

$$\sum_{p_i \in P} \mu'(p_i) = \sum_{p_i \in P} \mu(p_i).$$

A GRAFCET  $G=(S, T, I, O, \mu_0)$  is **conservative** if there exist a weight vector  $\omega$ , without null elements, for all  $\mu' \in R(C, \mu)$  such that it is constant the quantity

$$\sum_{p_i \in P} \omega(p_i) \mu(p_i).$$

## Petri Nets $\rightarrow$ GRAFCET

**Liveness of transitions:** The transition  $t_j$  is live of

**Level 0** - it can never be fired (dead transition).

**Level 1** - if it is potentially firable, e.g. if there exist  $\mu' \in R(C, \mu)$  such that  $t_j$  is enabled in  $\mu'$ .

**Level 2** - if, for each positive  $n$ , there exist a sequence of firings where occurs  $n$  firings of  $t_j$ .

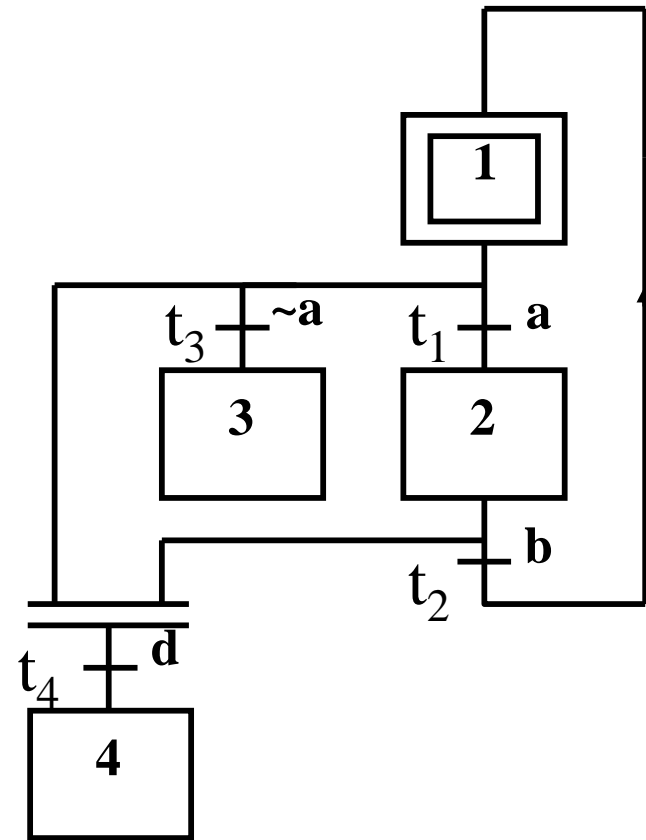
**Level 3** - if there exist a sequence of firings where an infinite number of firings of  $t_j$  occurs.

**Level 4** - if for each  $\mu' \in R(C, \mu)$  there exist a sequence  $\sigma$  that enables the firing of  $t_j$  (live transition).

