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Visual Servo Aircraft Control for Tracking Parallel Curves

Pedro Serra, Rita Cunha, Carlos Silvestre and Tarek Hamel

***Abstract*— This paper describes a nonlinear Image-Based Visual Servo (IBVS) controller for tracking nonlinear parallel 2-D curves, such as catenaries, with a fixed-wing aircraft. Image features are exploited from the image of the lines to design a feedback controller for the automatic manoeuvre. For curves in the vertical plane the proposed solution guarantees exponential stability in the horizontal direction and ultimate boundedness in the vertical direction, with bound proportional to the slope deviation with respect to a straight line. Simulation results are presented to illustrate the performance of the control approach.**

I. INTRODUCTION

Over the last decade, a growing interest in unmanned aerial vehicles (UAVs) has spread among the research community. Their potential to perform high precision tasks in challenging and uncertain operation scenarios is a motivation for the increasing effort on the development of new control algorithms. Moreover, the new sensor technology and the increasingly powerful computational systems are potentiating new and more challenging applications.

Inspection of infrastructures, such as bridges [1] or chimneys [2] and the vision-based tracking problems [3], has spawned a wide range of application with major interest within the research community where the use of cameras arises as natural tool for automatic inspection.

The research and development in the area of unmanned aerial vehicles has been paying off due to the many advantages When compared with traditional technologies that require human presence and intervention, UAVs can

- reduce costs by decreasing the need of manpower;
- reduce costs by decreasing the energy dependence;
- improve reliability of the results in some cases where human errors are frequent;
- give access to areas that are out-of-range for manned vehicles;
- expand the range of applications, including in indoor environments due to high manoeuvrability and small size;
- reduce risks of accidents and injuries.

The measurement of the vehicle's position with respect to the local environment is a major problem when designing control systems. Global Positioning System (GPS) is being widely used as the primary navigation aid, however there has been an increasing interest in developing alternative systems that provide robust relative pose information to be used in navigation algorithms. One alternative is the use of a vision system. Using cameras as primary sensors of relative position, the flight control problem can be cast into an Image-Based Visual Servo (IBVS) Control problem ([4], [5]), opening the possibility to perform autonomous tasks in low-structured environments with no external assistance, [6], [7].

This paper presents a solution to the problem of tracking nonlinear parallel 2-D curves, in particular, the typical catenary curve described by the overhead power lines. The proposed controller borrows from the work in [7], where the authors present an IBVS controller to track parallel straight lines, using the normalized Plücker coordinates of the straight lines, which can be readily obtained from the image measurements. The landing manoeuvre of an airplane is approached in [8] and [9], where the presented solution is also based in the Plücker coordinates of the lines delimiting the runway complemented with the Optical-Flow for the Flare phase. In this paper, we consider a modified version of the image error introduced in [7] that can be used for non-straight lines and is obtained by combining information from the vanishing point where the curves meet, with the image coordinates of points on the curves, which are judiciously chosen. For curves in the vertical plane, the proposed solution guarantees uniform exponential stability in the horizontal direction and ultimate boundedness in the vertical direction. The bound on this error is proportional to the maximum relative slope of the tangent to the curves, with the zero-slope direction defined by the vanishing point.

The control architecture is decoupled into an inner-loop and outer-loop controller. The outer-loop controller stabilizes the translational (or guidance) dynamics resorting to visual data and using the roll angle and the angle of attack as control inputs. The inner-loop controller actuates on the aircraft control surfaces and provides high-gain stabilization of the vehicle's angle of attack, side-slip and roll angles based on direct measurements of the IMU and pitot tubes. The time-scale separation between the two loops is considered sufficient so that the interaction terms can be ignored in the control design. Detailed studies on the inner/outer loop approach of controllers design can be found in [10] and [11].

This paper is structured in four sections. Section II presents the dynamic model of an airplane. Section III presents the image features that are used in the control design which, in turn, is presented in section IV. Section V presents simulation results for the full nonlinear dynamics of an aircraft. The final section provide a short summary of conclusions and future research directions.

