Nonlinear IBVS Controller for the Flare Maneuver of Fixed-Wing Aircraft using Optical Flow

Pedro Serra, Florent Le Bras, Tarek Hamel, Carlos Silvestre and Rita Cunha

***Abstract*— This paper describes a nonlinear Image-Based Visual Servo (IBVS) controller for the Flare phase of the landing maneuver of a fixed-wing aircraft in the presence of wind gust. Optic-Flow and 2D image features are exploited from the image of the runway to design a feedback controller for the automatic maneuver. The controller is divided into two parts. The first part guarantees the horizontal alignment with the center of the runway and uses the two lines delimiting the runway represented through a modification of the so-called Plücker coordinates. The second part takes advantage of the Optic-Flow measurements to ensure a smooth touchdown. Simulation results are presented to illustrate the performance of the control approach.**

I. Introduction

Unmanned Aerial Vehicles (UAVs) have matured into a major research topic over the last decade. Significant effort has been placed on the development of both fixed-wing and rotary-wing aircrafts. As new sensor technology and increasingly powerful computational systems become available, their potential to perform high precision tasks in challenging and uncertain operation scenarios increases, demanding efficient motion control algorithms to perform all kinds of challenging maneuvers autonomously.

One major problem when designing control systems is the difficulty to accurately measure the vehicle's position with respect to the local environment. Nowadays, GPS (Global Positioning System) is being widely used as the primary navigation aid in most algorithms, see for example [1] and [2]. However, this approach presents some drawbacks. The GPS provides positioning information in the Earth-Centered, Earth-Fixed frame (ECEF) without considering local topography. The GPS measurement rate is not sufficient for some applications and the quality of the height's measure is poor. Also GPS signals are subjected to shortages in environments with many occlusions, for example, urban environments, and are vulnerable to jamming effects. For these reasons there has been an increasing interest in developing alternative systems that provide robust relative pose information to be used instead of GPS in navigation algorithms. One alternative to GPS is the use of a vision system.

Using cameras as primary sensors for relative position, the flight control problem can be cast into an Image-Based Visual Servo (IBVS) Control problem ([3], [4]), opening the possibility to perform autonomous tasks in low-structured environments with no external assistance, [5], [6]. Aircraft landing is an example of such an application, for which it would be interesting to develop control algorithms to perform the maneuver without assistance and ground equipment. The landing maneuver of an airplane is composed of four phases, see Figure 1:

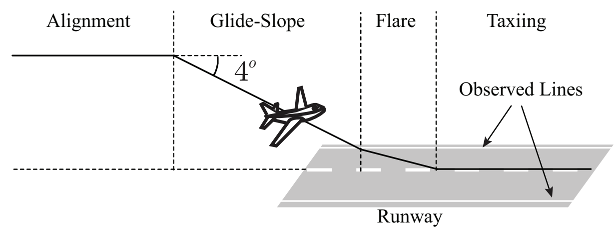

Fig. 1. Landing maneuvre

- Alignment: the airplane has to align with the runway and maintain a fixed desired altitude from the ground;
- Glide-slope: the airplane follows a straight-line descending path, while keeping the alignment with respect to the runway;
- Flare: when the airplane approaches the runway (at about a 20-meter distance for a Jet sized aircraft), a specific flare maneuver begins to lower the glide-path angle and ensure a touchdown with minimal vertical velocity;
- Taxiing: the last phase of the landing maneuver begins when the airplane touches the runway and acts as ground vehicle reducing its velocity.

This paper proposes a vision-based strategy to approach the problem of fixed-wing aircraft landing, addressing in particular the flare phase. This phase is the most critical and requires a well suited controller to ensure a smooth and damage-free touchdown.

The control architecture is decoupled into an inner-loop and outer-loop controller. The outer-loop controller stabilizes the translational (or guidance) dynamics resorting to visual data and using the sideslip angle and the angle of attack as control inputs. The inner-loop controller actuates on the aircraft control surfaces and provides high-gain stabilization of the vehicle's angle of attack, side-slip and roll angles based on direct measurements of the IMU and pitot tubes. The time-scale separation between the two loops is considered sufficient so that the interaction terms can be ignored in the control design. Detailed studies on the inner/outer loop approach of controllers design can be found in [7] and [8].

In [9] the authors present an IBVS controller for the Alignment and Glide-slope phases of the landing maneuver using the Plücker coordinates of the lines delimitating the runway. The proposed controller is divided in two components, the

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first uses new modified bi-normalized Plücker coordinates to maintain the airplane aligned with the center of the runway. The second component uses the vertical component of the optical-flow, also called optical-flow divergence, [10], and guarantees a smooth touchdown resorting to a specially suited Lyapunov function.

