

# **Industrial Automation**

## **(Automação de Processos Industriais)**

### **DES and Industrial Automation**

<http://users.isr.ist.utl.pt/~jag/courses/api1112/api1112.html>

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Rev. 2011/2012 Prof. José Gaspar

# Syllabus:

**Chap. 7 – Analysis of Discrete Event Systems [2 weeks]**

...

**Chap. 8 - DESs and Industrial Automation [1 week]**

GRAFCET / Petri Nets Relation

Model modification

Tools adaptation

Analysis of industrial automation solutions by analogy with  
Discrete Event Systems

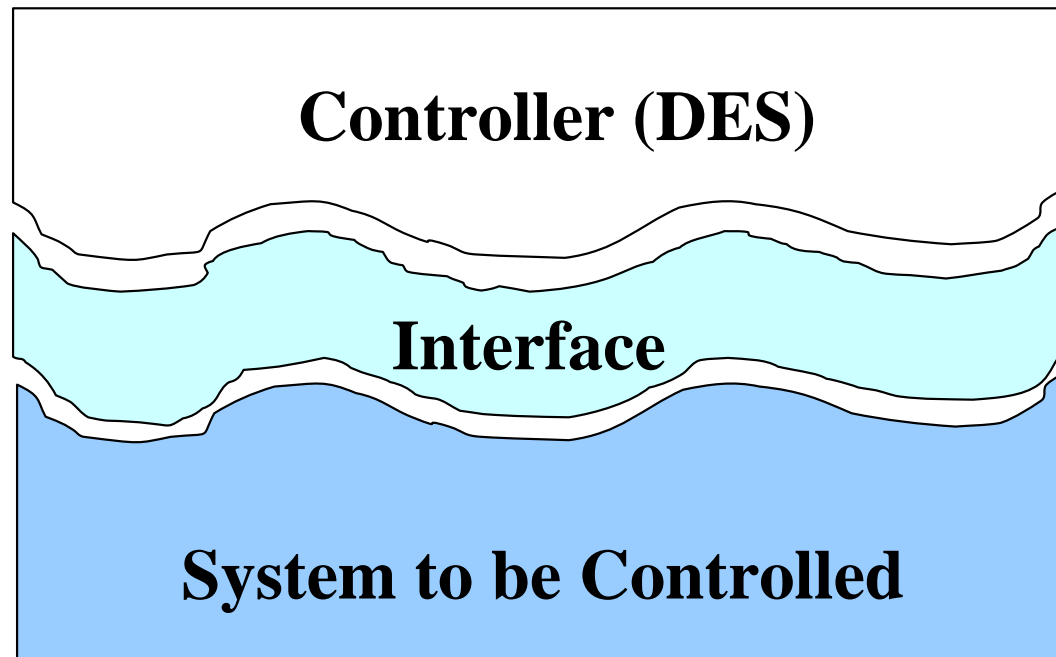
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**Chap. 9 – Supervision of DESs [1 week]**

## Some pointers to Discrete Event Systems

- History: <http://prosys.changwon.ac.kr/docs/petrinet/1.htm>
- Tutorial: <http://www.eit.uni-kl.de/litz/ENGLISH/members/frey/VnVSurvey.htm>  
<http://vita.bu.edu/cgc/MIDEDS/>  
<http://www.daimi.au.dk/PetriNets/>
- Analysers,  
and  
Simulators: <http://www.ppgia.pucpr.br/~maziero/petri/arp.html> (in Portuguese)  
<http://wiki.daimi.au.dk:8000/cpntools/cpntools.wiki>  
<http://www.informatik.hu-berlin.de/top/pnk/download.html>
- Bibliography: \* **Petri Nets and GRAFCET: Tools for Modelling Discrete Event Systems**  
R. DAVID, H. ALLA, New York : PRENTICE HALL Editions, 1992

## DES Implementation: **Models**



Besides modelling the **DES** it is required to design **models** of the **System to be controlled** and of the **Interface** to be used

## Given a Discrete Event System how to implement it?

### 1. Use a GRAFCET

- a) Less modeling ability
- b) Implementation in PLCs is straightforward
- c) **No analysis (or very scarce) methods available**

### 2. Use a Petri Net

- a) More modeling capacity
- b) **No direct implementation in PLCs** (therefore indirect or special software solutions required)
- c) Classical analysis methods available