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P. Serra, R. Cunha and C. Silvestre are with the Department of Electrical Engineering and Computer Science, and Institute for Systems and Robotics, Instituto Superior Técnico, 1049-001 Lisboa, Portugal. {pserra,cjs,rita}@isr.ist.utl.pt

C. Silvestre is also with the Department of Electrical and Computer Engineering, Faculty of Science and Technology of the University of Macau.

T. Hamel is with I3S UNSA-CNRS, Nice-Sophia Antipolis, France, thamel@i3s.unice.fr

II. Modelling

A. Aircraft dynamics

This section briefly describes the aircraft dynamic model adopted. For a comprehensive coverage of aircraft flight dynamics, the reader is referred to [12].

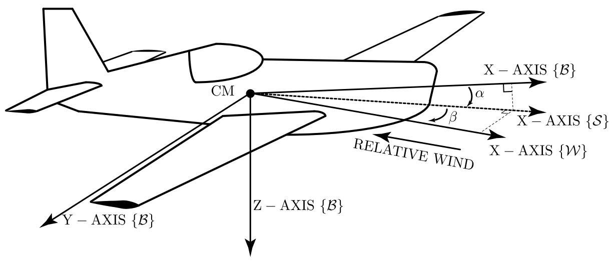

Fig. 1. Reference frames.

Let $\{\mathcal{I}\}$ denote the inertial reference frame and $\{\mathcal{B}\}$ the body reference frame, attached to the vehicle's center of mass. Two additional reference frames attached to the vehicle's center of mass are introduced, the airspeed reference frame $\{\mathcal{W}\}$ and the stability reference frame $\{\mathcal{S}\}$, see Fig. 1. The angle of attack α defines the orientation of $\{\mathcal{S}\}$ with respect to $\{\mathcal{B}\}$, which is used to analyse the effect of perturbations from steady-state flight. The airspeed frame $\{\mathcal{W}\}$ is obtained from the stability frame through a rotation about the z-axis by the side-slip angle β. Therefore, the rotation matrices from $\{\mathcal{S}\}$ to $\{\mathcal{W}\}$ and from $\{\mathcal{B}\}$ to $\{\mathcal{S}\}$ are given, respectively, by

$$ {}^{\mathcal{W}}_{\mathcal{S}}R = R_Z(-\beta), \qquad {}^{\mathcal{S}}_{\mathcal{B}}R = R_Y(\alpha). \tag{1}$$

The flight-path, heading, and bank angles (γ, χ, μ) are the so called wind angles and describe the orientation of the airspeed frame with respect to the inertial frame, such that

$$ {}^{\mathcal{I}}_{\mathcal{W}}R = R_Z(\chi)R_Y(\gamma)R_X(\mu). \tag{2}$$

Assuming the presence of wind with velocity v_w expressed in $\{\mathcal{I}\}$, let v_b denote the velocity of the aircraft relative to the wind expressed in $\{\mathcal{B}\}$, and let v denote the same vector expressed in $\{\mathcal{I}\}$. Let $\Omega = (p, q, r)^T$ denote the angular velocity also expressed in $\{\mathcal{B}\}$, $\xi = (x, y, z)^T$ the position in $\{\mathcal{I}\}$ and $R = {}^{\mathcal{I}}_{\mathcal{B}}R$ the rotation matrix from $\{\mathcal{B}\}$ to $\{\mathcal{I}\}$. For fixed-wing aircraft, it is convenient to write the equations of motion using a state vector naturally suited for flight control design. Instead of v_b, the variables V, α, and β are used to describe the magnitude and direction of the relative airspeed, which according to (1) satisfy

$$v_b = {}^{\mathcal{B}}_{\mathcal{W}}R \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = V \begin{bmatrix} \cos\beta\cos\alpha \\ \sin\beta \\ \cos\beta\sin\alpha \end{bmatrix}. \tag{3}$$

In addition, the wind angles (γ, χ, μ), which describe the orientation of the vehicle can be used to determine the direction of the relative velocity with respect to $\{\mathcal{I}\}$. According to (2)

$$v = {}^{\mathcal{I}}_{\mathcal{W}}R \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = V \begin{bmatrix} \cos\chi\cos\gamma \\ \sin\chi\cos\gamma \\ -\sin\gamma \end{bmatrix}. \tag{4}$$