This paper is structured in four sections. Section II presents the dynamic model of an airplane. Section III presents the image features and derives a Lyapunov based controller for the considered problem. Section IV presents simulation results for the full nonlinear dynamics of an aircraft. The final sections provide a short summary of conclusions and future research directions.

II. Modeling

A. Aircraft dynamics

To describe the motion of the UAV, two reference frames are introduced: a fixed inertial frame $\mathcal{I}$ associated with the vector basis $[e_x, e_y, e_z]$ and a body-fixed frame, $\mathcal{B}$, attached to the vehicle's center of mass and associated with the vector basis $[e_x^b, e_y^b, e_z^b]$. The orientation of the aircraft is given by the rotation matrix $R \in \mathrm{SO}(3)$ from $\mathcal{B}$ to $\mathcal{I}$, which can be parameterized by the yaw, pitch and roll Euler's angles, denoted by ψ, θ and ϕ, respectively. The position of the vehicle's center of mass expressed in $\mathcal{I}$ is denoted by $\xi = (x, y, z)^T$ and the linear velocity, expressed in $\mathcal{B}$ is denoted by v and defined as the sum of the wind velocity v_w and the so-called airspeed v_a:

$$v = v_a + v_w. \tag{1}$$

The wind is assumed to be constant with respect to the inertial frame. Finally the angular velocity defined in $\mathcal{B}$ is denoted by $\Omega = (p, q, r)^T$. The kinematic and dynamic equations of motion for the vehicle can be written as

$$\dot{\xi} = Rv \tag{2}$$

$$m\dot{v} = -\mathrm{sk}(\Omega)mv + F \tag{3}$$

$$\dot{R} = R\,\mathrm{sk}(\Omega) \tag{4}$$

$$I\dot{\Omega} = -\mathrm{sk}(\Omega)I\Omega + \Gamma. \tag{5}$$

where m is the vehicle's mass, I is the moment of inertia and $\mathrm{sk}(.) : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ denotes the matrix realization of the vectorial cross product: $\mathrm{sk}(\Omega)x = \Omega \times x$. The exogenous torque is denoted by Γ and the exogenous force is denoted by F and can be decomposed as

$$F = F_{\text{earth}} + F_{\text{engine}} + {}^{\mathcal{A}}_{\mathcal{B}}R^T F_{\text{aero}},$$

where $F_{\text{earth}} = mgR^T e_z^b$ is the gravitational force, $F_{\text{engine}} = Te_x^b$ where T is the thrust of engine turbines. F_{aero} is due to aerodynamical effects and is expressed in the aerodynamic airframe $\mathcal{A}$ as a function of the dynamic pressure $\bar{Q}$, angle of attack α, and sideslip angle β:

$$\frac{F_{\text{aero}}}{\bar{Q}S} = -C_X(\alpha, \beta)E_x^a + C_{Y,\beta}\beta E_y^a - C_{Z,\alpha}(\alpha - \alpha_0)E_z^a,$$

where $E_x^a = \frac{v_a}{\|v_a\|}$, $E_z^a = \frac{E_x^a \times e_y}{\|E_x^a \times e_y\|}$ and $E_y^a = E_z^a \times E_x^a$. The matrix ${}^{\mathcal{A}}_{\mathcal{B}}R \in \mathrm{SO}(3)$ is the rotation matrix from $\mathcal{B}$ to $\mathcal{A}$. The reference surface of the airplane is denoted by S, $(C_X, C_{Y,\beta}, C_{Z,\alpha})$ are the so-called aerodynamic coefficients and α_0 is the angle-of-attack that nullifies aircraft's lift.

The actuators for the dynamics (2)-(5) are the thrust of engine turbines T and orientation of control surfaces $(\delta_l, \delta_m, \delta_n)$ that allow to design the torque Γ as desired.

The approach used for the Flare phase of the landing maneuver consists in:

1) regulating the norm of the airspeed $V_a = \|v_a\|$ to a desired forward velocity V_a^d,
2) stabilizing the attitude dynamics (4)-(5) through a high gain inner loop controller such that assignments in (ϕ, α, β) are correctly performed,
3) stabilizing the translational dynamics (2)-(3) using (β, α) as guidance control inputs and considering $\phi = 0$ and V_a constant. This approach is particulary adapted for the flare maneuver because a landing system is used. For the other phases, the so-called bank-to-turn maneuver which consists in considering (α, ϕ) as guidance inputs along with the constraint $\beta = 0$, is classically used.