## Petri Nets → GRAFCET

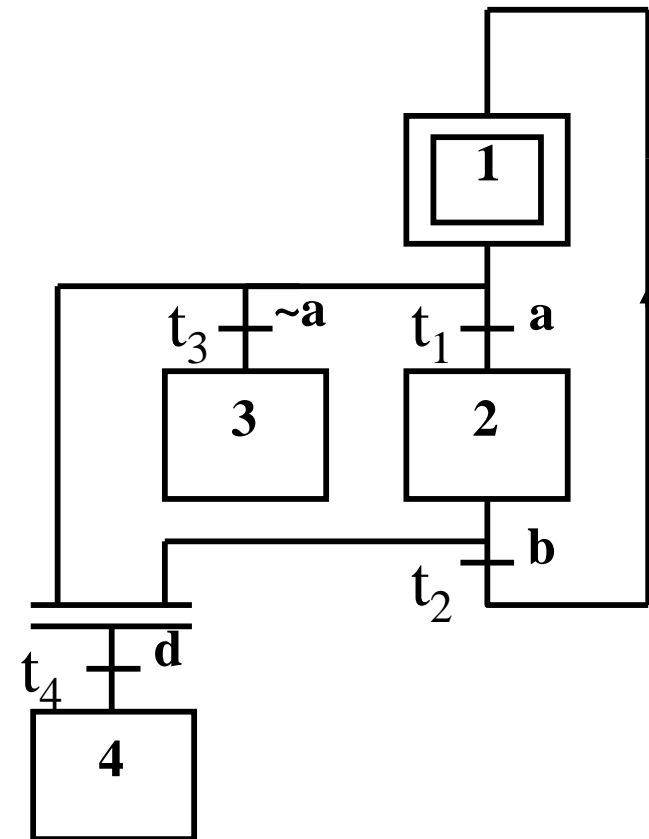
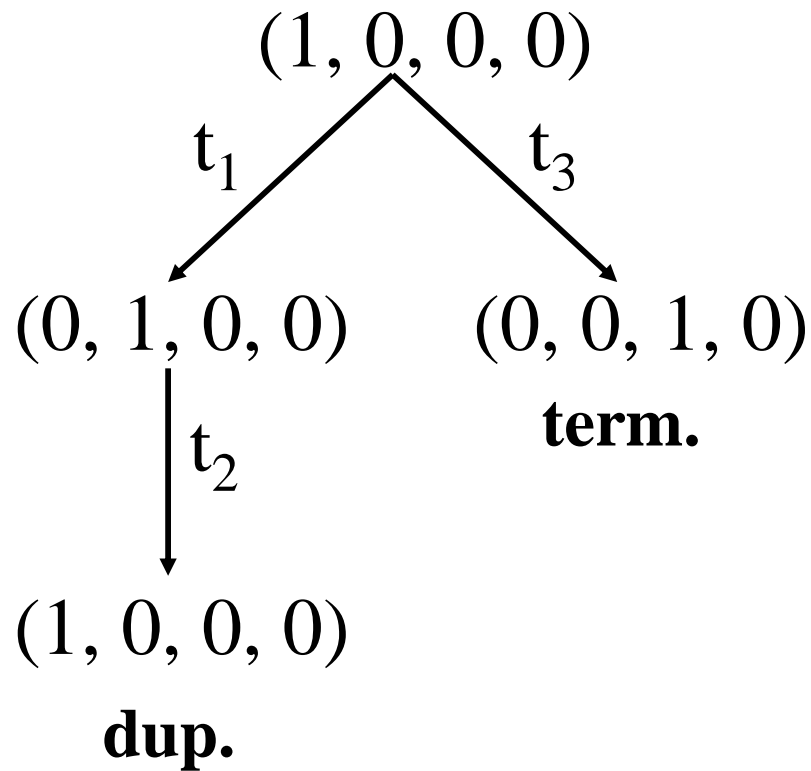
### Example of GRAFCET

- $t_1$  is level 3.
- $t_2$  is level 3.
- $t_3$  is level 1.
- $t_4$  is level 0.



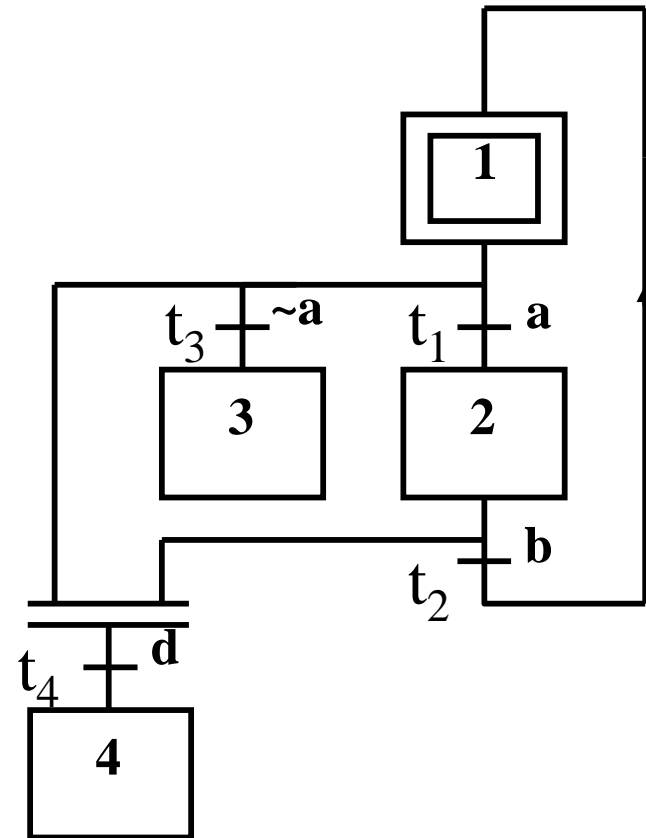
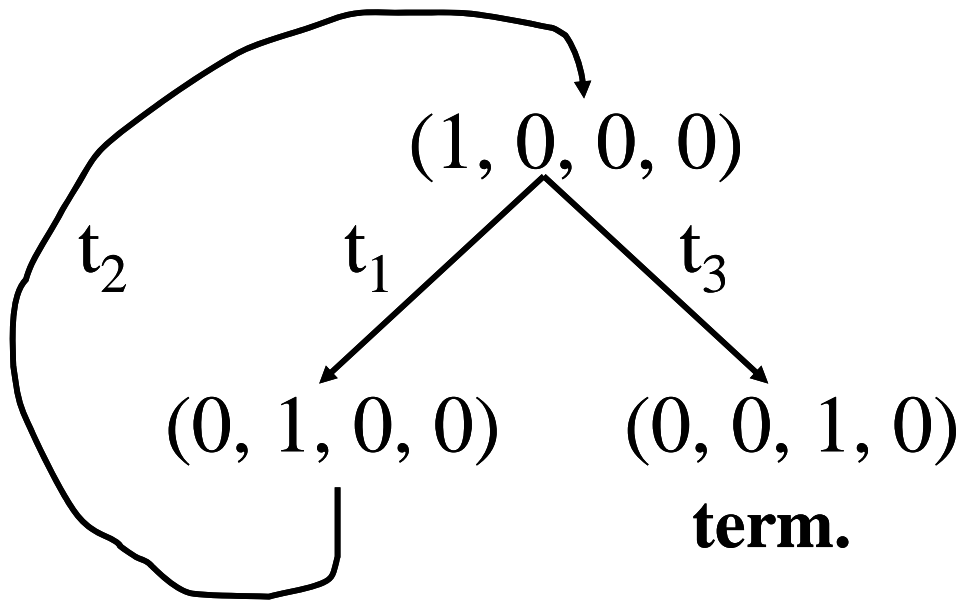
## Petri Nets → GRAFCET