### (3. Use an Automaton)

## Analysis of solutions

### GRAFCET and Petri Nets

#### Similarities to exploit:

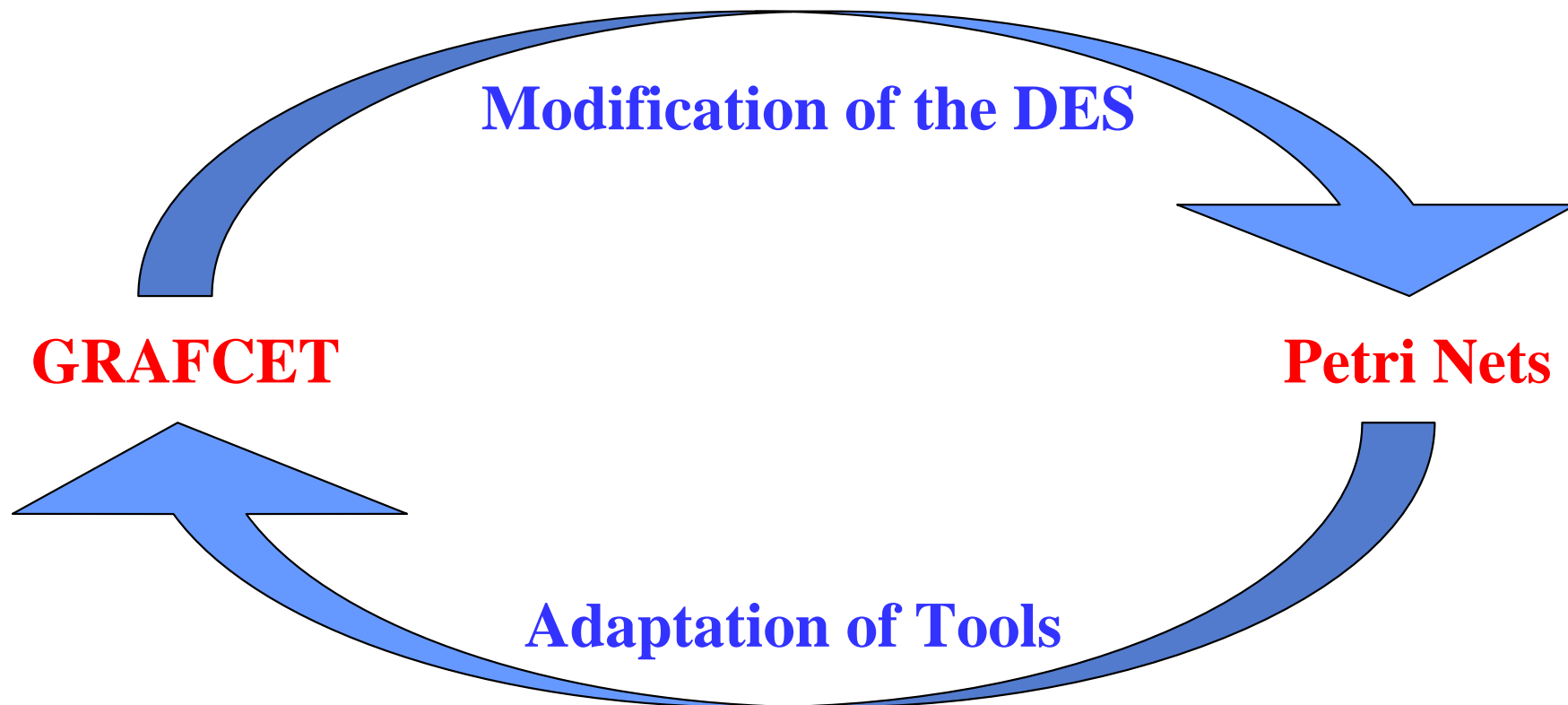
- a) Places and steps are similar
- b) Transitions compose both tools
- c) Places can be used to implement counters and binary variables
- d) Logic functions can be rewritten resorting to the firing of transitions