The first item requires the use of the propeller thrust to regulate the airspeed V_a towards the desired value V_a^d. Note that, in order to guarantee that the aircraft's dynamics (2)-(5) are controllable, the desired airspeed must be larger than the maximum between the wind amplitude $\|v_w\|$ and the lower speed threshold of the aircraft V_a^l:

$$V_a^d > \max\{\|v_w\|, V_a^l\}.$$

In practice, this limitation comes from the airplane design and characteristics. The airplane should not be used when the wind conditions are higher than a limit identified upon the airplane conception. Hence, the following assumption is done on wind velocity.

Assumption 1: There exists $\varepsilon \in [0, 1]$ such that:

$$\|v_w\| < \varepsilon V_a^d.$$

The second item is accomplished through a standard high-gain inner-loop whose description is omitted from this paper.

Finally to achieve the goal described in 3), the guidance dynamics, can be simplified by considering that the corresponding time constant is larger than those of the inner-loop controller and of the airspeed regulation. As such, it can be assumed that the airspeed is constant and the roll angle is null. Therefore the dynamics for the guidance control problem are described as

$$\begin{aligned} \dot{\xi} &= R(v_a + v_w) \\ \dot{v}_w &= -\mathrm{sk}(\Omega)v_w \\ \dot{v}_a &= -\mathrm{sk}(\Omega)v_a + \pi_{v_a}u_a(\alpha, \beta) \end{aligned}$$

where $\pi_{v_a} = I_d - \frac{v_a v_a^T}{V_a^2}$ yields the projection on the plane orthogonal to v_a, and $u_a(\alpha, \beta)$ is the actuation provided by the guidance controller. The angle commands (α^c, β^c) to the inner-loop controller are determined by the nonlinear inversion of the equation for $u_a(\alpha, \beta)$.

The proposed control strategy will depend only on the measurement of the following variables:

- the Euler angles (ϕ, θ, ψ) and angular velocity Ω, both provided by an Inertial Measurement Unit - IMU,
- the norm of the airspeed, angle of attack and sideslip angle $(|V_a|, \alpha, \beta)$, measured by pitot tubes and pressure intakes, and providing a direct measure of v_a
- and visual features extracted by a vision system.

The wind velocity cannot be measured but is estimated by the proposed control algorithm. The aircraft position is unknown, however the visual features used provide sufficient information to align the aircraft with the runway and perform the maneuver without complementary position measurements.

III. Choice of Image features

In this section image features are derived. It is assumed that the target is the runway on a textured ground. The borders of the runway are used to perform the alignment of the aircraft while textures are exploited to perform the vertical landing.

A. Modified Plücker coordinates

Consider a collection of $n \geq 2$ parallel lines. Let $u \in \mathcal{I}$ and $U \in \mathcal{B}$ ($U = R^T u$) denote the unit direction of the lines. The camera-fixed frame is assumed to coincide with $\mathcal{B}$ and the image features are assumed to remain in the camera's field of view during flight.

The visual features are represented through a modification of the so-called bi-normalized Plücker coordinates. Plücker coordinates are an explicit representation of straight lines in 3-D space, which simplify technically the development of the proposed control approach, [6], [9]. Using these coordinates, a line is represented by the unit vector $h_i \in \mathcal{B}$ orthogonal to the plane containing both the line and the origin of the reference frame $\mathcal{B}$. Note that $h_i \in \mathcal{B}$ can be written as

$$h_i = \frac{H_i}{\|H_i\|} = \frac{P_i \times U}{\|P_i \times U\|}$$

where $H_i = P_i \times U$ and P_i denotes the vector between the camera and an arbitrary point on the image of the *i*-th parallel line, see Figure 2. The images provide direct measurements

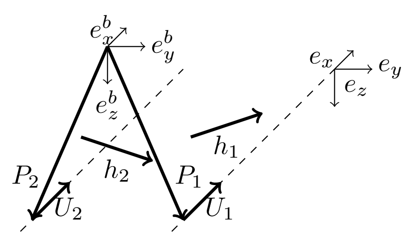

Fig. 2. Runway lines and Plücker coordinates.

of h_i and U can be obtained from $U = \frac{h_i \times h_j}{\|h_i \times h_j\|}$, for $i \neq j$.

In previous work, measurements h_i were directly used to design a centroid vector[1] $q := \sum h_i$ which encoded the 2D pose information needed for stabilization on the trajectory parallel to the runway [9]. The IBVS task consisted therefore in stabilizing q on a desired centroid vector q^*.