From these definitions, it follows that the dynamic and kinematic equations of motion for the aircraft can be written as

$$I\dot{\Omega} = -\mathrm{s_k}(\Omega)\, I\Omega + \Gamma \tag{5}$$

$$\dot{\xi} = v + v_w \tag{6}$$

$$\dot{R} = R\mathrm{s_k}(\Omega) \tag{7}$$

$$\dot{V} = \frac{1}{m}\left(F_T\cos\alpha\cos\beta - D\right) - g\sin\gamma \tag{8}$$

$$\dot{\beta} = -\frac{1}{mV}(F_T\cos\alpha\sin\beta + C) + \frac{1}{m}g\sin\mu\cos\gamma - r_s \tag{9}$$

$$\begin{aligned}\dot{\alpha} = &-\frac{1}{mV\cos\beta}(F_T\sin\alpha + L) + \frac{1}{V\cos\beta}g\cos\mu\cos\gamma + \\ &+ (q - p_s\tan\beta)\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{\gamma} \quad &= -\tfrac{1}{V}g\cos\gamma + \tfrac{1}{mV}(C\sin\mu + L\cos\mu) + \\ &+ \tfrac{F_T}{mV}\left[\sin\alpha\cos\mu + \cos\alpha\sin\beta\sin\mu\right]\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{\chi} \quad &= \tfrac{1}{mV\cos\gamma}(L\sin\mu - C\cos\mu) + \\ &+ \tfrac{F_T}{mV\cos\gamma}\left[\sin\alpha\sin\mu - \cos\alpha\sin\beta\cos\mu\right]\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{\mu} \quad &= L\left[\tan\beta + \tan\gamma\sin\mu\right]/(mV) - C\tan\gamma\cos\mu/(mV) + \\ &+ F_T\left[\sin\alpha\sin\mu\tan\gamma + \sin\alpha\tan\beta\right]/(mV) + \\ &- F_T\cos\alpha\sin\beta\cos\mu\tan\gamma/(mV) - \tfrac{g}{V}\tan\beta\cos\gamma\cos\mu + \\ &- \cos\alpha\sin\beta\cos\mu\tan\gamma/(mV) + \tfrac{p_s}{mV\cos\beta}\end{aligned} \tag{13}$$

where m is vehicle's mass, I the moment of inertia, Γ the exogenous torque, F_T is the thrust input and g the gravitational acceleration. The aerodynamic force components due to the drag, D, crosswind C and lift L as well as the exogenous moments in (5) can be described as functions of $V, \alpha, \beta, p, q, r$ and also the actuation inputs given by the thrust force F_T and the elevator, aileron, and rudder angles denoted by δ_e, δ_a, and δ_r, respectively, [12].

Note that the airplane should not be used when the wind conditions are higher than a limit identified upon the airplane conception. Hence, the following assumption is made on wind velocity.

Assumption 1: There exists $\varepsilon \in [0, 1]$ such that:

$$\|v_w\| < \varepsilon V. \tag{14}$$

The airplane considered in this paper, is a $1/4$ scale Extra-330 since it is a highly aerobatic aircraft. The parameters for this aircraft were estimated resorting to a wind tunnel simulator.

B. The Inner-Loop

The inner-loop is designed resorting to time-scale separation, that allows for the partitioning of the aircraft dynamics into slow states and fast states, with the fast states used as control inputs for the slow states, [13]. It is composed by: a propulsion controller that uses the thrust command in equation (8) to drive the norm of the airspeed V to a desired constant value V_d; a surface controller which is designed in order to drive the angular velocity Ω to a desired velocity Ω^d, using the control surfaces δ_e, δ_a and δ_r in equation (5); and an angular velocity controller that uses Ω^d as control input in equations (9), (10) and (13) in order to drive the angles α, β and μ to the desired values given as references by the outer-loop.

Due to space limitations and since the inner-loop design is not the main issue addressed by this paper, the interested reader is refered to [13] and [14].