#### Differences to be taken into account:

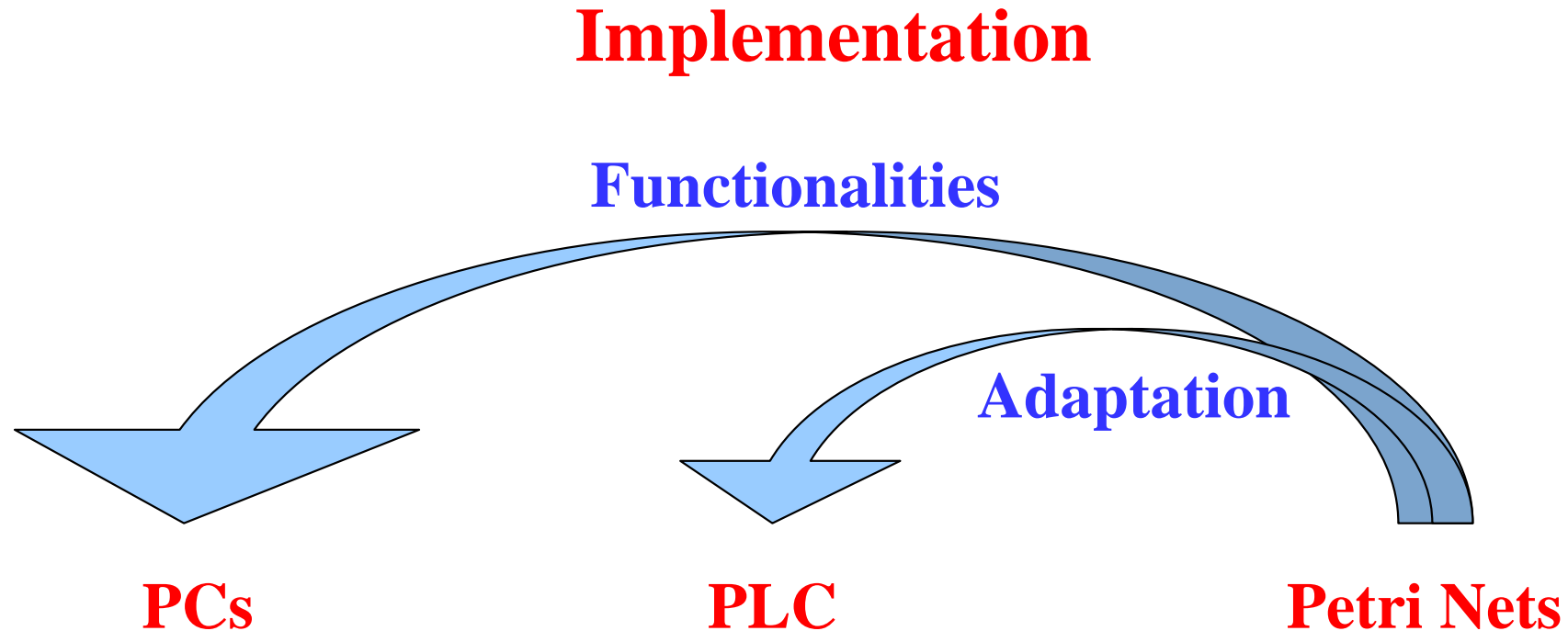
- a) Firing rules (mutual exclusion)
- b) Conflicts
- c) Binary activation of stages
- d) Interface with the system to be controlled
- e) Activation functions

# Implementation of DES using GRAFCET

## Analysis



# Implementation of DES using Petri Nets



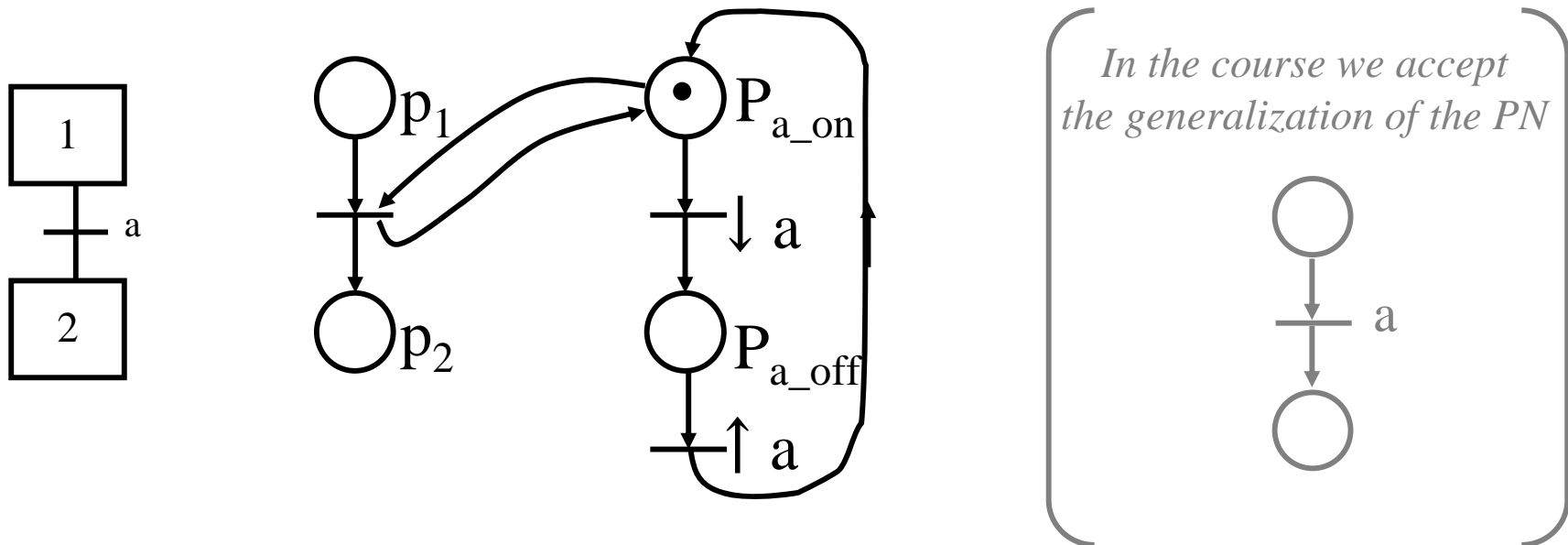
**Both solutions are valid.  
Out of the scope of this course.**