### Example of GRAFCET



**Petri Nets → GRAFCET**

**Example of GRAFCET**



**Strictly conservative.**

## Petri Nets → GRAFCET

### Method of Matrix Equation (for the state evolution)

The evolution of a GRAFCET can be written in compact form as:

$$\mu' = \mu + Dq$$

where:

- $\mu'$  - Desired marking vector (column vector)
- $\mu$  - Initial marking
- $q$  - Transition firing vector (column vector)
- $D$  - Incidence matrix. Accounts for the token evolution as a consequence of transitions firing.

## Petri Nets → GRAFCET

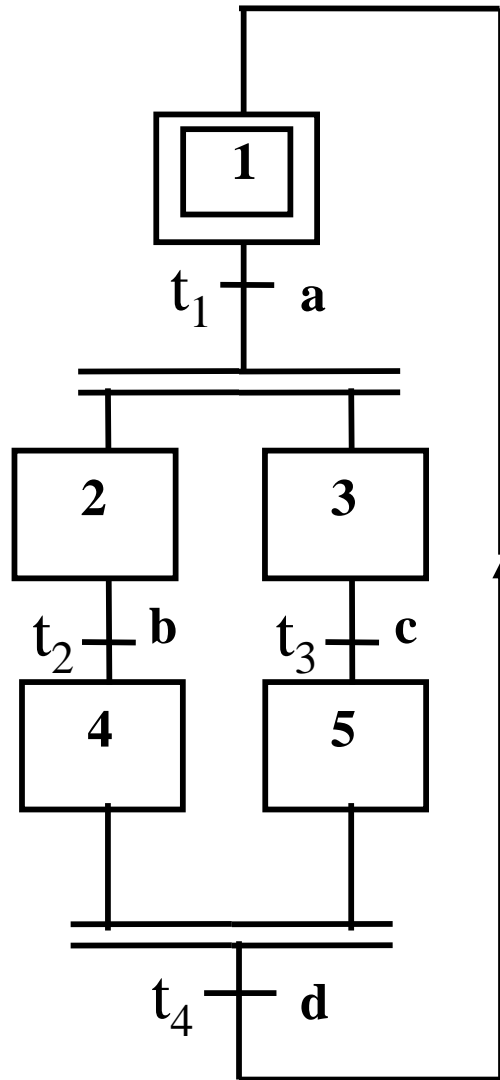
### Additional problems that can be addressed resorting to the Method of Matrix Equations:

- **Reachability** (sufficient condition)

**Theorem** – if the problem of finding the vector of firings, for a GRAFCET without conflicts, from the state  $\mu$  to the state  $\mu'$  has no solution using the Method of Matrix Equations, then the problem of reachability of  $\mu'$  is impossible.

- **Conservation** – the conservation vector can be computed automatically.
- **Temporal invariance** – cycles of operation can be found.