C. The outer-loop

The outer-loop controller is responsible for controlling the guidance dynamics described by the flight-path angle γ and heading angle χ, which according to (4) define the direction of the relative velocity vector expressed in $\{\mathcal{I}\}$.

Due to time-scale separation between the inner and outer loops, we can assume that the airspeed is constant $V = V_d$ and consequently the velocity dynamics can be written as

$$\dot{v} = \mathrm{s_k}\,(v)\, R_z(\chi) \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\gamma} \\ \dot{\chi} \end{bmatrix} \tag{15}$$

where $\dot{\gamma}$ and $\dot{\chi}$ are presented in equations (11) and (12) respectively.

For the application at hand, it is more suited to use the bank-to-turn manoeuvre and drive the sideslip angle to zero. Therefore, the desired sideslip angle passed to the inner-loop is $\beta^d = 0$ and the guidance dynamics becomes

$$\dot{v} = \frac{1}{mV} \mathrm{s_k}\,(v)\, u_a(0, \alpha^d, \mu^d) \tag{16}$$

where the function $u_a(.)$ is obtained from (15) assuming that V is constant and $\beta = 0$. The virtual control input u_a is determined by the IBVS controller presented in the next sections and α^d and μ^d are obtained from the nonlinear inversion of $u_a(0, \alpha^d, \mu^d)$.

III. Visual Features

The goal of this paper is to introduce a new technique for tracking non-straight lines in 3D space based on visual features. For a better understanding of this methodology, it is convenient to first consider the simpler case of tracking straight lines.

A. Vision-based tracking of straight lines

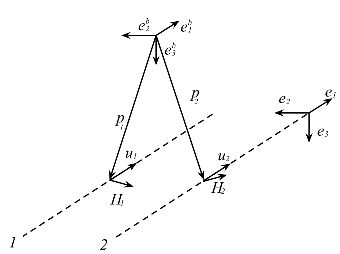

Fig. 2. Parallel lines and Plücker coordinates.

Consider a set of $n \geq 2$ straight lines, as illustrated in Fig. 2, with unit direction described by u expressed in $\{\mathcal{I}\}$[1]. Consider also a point on the line i ($i = 1, 2, \ldots, n$), l_i expressed in $\{\mathcal{I}\}$. Each line can be described in a convenient way resorting to the Plücker coordinates, [7], [8], [9], which comprise the direction u and the vector H_i given by

$$H_i = p_i \times u, \tag{17}$$

where $p_i = l_i - \xi$. Although H_i cannot be extracted directly from the image, it is possible to obtain a normalization of this variable, [7], (as detailed further ahead)

$$h_i = H_i / \|H_i\|. \tag{18}$$

Suppose the goal is to steer the airplane along the direction u, keeping a constant position with respect to the lines. One possible solution to this problem relies on a single image feature given by the centroid

$$q = \sum h_i. \tag{19}$$

As required, q encodes all the information about the position of the vehicle with respect to the straight lines, apart from the coordinate along the direction u (see [7] for further details). Hence, to achieve the tracking objective, we can define the error to be driven to zero through a feedback control system as

$$\delta := q - q^* \tag{20}$$

where q^* is constant in $\{\mathcal{I}\}$ and defined as desired centroid.

B. Vision-based tracking of nonlinear curves

Consider a set of $n \geq 2$ nonlinear parallel curves, whose coordinates in $\{\mathcal{I}\}$ can be parametrized by a scalar τ such that

$$\bar{l}_i(\tau) = u\tau + \sigma(\tau) u_0 + l_{i0} \tag{21}$$

where $\sigma(\tau)$ is a bounded scalar function with bounded derivative, u_0 and l_{i0} are constant vectors in $\mathbb{R}^3$, and u_0 satisfies $u^T u_0 = 0$. In particular, we are interested in catenary lines of the form

$$\bar{l}_i(\tau) = (\tau, l_{iy}, -a\cosh\{[\tau - (\mathrm{ceil}(\tau/L) - 1)L - L/2]/a\})^T, \tag{22}$$

where l_{iy} and a are constant scalars, L is the length of the line and ceil(.) is the round toward positive infinity. For this particular case, we have $u = [1\ 0\ 0]^T$, $u_0 = [0\ 0\ 1]^T$, $l_{i0} = [0\ l_{iy}\ 0]^T$, and $\sigma(\tau) = -a\cosh\{[\tau - (\mathrm{ceil}(\tau/L) - 1)L - L/2]/a\}$. Typically, this is the equation obeyed by electric power lines between towers, see Figure 3.