# Analysis of solutions

## GRAFCET → Petri Nets

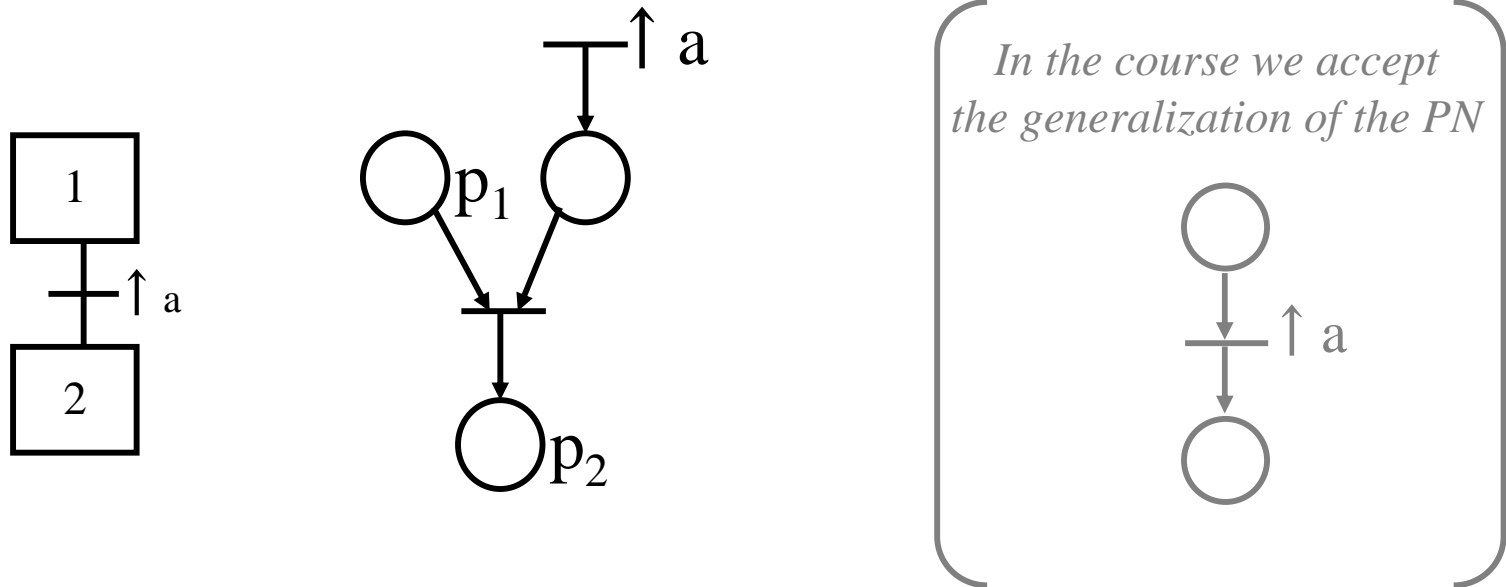
### Representation of variables active on level



## Analysis of solutions

### GRAFCET → Petri Nets

### Representation of variables active at edge



# Analysis of solutions

Petri Nets → GRAFCET

## Adaptation of Tools:

- 1) Reachability Tree → Reachability Graph
- 2) Method of the Matrix Equations  
to describe the state evolution

## Petri Nets → GRAFCET

### Reachability Graph

Is a graph containing the **reachable makings**.

Is composed by two types of nodes:

- terminal
- interior

The duplicated nodes are not represented.

They become connected to the respective copies.

~~The symbol infinity ( $\omega$ ) is introduced,  
to obtain finite trees, when a marking covers other(s).~~

## Petri Nets → GRAFCET

### Reachability Graph

Theorem - If a reachability graph has terminal nodes then the corresponding GRAFCET has deadlocks.

This method will be used to study the properties introduced in Chapter 6.

## Petri Nets → GRAFCET

### Reachable Set

Given the GRAFCET  $G=(S, T, I, O, \mu_0)$  with initial marking  $\mu_0$ , the set of all markings that are reachable is the **reachable set**  $\mu' \in R(C, \mu)$ .