In this approach, we propose a modification to the Plücker coordinates that leads to a new centroid vector used to stabilize only the horizontal movement. Hence, let the desired centroid vector be defined as $q^* := R^T b^*$, where b^* is a constant vector that encodes the desired position information. For this case we consider $b^* = e_y$. Thus let the new modified Plücker coordinates, g_i, be defined as the projection of h_i in the orthogonal plane to q^*:

$$g_i = \pi_{q^*} h_i$$

where $\pi_{q^*} = I_d - \frac{q^* q^{*T}}{\|q^*\|^2}$. And the new centroid vector is the sum of the modified Plücker coordinates $q := \pi_{q^*} \sum g_i$. Note that when $q = 0$ the airplane must be at center of the lines, i.e. the sum of the h_i vectors is in the direction of q^*, see Figure 2.

The time derivative of H_i is given by [6]

$$\dot{H}_i = -\mathrm{sk}(\Omega) H_i - v \times U,$$

thus the dynamics of g_i can be described as

$$\dot{g}_i = -\mathrm{sk}(\Omega) g_i - \frac{1}{\|H_i\|} \pi_{q^*} \pi_{h_i} (v \times U).$$

Finally the time derivative of the centroid vector is given by

$$\dot{q} = -\mathrm{sk}(\Omega) q - \pi_{q^*} Q (v \times U) \tag{6}$$

where

$$Q = \sum \frac{1}{\|H_i\|} \pi_{h_i}. \tag{7}$$

Q is a positive definite matrix as long as there are at least two ($n \geq 2$) visible features [6]. This property is exploited in the control design and avoids the need to estimate it. Nevertheless, some bounds are required on the trajectories considered, to avoid ill-conditioning of the eigenvalues of Q. In this development, we define a region of space by a pair of uniform bounds on the matrix

Assumption 2: There exist two positive scalars $(q_m, q_x) > 0$ such that:

$$q_m < \{\lambda_i(Q)\} < q_x.$$

This is a classical assumption for IBVS control schemes. Recalling (7), q_m limits the distance between the airplane and the runway, while q_x, the upper-bound, implies that the ground is not touched by the camera.

B. Translational Optical Flow

Consider the dynamics of an image point, also called optical-flow, under spherical projection of a camera with unit image radius, [12]:

$$\dot{p} = -\mathrm{sk}(\Omega) p - \frac{\cos\theta_p}{d(t)} \pi_p v \tag{8}$$

where $d := d(t)$ is the orthogonal distance from the target surface to the origin of frame $\mathcal{B}$ measured as a positive scalar, and θ_p is the angle between the inertial direction η and the observed target point p.

The visual velocity measure that is used is the translational optical flow w, expressed in the inertial frame:

$$w(t) = \frac{V}{d} = \frac{Rv}{d}.$$

When the observed world is a flat planar surface, translational optical flow will have three components, flow in the two planar directions, analogous to classical optical flow, and flow in the normal direction to the plane, analogous to optical divergence.

Measuring the translational optical flow is a key aspect of the practical implementation of control algorithms proposed.

[1]The centroid information is commonly used in visual servo control [11], [5].

The optical flow $\dot{p}$ can be computed using a range of algorithms (correlation-based technique, features-based approaches, differential techniques, etc) [18]. Note that due to the rotational ego-motion of the camera, (8) involves the angular velocity as well as the linear velocity [17].

An effective measurement of w is obtained by integrating the observed optical flow over a section $\mathcal{W}^2$ of the sphere around the pole normal to the target plane, [12]:

$$\phi = \int\int_{\mathcal{W}^2} \dot{p} = -\beta_f \Omega \times \eta - \frac{Q_f v}{d}, \tag{9}$$

where β_f represents the angle of the field of view of the window $\mathcal{W}^2$ and $Q_f = R^T(R_t \Lambda R_t^T)R$ is a symmetric positive definite matrix. The matrix Λ is a constant diagonal matrix depending on the window parameters and R_t represents the rotational matrix from the target plane to the inertial frame.

From (9) it is straightforward to obtain the translational optical flow:

$$w = (w_x, w_y, w_z)^T = -(R_t \Lambda^{-1} R_t^T) R(\phi + \mu\Omega \times \eta)$$

Note that if the target frame coincides with the inertial plane, $R_t = I_d$, then the normal direction to the target becomes e_z (observed from the camera-frame as vector pointing towards the plane). Moreover, if one assumes that if the target plane is in the plane $x - y$ of the inertial frame, the variable d becomes the height h (or $|z|$) from the camera to the target.

IV. Problem formulation

Let w^* be the constant desired translational optical flow divergence. It is straightforward to show that when $w_z = w^*$ one has $\dot{h} = -h_0 w^* \exp(-w^* t)$ and $h = h_0 \exp(-w^* t)$ which converges to zero and ensuring a smooth landing.