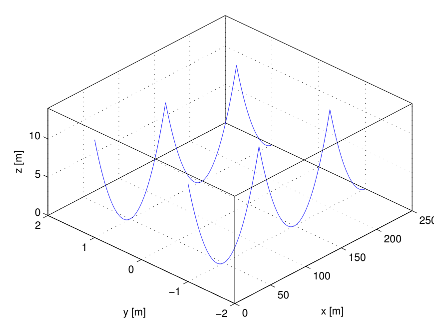

Fig. 3. Catenary lines.

Consider the plane that passes through the aircraft's current position ξ and is perpendicular to u. The point of intersection between this plane and each line i ($i = 1, 2, \ldots, n$) can be obtained with $\tau_\xi = -u^T(l_{i0} - \xi)$ and has coordinates in $\{\mathcal{B}\}$ given by

$$\bar{p}_i = \bar{l}_i(\tau_\xi) - \xi = \pi_u(l_{i0} - \xi) + \sigma(\tau_\xi) u_0 \tag{23}$$

where π_x denotes the projection matrix given by

$$\pi_x = I - xx^T / \|x\|^2. \tag{24}$$

Following the line of thought used for straight lines, consider the vectors $\bar{H}_i = \bar{p}_i \times u$ and $\bar{h}_i = \bar{H}_i / \|\bar{H}_i\|$, which can be associated to the Plucker coordinates of virtual straight lines passing through each $\bar{p}_i$ and with direction u. Consider also a new single image feature given by $\bar{q} = \sum \bar{h}_i$ and finally the new error $\bar{\delta} = \bar{q} - q^*$.

[1] Assume, for now, that this vector is known. In fact, it can be extracted from the image using different methods that are presented further ahead.

C. Extracting the visual features from the image information

Considering the case where parallel straight lines are being viewed by the camera, the normalized Plücker coordinates h_i can be extracted using the external product between two points in the same line

$$h_i = R\frac{im(P_i^1)\times im(P_i^2)}{\|im(P_i^1)\times im(P_i^2)\|}, \tag{25}$$

where $im(P_i^j) = P_i^j/\|P_i^j\|$ denotes the image of P_i^j for an ideal spherical camera and P_i^j denotes the 3-D coordinates of a point j on the line i, expressed in the body frame $\{\mathcal{B}\}$. Notice that $im(P_i^j)$ can be readily obtained from the image acquired using a calibrated CCD camera.

Since u is orthogonal to each h_i, it can be written as

$$u = \frac{h_1\times h_2}{\|h_1\times h_2\|}. \tag{26}$$

Consider now the case where the lines are not straight. Although (26) no longer holds, we can still extract the direction u from the image, using the vanishing point where the lines meet [15]. Assuming that the vanishing point is visible in image plane, it can be shown that it represents a direct measure of u, since

$$u = R\lim_{\tau\to\infty} im(R^T(\bar{l}_i(\tau)-\xi)), \tag{27}$$

where $\bar{l}_i(\tau)$ describes the line coordinates in $\{\mathcal{I}\}$, as introduced in (21). Then, the vectors $\bar{h}_i$ can be obtained using

$$\bar{h}_i = R\frac{im(\bar{P}_i)\times R^Tu}{\|im(\bar{P}_i)\times R^Tu\|}, \tag{28}$$

with $\bar{P}_i = R^T\bar{p}_i$.