Remark: the Reachable Set **is not infinite!**

Given a GRAFCET with  $m$  steps it has at most  $2^m$  nodes.

## Petri Nets → GRAFCET

### Boundness and Limitation

The GRAFCET  $G=(S, T, I, O, \mu_0)$  is always secure!

The same does not occur with some auxiliary elements of the GRAFCET, e.g., counters and buffers.

For those elements the analysis methods studied for Petri Nets can be used directly.

## Petri Nets → GRAFCET

### Conservation

A GRAFCET  $G=(S, T, I, O, \mu_0)$  is **strictly conservative** if for all  $\mu' \in R(C, \mu)$

$$\sum_{p_i \in P} \mu'(p_i) = \sum_{p_i \in P} \mu(p_i).$$

A GRAFCET  $G=(S, T, I, O, \mu_0)$  is **conservative** if there exist a weight vector  $\omega$ , without null elements, for all  $\mu' \in R(C, \mu)$  such that it is constant the quantity

$$\sum_{p_i \in P} \omega(p_i) \mu(p_i).$$



## Petri Nets → GRAFCET

**Liveness of transitions:** The transition  $t_j$  is live of

**Level 0** - it can never be fired.

**Level 1** - if it is potentially firable, e.g. if there exist  $\mu' \in R(C, \mu)$  such that  $t_j$  is enabled in  $\mu'$ .

**Level 2** - if, for each positive  $n$ , there exist a sequence of firings where occurs  $n$  firings of  $t_j$ .

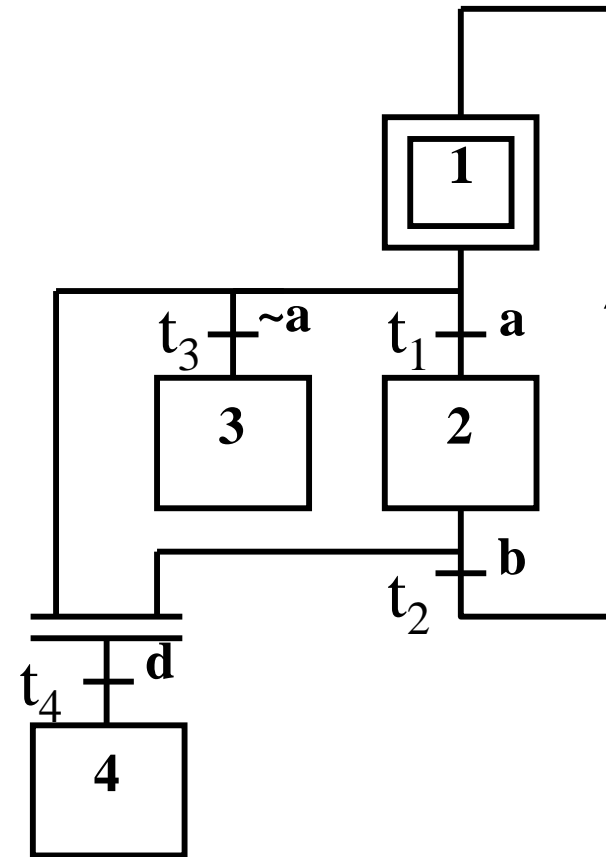
**Level 3** - if there exist a sequence of firings where an infinite number of firings of  $t_j$  occurs.