The IBVS stabilization task consists in driving exponentially the centroid vector q to the desired one, q^* (i.e. $q - q^* \to 0$) in the direction orthogonal to q^* resulting in the alignment of the airplane in the center of the runway, and also regulate $\frac{\dot{h}}{h} + w^*$. Thus, two error terms are introduced:

$$\begin{aligned} \delta &= \pi_{q^*} q \\ \delta_3 &= q_0^* \dot{h} = q_0^* q_0^{*T} \mathrm{sk}(U) v \end{aligned} \tag{10}$$

where $q_0^* = \frac{q^*}{|q^*|}$ and $\dot{h} = q_0^{*T}(v \times U)$ is the time derivative of the height of the aircraft's center of mass. Hence the control approach is divided in two parts. In the first part, δ is driven to zero ensuring the horizontal alignment and in the second part a control law is chosen to guarantee the touchdown.

A. Horizontal alignment

Combining the dynamics of equation (6) and an estimation of the wind velocity, the dynamics of the error term δ can be described as

$$\begin{aligned} \dot{\delta} &= -\mathrm{sk}(\Omega)\delta + \pi_{q^*} Q \pi_{q^*} \mathrm{sk}(U) v + \pi_{q^*} Q \delta_3 \qquad (11) \\ &= -\mathrm{sk}(\Omega)\delta + \pi_{q^*} Q \pi_{q^*} [\mathrm{sk}(U) v_a - \hat{v}_w - \tilde{v}_w] + \bar{\delta}_3 \end{aligned}$$

where $\bar{\delta}_3 = \pi_{q^*} Q \delta_3$, $\tilde{v}_w = v_w \times U - \hat{v}_w$ and $\hat{v}_w$ is an estimate of $v_w \times U$.

Choose the following dynamics for $\hat{v}_w$:

$$\dot{\hat{v}}_w = -\mathrm{sk}(\Omega)\hat{v}_w + \mathbf{P}\pi_U u_w, \qquad \hat{v}_w(0) = 0 \tag{12}$$

where u_w acts as the input of the wind estimator and

$$\mathbf{P} = \varepsilon' V_a \sqrt{1 - \frac{\|\hat{v}_w\|^2}{\varepsilon'^2 V_a^2}} \left(I - \frac{\hat{v}_w \hat{v}_w^T}{\varepsilon'^2 V_a^2} \right), \quad \varepsilon' \in \left(\frac{1+\varepsilon}{2}, 1 \right).$$

This wind velocity estimation remains in the plan orthogonal to U and ensures the norm of the wind estimate $\|v_w\|$ is strictly lower than $\varepsilon' V_a$. More details on this estimator can be found in [9].

Consider the following Lyapunov function, where k_1 is a positive control gain:

$$S_1 = \|\delta\|^2 + \frac{2}{k_1}(\pi_{q^*}\tilde{v}_w)^T \delta + \frac{4}{k_1^2}\|\pi_{q^*}\tilde{v}_w\|^2$$

choosing u_w as follows:

$$u_w = -k_2 \delta, \qquad k_2 > 0 \tag{13}$$

let v_a^d be the virtual commanded airspeed defined as

$$v_a^d := \mathrm{sk}(U)(k_1\delta - \hat{v}_w) + \sqrt{V_a^2 - \|k_1\delta - \hat{v}_w\|^2} U. \tag{14}$$

It can be verified that $\|v_a^d\| = V_a$. Knowing that $\delta < 2n$ and $\|\hat{v}_w\| < \varepsilon' V_a$, and choosing k_1 such that

$$k_1 < \frac{1 - \varepsilon'}{2n} V_a$$

it can be ensured that v_a^d is correctly defined since $\|k_1\delta - \hat{v}_w\| < V_a$.

Introducing a new error term defined as:

$$\delta_2 = \pi_{q^*} \mathrm{sk}(U)(v_a - v_a^d)$$

and recalling (11), (12), (13) and (14), the derivative of S_1 can be described as

$$\begin{aligned} \dot{S}_1 = {} & 2\left(\delta + \frac{\pi_{q^*}\tilde{v}_w}{k_1}\right)^T Q\delta_2 + 2\left(\delta + \frac{\pi_{q^*}\tilde{v}_w}{k_1}\right)^T \bar{\delta}_3 + \\ & + \frac{2k_2}{k_1}\left(\delta + \frac{4}{k_1}(\pi_{q^*}\tilde{v}_w)\right)^T \mathbf{P}\delta + \\ & - 2k_1\left(\delta + \frac{\pi_{q^*}\tilde{v}_w}{k_1}\right)^T Q \left(\delta + \frac{\pi_{q^*}\tilde{v}_w}{k_1}\right) \end{aligned}$$