IV. Control

Assuming, for now, that the wind velocity is negligible and considering the camera is pointing downwards and is fixed in the aircraft's center of gravity, the dynamics of p_i and h_i are given by

$$\dot{p}_i = -v, \tag{29}$$

and

$$\dot{h}_i = \pi_{h_i}\mathrm{s_k}\,(u)\,v/\|H_i\| \tag{30}$$

respectively. Then, for the case of straight lines, the dynamics of the error δ can be written as

$$\dot{\delta} = Q\mathrm{s_k}\,(u)\,v \tag{31}$$

where the image Jacobian matrix Q is given by

$$Q = \sum \pi_{h_i}/\|H_i\|. \tag{32}$$

Given that $\|H_i\|$ cannot be directly extracted from the image, obtaining measurements for Q is a difficult task. Nonetheless, as carefully described in [7], it is always possible to define Q as a positive definite matrix, which allows for the synthesis of controllers without an explicit estimate of Q. However, the trajectories considered must be subject to some bounds in order to avoid ill-conditioning of the control system. Hence, a region of space is defined by a pair of uniform bounds on the matrix Q, [7]

$$\lambda_{\min}(Q) < \{\lambda_i(Q)\} < \lambda_{\max}(Q). \tag{33}$$

Similar derivations can be done for the case of nonlinear curves. Namely, the time derivative of $\bar{p}_i$ can be written as

$$\dot{\bar{p}}_i = -\pi_u v + (\partial\sigma(\tau_\xi)/\partial\tau)u_0u^Tv, \tag{34}$$

where $\tau_\xi = -u^T(l_{i0}-\xi)$. Notice that the partial derivative $\frac{\partial\sigma(\tau_\xi)}{\partial\tau}$ is unknown since the position of the aircraft relatively to the lines is unknown.

Using (34) to compute $\dot{\bar{H}}_i$ and consequently $\dot{\bar{h}}_i$, it follows that

$$\dot{\bar{\delta}} = Q\mathrm{s_k}\,(u)\,(I-(\partial\sigma(\tau_\xi)/\partial\tau)u_0u^T)v. \tag{35}$$

A comparison between (35) and (31) highlights the fact that $\bar{\delta}$ can be interpreted as a generalization of δ, since (35) reduces to (31) when the lines are straight, or, equivalently, when $\frac{\partial\sigma(\tau_\xi)}{\partial\tau}=0$. In view of this result, we proceed with the design of a control law for the general case of nonlinear curves, which is also applicable to straight lines.

Given that the error dynamics (35) has no actuation input, let v_d be the desired velocity defined as

$$v_d = k_1\mathrm{s_k}\,(u)\,\bar{\delta} + u\sqrt{V^2 - k_1^2\bar{\delta}^T\bar{\delta}}, \tag{36}$$

and consider the new error term given by

$$\delta_v = \mathrm{s_k}\,(u)\,(v-v_d). \tag{37}$$

Using (37), the error dynamics can be rewritten as

$$\dot{\bar{\delta}} = -k_1Q\bar{\delta} + Q\delta_v - \frac{\partial\sigma(\tau_\xi)}{\partial\tau}u^TvQ\mathrm{s_k}\,(u)\,u_0 \tag{38}$$

and with (16)

$$\begin{aligned}\dot{\delta}_v &= -\tfrac{1}{mV}\mathrm{s_k}\,(u)\,\mathrm{s_k}\,(v)\,u_a + k_1Q\delta_v - k_1^2Q\bar{\delta} + \\ &\quad -k_1\tfrac{\partial\sigma(\tau_\xi)}{\partial\tau}u^TvQ\mathrm{s_k}\,(u)\,u_0\end{aligned} \tag{39}$$

The following theorem shows that if $\sigma(\tau)$ has a bounded derivative the state $(\bar{\delta},\delta_v)$ is uniformly ultimately bounded.