**Level 4** - if for each  $\mu' \in R(C, \mu)$  there exist a sequence  $\sigma$  that enables the firing of  $t_j$ .

## Petri Nets $\rightarrow$ GRAFCET

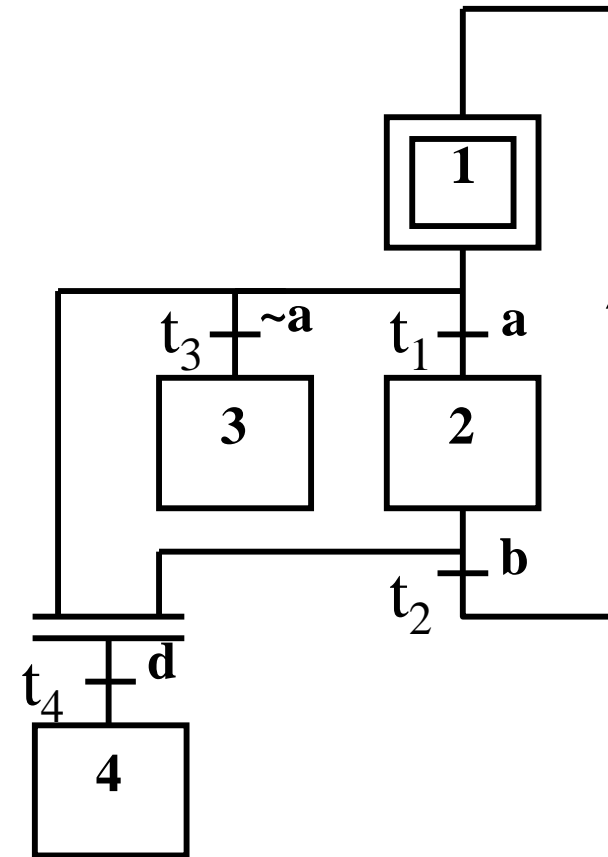
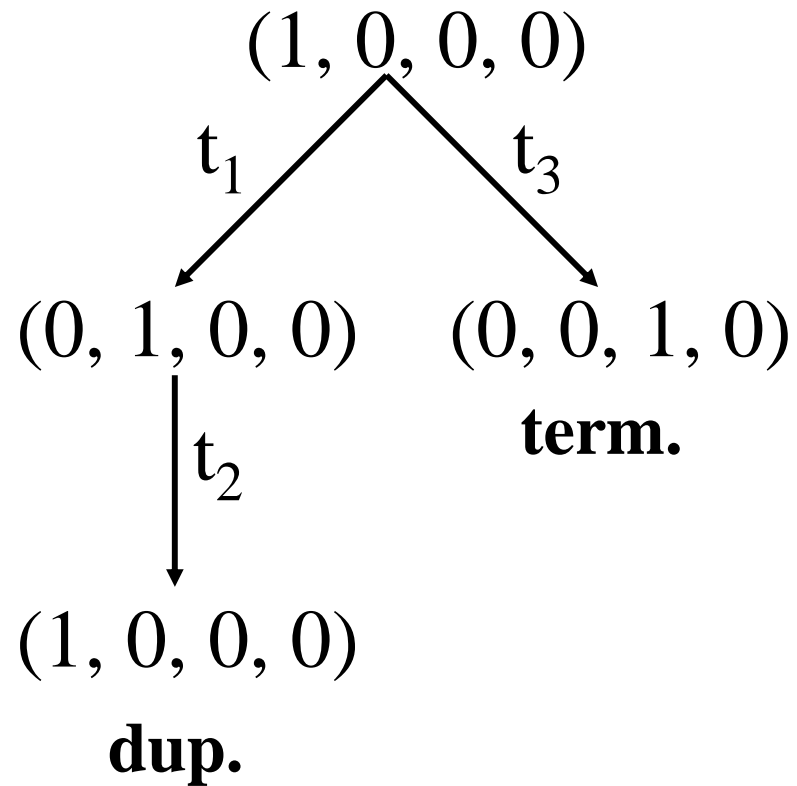
### Example of GRAFCET

- $t_4$  is level 0.
- $t_1$  is level 3.
- $t_2$  is level 3.
- $t_3$  is level 1.