Consider a second Lyapunov function

$$S_2 = \frac{1}{2}\|\delta_2\|^2,$$

along with the dynamics of δ_2

$$\begin{aligned} \dot{\delta}_2 = {} & -\mathrm{sk}(\Omega)\delta_2 + \pi_{q^*}\mathrm{sk}(U)\pi_{v_a} u_a + k_1 \pi_{q^*} Q \delta_2 + \\ & - k_1^2 \pi_{q^*} Q \left(\delta + \frac{\pi_{q^*}\tilde{v}_w}{k_1}\right) - k_2 \pi_{q^*} P\delta + k_1 \bar{\delta}_3, \end{aligned} \tag{15}$$

one can verify that the time derivative of the second storage function is given by:

$$\begin{aligned} \dot{S}_2 = {} & \delta_2^T \pi_{q^*}\mathrm{sk}(U)\pi_{v_a} u_a + k_1 \delta_2^T Q \delta_2 + k_1 \delta_2^T \bar{\delta}_3 \\ & - k_1^2 \delta_2^T Q \left(\delta + \frac{\pi_{q^*}\tilde{v}_w}{k_1}\right) - k_2 \delta_2^T \mathbf{P}\delta. \end{aligned} \tag{16}$$

The following proposition presents the controller that guarantees the horizontal alignment.

Theorem 1: Consider the dynamics defined by (11) and (15) along with (14). Assume that v_a^d is not in the opposite direction of v_a, i.e.

$$\exists \varepsilon_a > 0 \quad | \quad 1 - \cos(v_a, v_a^d) < 2 - \varepsilon_a.$$

and

$$\bar{\delta}_3 \to 0 \quad \text{as} \quad t \to \infty. \tag{17}$$

Then choosing the control

$$\pi_{q^*}\mathrm{sk}(U)\pi_{v_a}u_a = -k_3\delta_2 \quad (18)$$

positive gains (k_1, k_2, k_3, K) exist, such that the function

$$\mathcal{L} = S_1 + KS_2$$

is a Lyapunov function for the guidance dynamics that guarantees that the closed-loop solution exists for all time and the error signals $(\delta, \delta_2, \tilde{v}_w)$ converge to zero.

Proof: Consider the first term of (16) and introduce the control (18). Given that $\|v_a^d\| = V_a$, it can be written as:

$$\delta_2^T\pi_{q^*}\mathrm{sk}(U)\pi_{v_a}u_a = -k_3\|\delta_2\|^2$$

Then using Schwarz inequality $X^TY \leq \frac{1}{2}(|x|^2 + |y|^2)$, and noticing that

$$\|\delta\| \leq \sqrt{\mathcal{L}}, \quad \|\pi_{q^*}\tilde{v}_w\| \leq \tfrac{k_1}{2}\sqrt{\mathcal{L}}, \quad \|\delta_2\| \leq \sqrt{\tfrac{2\mathcal{L}}{K}}$$

the derivative of $\mathcal{L}$ can be written as $\dot{\mathcal{L}} \leq f_1 + f_2$ where

$$\begin{aligned} f_1 \leq & -k_3K\|\delta_2\|^2 + K\delta_2^T\left[\left(\frac{1}{K} + \frac{k_1^2}{2} + k_1\right)Q - \frac{k_2}{2}\mathbf{P}\right]\delta_2 \\ & + y^T\left[\left(-2k_1 + 1 + \frac{k_1^2K}{2}\right)Q + \frac{4k_2}{k_1}\mathbf{P}\right]y + \\ & - k_2\left(\frac{K}{2} + \frac{2}{k_1}\right)\delta^T\mathbf{P}\delta \end{aligned}$$

with $y = \left(\delta + \frac{\pi_{q^*}\tilde{v}_w}{k_1}\right)$ and

$$f_2 = (2y + \delta_2)^T\,\bar{\delta}_3 \leq c_1\sqrt{\mathcal{L}}\|\bar{\delta}_3\|, \quad c_1 = 3 + \sqrt{2/K}. \quad (19)$$