Theorem 1: Consider the dynamics given by (38) and (39) and assume that $|\frac{\partial\sigma(\tau)}{\partial\tau}| \le c_0$, for all $\tau\in\mathbb{R}$, the initial velocity satisfies $v(0)^Tu>0$, the desired position is the middle of the curves, i.e.

$$q^*/\|q^*\| = \mathrm{s_k}\,(u)\,u_0. \tag{40}$$

Then, with gains k_1 and k_3 that satisfy

$$\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)} < k_1 < \frac{V}{2\max\{\|\bar{\delta}\|\}} \tag{41}$$

$$k_3 > (k_1^4 + 2k_1 + 1)\lambda_{\max}(Q)/2 \tag{42}$$

the control law

$$u_a = -k_3mV\delta_v/(v^Tu) \tag{43}$$

ensures uniform ultimate boundedness of the solution $(\bar{\delta}(t),\delta_v(t))$, with bound given by

$$b = \frac{c_0\lambda_{\max}(Q)Vk_1}{\rho(k_1\lambda_{\min}(Q)-1)},\ 0<\rho<1. \tag{44}$$

Proof: Applying the control law (43) to the first term of (39) yields

$$-\frac{1}{mV}\delta_v^T\mathrm{s_k}\,(v)\,u_a = -k_3\|\delta_v\|^2. \tag{45}$$

Consider the Lyapunov candidate function

$$W = 1/2\|\bar{\delta}\|^2 + 1/2\|\delta_v\|^2. \tag{46}$$

Its derivative can be written as

$$\begin{aligned}\dot{W} &= -k_1\bar{\delta}^TQ\bar{\delta} - \delta_v^T(k_3I - k_1Q)\delta_v + \bar{\delta}^T(I - k_1^2)Q\delta_v + \\ &-(\bar{\delta} + k_1\delta_v)^T\frac{\partial\sigma(\tau_\xi)}{\partial\tau}u^Tv.\end{aligned} \tag{47}$$

Knowing that $v^Tu \leq V$ and applying the Cauchy-Schwartz inequality, it is possible to define an upper bound for $\dot{V}$ as

$$\begin{aligned}\dot{W} &\leq -(k_1\lambda_{\min}(Q) - 1)\|\bar{\delta}\|^2 + c_0\lambda_{\max}(Q)V(\|\bar{\delta}\| + k_1\|\delta_v\|) \\ &-\left(k_3 - \frac{k_1^4 + 2k_1 + 1}{2}\lambda_{\max}(Q)\right)\|\delta_v\|^2\end{aligned} \tag{48}$$

therefore the system is uniformly ultimately bounded provided that the gains k_1 and k_3 satisfy (41) and (42), respectively.

It remains to be proven that $v(t)^Tu > 0$ for all $t > t_0$. Consider the following storage function

$$S = V - v^Tu = 1/(2V)\|v - Vu\|^2. \tag{49}$$

The derivative is given by

$$\dot{S} = -\frac{k_3}{v^Tu}(V + v^Tu)S + \frac{k_1k_3}{v^Tu}(v - Vu)^T\mathrm{s_k}(u)\,\bar{\delta} \tag{50}$$

and can be upper bounded by

$$\dot{S} \leq -\frac{k_3}{v^Tu}\|v - Vu\|\left((V + v^Tu)\frac{\|v - Vu\|}{2V} - k_1\|\bar{\delta}\|\right) \tag{51}$$

which is negative definite as long as $k_1^2\|\bar{\delta}\|^2 \leq \frac{V^2}{2}$ which falls onto the conditions of the proposed theorem. ■

A. Horizontal alignment

Although the proposed controlled only guarantees that the error is ultimately bounded, it is possible to prove that the error in the lateral direction converges to zero exponentially fast. The horizontal alignment error is given by

$$\delta_2 = \pi_{q^*}\bar{\delta} = \pi_{q^*}\sum\bar{H}_i/\|\bar{H}_i\|. \tag{52}$$

Assuming that q^* satisfies (40), δ_2 can also be expressed as

$$\delta_2 = -u_0u_0^T\sum\bar{H}_i/\|\bar{H}_i\|. \tag{53}$$

The following lemma establishes a relation between $\sum\frac{\bar{H}_i}{\|\bar{H}_i\|}$ and $\sum\bar{H}_i$ and shows that we can use a new representation for the horizontal alignment error given by

$$\phi = u_0^T/n\sum\bar{H}_i. \tag{54}$$

Lemma 1: Assume that $q(t)^Tq^* > 0$ for all $t > t_0$ and let

$$\begin{cases}\theta = \dfrac{u_0^T\sum\frac{\bar{H}_i}{\|\bar{H}_i\|}}{\phi}, & \text{for} \quad \phi \neq 0 \\ \theta = \frac{n^2}{\sum\bar{H}_i}, & \text{for} \quad \phi = 0\end{cases} \tag{55}$$

then, $\delta_2 = -\phi\theta u_0$, $\theta > 0$ and $\phi = 0 \Leftrightarrow \delta_2 = 0$.