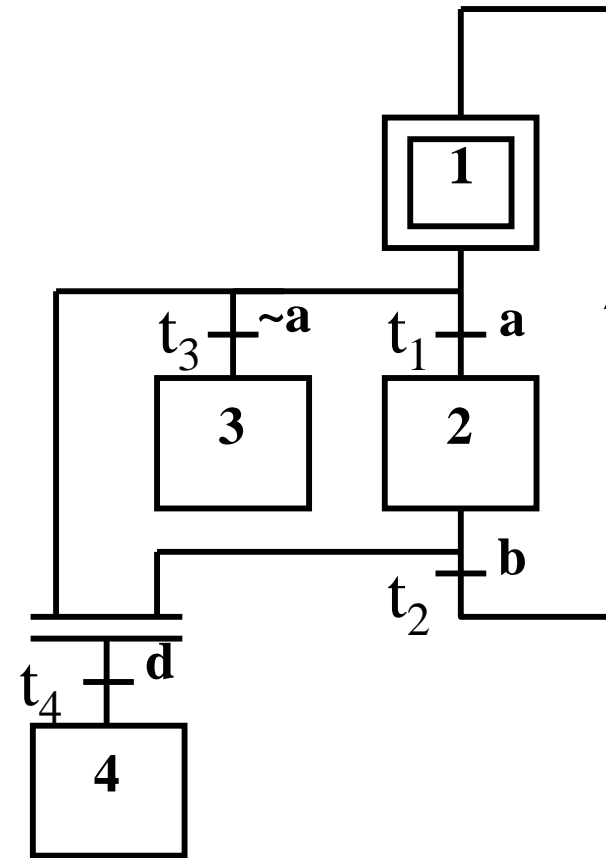
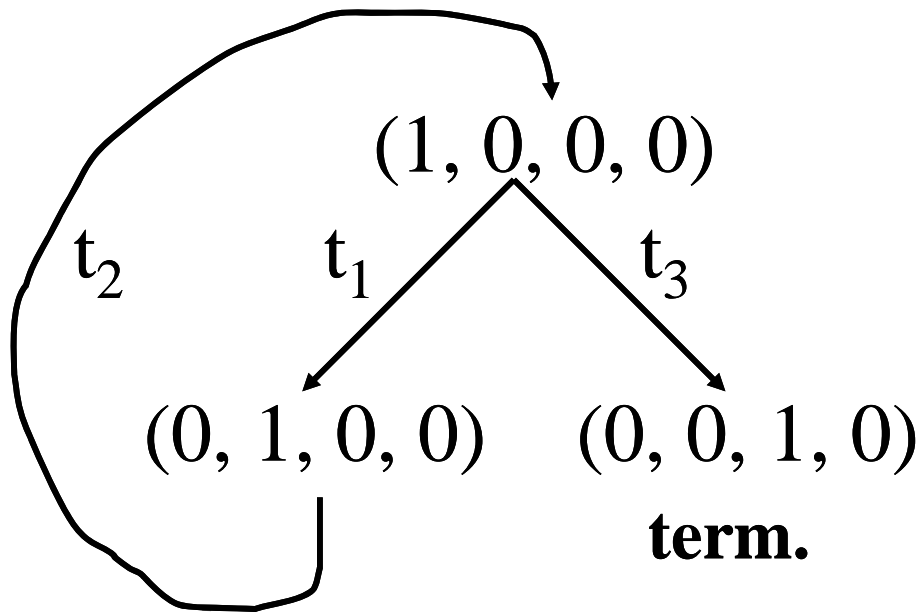
## Petri Nets $\rightarrow$ GRAFCET

### Example of GRAFCET



Petri Nets → GRAFCET

Example of GRAFCET



Strictly conservative.

## Petri Nets → GRAFCET

### Method of Matrix Equation (for the state evolution)

The evolution of a GRAFCET can be written in compact form as:

$$\mu' = \mu + Dq$$

where:

- $\mu'$  - desired marking (vector column vector)
- $\mu$  - initial marking
- $q$  - column vector of the transition firings
- $D$  - incidence matrix. Accounts for the token evolution as a consequence of transitions firing.

## Petri Nets → GRAFCET

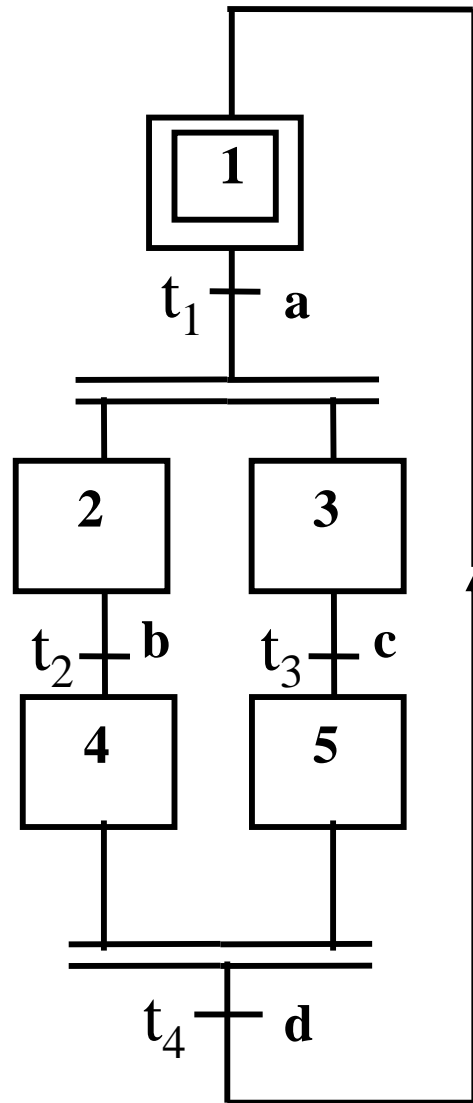
### Problems that can be addressed resorting to the Method of Matrix Equations

- **Reachability** (sufficient condition)

Theorem – if the problem of finding the vector of firings, for a GRAFCET without conflicts, from the state  $\mu$  to the state  $\mu'$  has no solution using the Method of Matrix Equations, then the problem of reachability of  $\mu'$  is impossible.

- **Conservation** – the conservation vector can be computed automatically.
- **Temporal invariance** – cycles of operation can be found.