### Example of GRAFCET



$$\mu' = \mu + Dq$$

$$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Conservation

$$x^T D = 0$$

$$\begin{cases} -x_1 + x_2 + x_3 = 0 \\ -x_2 + x_4 = 0 \\ -x_3 + x_5 = 0 \\ x_1 - x_4 - x_5 = 0 \end{cases}$$

$$x_1 = x_3 + x_4$$

$$x_1 = x_2 + x_5$$

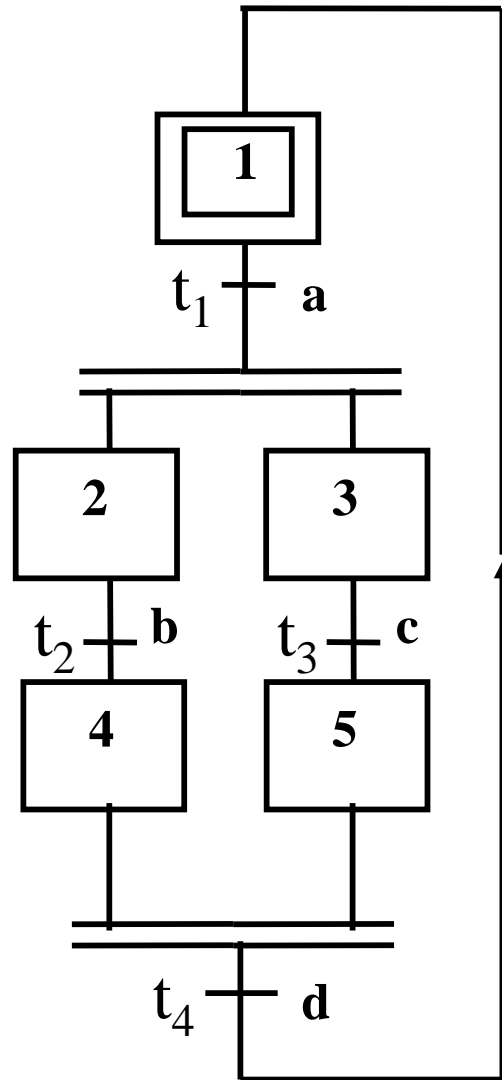
$$x_2 + x_3 = x_4 + x_5$$

$$x = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Solution:  
Undetermined  
set of equations



Example of GRAFCET



$$\mu' = \mu + Dq$$

$$Dq = 0$$

$$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad q = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{bmatrix}$$

Temporal invariance

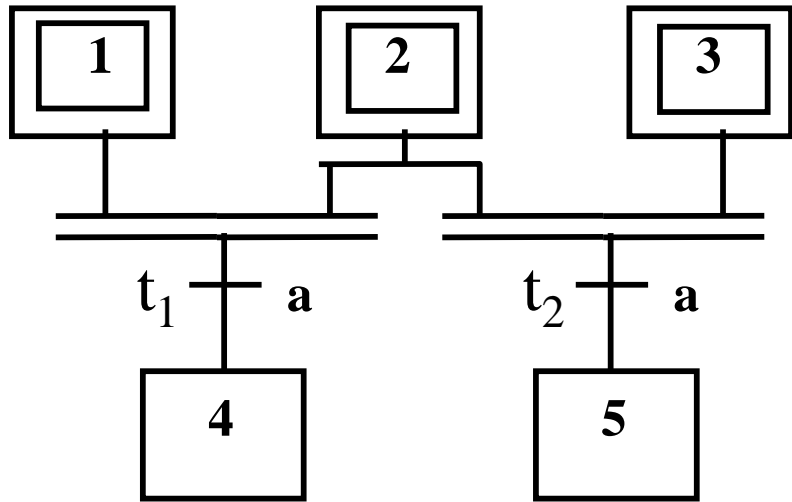
Solution:  
Set of equations  
with solution

$$\begin{cases} -\sigma_1 + \sigma_4 = 0 \\ \sigma_1 - \sigma_2 = 0 \\ \sigma_1 - \sigma_3 = 0 \\ \sigma_2 - \sigma_4 = 0 \\ \sigma_3 - \sigma_4 = 0 \end{cases}$$

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 1.$$

Example of GRAFCET

$$\mu' = \mu + Dq$$



$$\mu' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 & 0 \\ -1 & -1 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$q = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$$

Set of Equations is **impossible**  
therefore the marking is **not reachable...**

**WRONG !**

***The method fails if there are conflicts!***

$$\left\{ \begin{array}{l} 0 = 1 - \sigma_1 \\ 0 = 1 - \sigma_1 - \sigma_2 \\ 0 = 1 - \sigma_2 \\ 1 = \sigma_1 \\ 1 = \sigma_2 \end{array} \right.$$