Recalling Assumption 2, and noticing that one can ensure that f_1 is upper-bounded by a definite negative expression of $(\mathbf{P}\delta, \delta + \frac{\pi_{q^*}\tilde{v}_w}{k_1}, \delta_2)$ as soon as the control gains satisfy:

$$K < \tfrac{4k_1 - 2}{k_1^2}, \quad k_2 < \tfrac{k_1 q_m}{4\varepsilon' V_a}\left(2k_1 - 1 - \tfrac{k_1^2K}{2}\right),$$
$$k_3 > \left(\tfrac{1}{K} + \tfrac{k_1^2}{2} + k_1\right)q_x + \tfrac{k_2}{2}\varepsilon' V_a.$$

then $f_1 \leq -c_2\mathcal{L}$ where $c_2 > 0$ is a function of the gains $(k_1, k_2, k3, K)$. Using this property along with (19), one can write

$$\dot{\mathcal{L}} \leq -c_2\mathcal{L} + c_1\sqrt{\mathcal{L}}\|\bar{\delta}_3\|.$$

Let $W = \sqrt{\mathcal{L}}$, then

$$\dot{W} = \frac{\dot{\mathcal{L}}}{\sqrt{\mathcal{L}}} \leq -c_2W + c_1\|\bar{\delta}_3\|.$$

It is simple to show that

$$W(t) \leq e^{-c_2(t-t_0)}W(t_0) + \int_{t_0}^{t} c_1 e^{-c_2(t-\tau)}\|\bar{\delta}_3(\tau)\|d\tau \quad (20)$$

the use of (17) in (20) shows that $W(t) \to 0$ as $t \to \infty$. Given the definition of $\mathbf{P}$, we note that it is positive definite as soon as $\hat{v}_w < \varepsilon' V_a$, then $(\delta, \pi_{q^*}\tilde{v}_w, \delta_2) \to 0$ as $t \to \infty$ and the solution is uniformly bounded for all $t \geq t_0 \geq 0$, [13]. ■

Remark 1: The proof is based upon the assumption that δ_3 is convergent towards zero. This is proved to be true in the next section.

Remark 2: Note that although the proposed controller guarantees convergence for the estimation error $\tilde{v}_w$, the initial condition of the estimator should not be arbitrary due to the intrinsic risk of the Flare maneuver, any kind of transient responses must be avoided. Thus, the initial condition of the estimator is inherited from the glide-slope controller, which has already a steady estimative for the wind velocity.

B. Touchdown control

The time derivative of (10) is given by

$$\dot{\delta}_3 = q_0^* q_0^{*T}\mathrm{sk}(U)\pi_{v_a}u_a(\alpha, \phi) \quad (21)$$

Theorem 2: Consider the dynamics of equation (21) with the control input defined as

$$q_0^* q_0^{*T}\mathrm{sk}(U)\pi_{v_a}u_a = -k_4 q_0^*\left(\frac{\dot{h}}{h} + w^*\right) \quad (22)$$

Then for any initial condition such that $h(0) > 0$, the variable δ_3 and the states $(h, \dot{h})$ converge exponentially fast to zero.

Proof: [Sketch] Introducing (22) in (21) yields the following closed-loop system

$$\ddot{h} = -k_4\left(\frac{\dot{h}}{h} + w*\right). \quad (23)$$

To analyze the stability of (23), consider the auxiliary state:

$$\chi = h\exp\left\{\frac{\dot{h}}{k_4}\right\}$$

Differentiating χ yields:

$$\dot{\chi} = -w^* h\exp\left\{\frac{\dot{h}}{k_4}\right\} = -w^*\chi$$

It is straightforward to verify that $h(t) > 0$ for all time and as long as $\dot{h}$ is bounded, one can insure that χ converges exponentially to zero and consequently, $h(t)$ converges exponentially towards zero.

To insure that $\dot{h}$ is bounded, consider the following Lyapunov function candidate:

$$V = \frac{1}{2}\dot{h}^2 + k_4 w^* h$$

and note that $\dot{V}$ can be written as:

$$\dot{V} = -k_4\frac{\dot{h}^2}{h}$$

This insures that $\dot{h}$ is bounded and converges to zero. To obtain the largest invariant set such that $\dot{V} = 0$, or equivalently $\dot{h} = 0$ and $\frac{\dot{h}}{h} \neq 0$, note that

$$\dot{h} \equiv 0 \Rightarrow \ddot{h} \equiv 0 \Leftrightarrow \frac{\dot{h}}{h} \equiv w*$$

From LaSalle's Invariance Principle, it follows that $\dot{h}(t) \to 0$ and $\frac{\dot{h}(t)}{h(t)} \to w*$ as $t \to \infty$. Since h is exponentially decaying to zero, we can conclude that $\dot{h}$ is also exponentially decaying to zero.