### Example of GRAFCET



$$\mu' = \mu + Dq$$

$$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Conservation  $x^T D = 0$

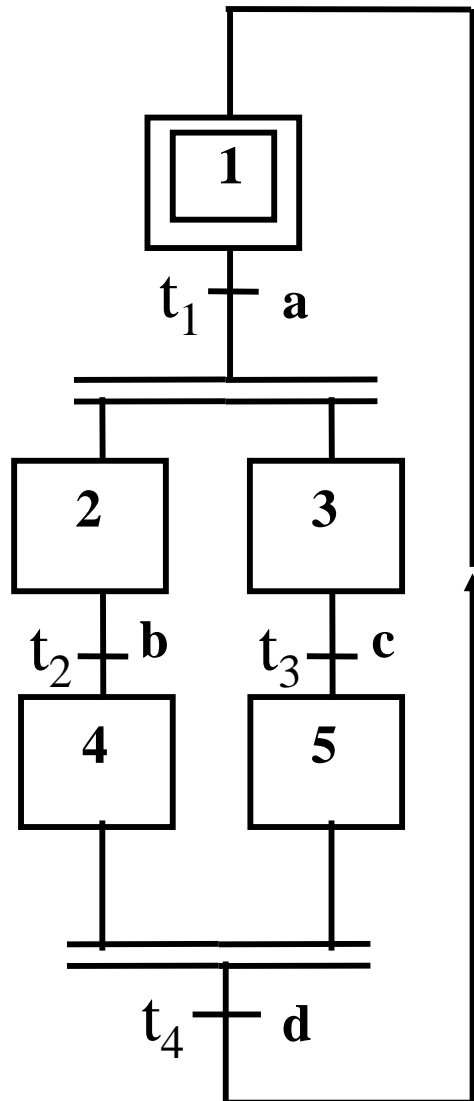
$$\begin{cases} -x_1 + x_2 + x_3 = 0 \\ -x_2 + x_4 = 0 \\ -x_3 + x_5 = 0 \\ x_1 - x_4 - x_5 = 0 \end{cases}$$

$$\begin{aligned} x_1 &= x_3 + x_4 \\ x_1 &= x_2 + x_5 \\ x_2 + x_3 &= x_4 + x_5 \end{aligned}$$

Solution:  
Undetermined  
set of equations

$$x = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

**Example of GRAFCET**



$$\mu' = \mu + Dq$$

$$Dq = 0$$

$$D = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad q = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{bmatrix}$$

**Temporal invariance**

Solution:  
Set of equation  
with one solution

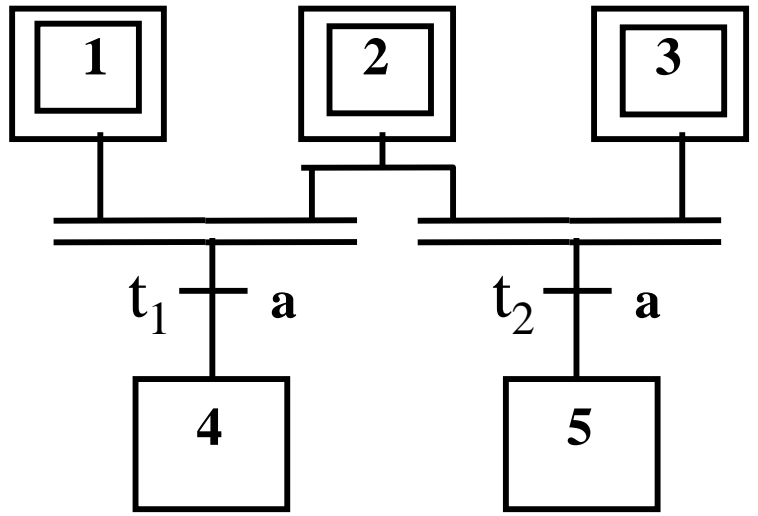
$$\begin{cases} -\sigma_1 + \sigma_4 = 0 \\ \sigma_1 - \sigma_2 = 0 \\ \sigma_1 - \sigma_3 = 0 \\ \sigma_2 - \sigma_4 = 0 \\ \sigma_3 - \sigma_4 = 0 \end{cases}$$

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 1.$$



**Example of GRAFCET**

$$\mu' = \mu + Dq$$



$$\mu' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 & 0 \\ -1 & -1 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$q = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$$

**Set of Equations impossible  
Therefore marking not reachable.**

**WRONG!**

$$\left\{ \begin{array}{l} 0 = 1 - \sigma_1 \\ 0 = 1 - \sigma_1 - \sigma_2 \\ 0 = 1 - \sigma_2 \\ 1 = \sigma_1 \\ 1 = \sigma_2 \end{array} \right.$$

**The method fails if there are conflicts!**