Finally to insure that χ and the controller are well defined $\forall t > 0$, it suffices to guarantee that there exists a time T, such that $h(t)$ is positive and decreasing, $\dot{h}$ is negative and increasing and finally $\ddot{h}$ is positive, $\forall t > T$, [14]. Due to the lack of space, this part is not included in the paper but we invite the reader to ask directly the authors for the complete proof. ■

Equations (18) and (22) can be added resulting in the following control law

$$\pi_U u_\alpha = \mathrm{sk}(U)k_3\delta_2 + \mathrm{sk}(U)k_4 q_0^*\left(\frac{\dot{h}}{h} + w^*\right).$$

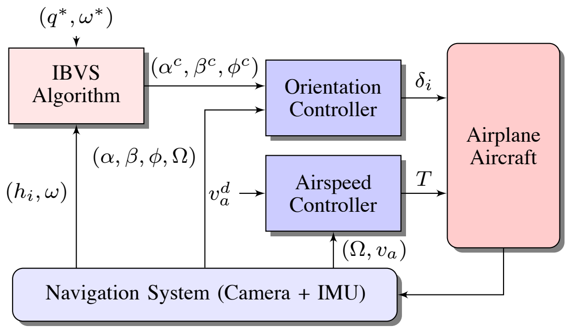

Fig. 3. Implementation block diagram.

V. Simulation Results

In this section, full dynamics of the jet-sized aircraft described in section II, are simulated and the visual guidance control law is tested in presence of wind. The aircraft model incorporates the nonlinear flight dynamics including aerodynamic effects and saturation on control surfaces deflection and thrust. Simulations have been undertaken with a specific simulation architecture of the LRBA, termed A^3. The control scheme used is represented in Figure 3.

The results presented include the full landing mission, alignment, glide-slope and flare phases, although this paper is focused only on the flare. Details about the controller used for the alignment and glide-slope can be found in [9]. The runway is aligned with the e_x axis and is 60 meters width. The desired trajectory consists in an alignment in the runway axis, 350m above the ground level, followed by a $4°$ glide-path maneuver starting when the aircraft is 4000m far from the runway. Finally the flare maneuver starts to ensure a smooth touchdown. The initial position is about 60m along the lateral direction, 25m along the vertical axis, and 7000m from the beginning of the runway (longitudinal position). For this simulation, the desired aerial velocity is $V_a = 80ms^{-1}$. Figures 4 and 5 show the aircraft position and attitude along the forward motion. The results were obtained submitting the aircraft to lateral wind of $10m/s$.

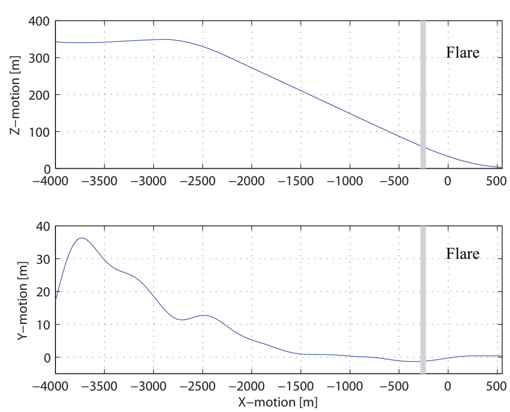

Fig. 4. Airplane position.

VI. Concluding Remarks

This paper proposed a robust nonlinear IBVS controller for fixed-wing aircraft, without direct measurement of the aircraft position. The proposed controller allows the airplane to perform the Flare phase of the landing maneuver autonomously through a feedback on visual features. The controller performs the stabilization task along with bounded estimation of the wind. The control algorithm has been theoretically proved and tested in simulation with a nonlinear aircraft model. Results show that the control approach is suitable for the task and is robust to wind gust. Future work includes simulations with other types of wind gust models, such as Dryden spectrum, image treatment in the simulation architecture along with pan & tilt camera to ensure that the target surface is always visible.

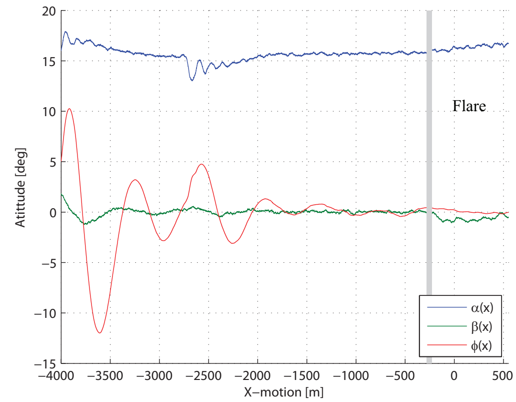

Fig. 5. Airplane attitude (ϕ, α, β).

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