Proof: The proof involves exhaustive, but straightforward computations which are omitted for reasons of compactness. Nonetheless the interested reader is referred to [16]. ■

To show that ϕ converges to zero we can apply the backstepping technique to obtain

$$\dot{\phi} = -k_4\phi + \phi_v \tag{56}$$

where $\phi_v = u_0^T\mathrm{s_k}(u)\,v + k_4\phi$. The derivative of ϕ_v is given by

$$\dot{\phi}_v = u_0^T\mathrm{s_k}(u)\,\mathrm{s_k}(v)\,u_a - k_4^2\phi + k_4\phi_v. \tag{57}$$

Using the control law for u_a defined in Theorem 1, and noting that

$$u_0^T\mathrm{s_k}(u)\,\mathrm{s_k}(v)\,u_a = -k_3\left(u_0^T\mathrm{s_k}(u)\,v - \theta k_1\phi\right) \tag{58}$$

we can write

$$\dot{\phi}_v = -(k_3(k_1\theta - k_4) + k_4^2)\phi - (k_3 - k_4)\phi_v. \tag{59}$$

Proposition 1: Consider the dynamics given by (56) and (59). Then, there exist gains k_3 and k_4 such that origin of the system is exponentially stable.

The proof of this proposition is based on the following Lyapunov function

$$V_y = (k_4^2/2)\|\phi\|^2 + (1/2)\|\phi_v\|^2 \tag{60}$$

whose derivative is given by

$$\dot{V}_y = -k_4^4\|\phi\|^2 + -k_3\left(k_1\theta - k_4\right)\phi_v^T\phi - (k_3 - k_4)\|\phi_v\|^2. \tag{61}$$

From Theorem 1, $\bar{\delta}$ is bounded and consequently δ_2 and ϕ are also bounded, hence θ is bounded. Therefore it can be shown that there exist a set of gains that ensure that $\dot{V}_y$ is negative definite.

V. Simulation Results

The IBVS control scheme described is simulated using the UAV model described in Section II. The aircraft model incorporates the nonlinear flight dynamics including aerodynamic effects and saturation on control surfaces deflection and thrust. The control scheme used includes a camera simulator which generates the image seen by the airplane's camera at a fixed frequency of 20 frames per second. Every time that each new frame is generated, the IBVS controller is triggered and the algorithm is applied.

In the simulations presented, the airplane is set to follow a pair of catenary lines in the presence of a constant wind disturbance. The estimator for the wind velocity described in [8] was used in order to improve the results in the presence of wind.

The lines are aligned with the x-axis of the inertial coordinate frame $\{\mathcal{I}\}$ at an average height of 10 meters above the ground. Figures 4, 5 and 6 show the results obtained with $k_1 = 1$ and $k_2 = 0.01$. The desired velocity is set to 9.11 ms^{-1}, which is the nominal velocity for the aircraft model used in the simulation. The wind intensity throughout the simulations is $1\ ms^1$ which is more than 10 % of the relative airspeed velocity.

As shown in Fig. 4, after an initial transient, the position error converges to zero in the horizontal direction and is bounded in the vertical direction. The trajectory described by the vehicle reflects the averaging effect of the IBVS guidance controller, which is acting as a first order filter on the curvature of the lines given by $|\frac{\partial\sigma(\tau_\xi)}{\partial\tau}|$, which is illustrated in Figure 7. Reducing the curvature's variation allows for considerable a improvement in position tracking.

VI. Concluding remarks

This paper proposed a nonlinear IBVS controller scheme for fixed-wing aircraft, assuming no direct measurement of the aircraft position is available. The proposed controller allows the UAV to follow a set of parallel non-straight lines in 3-D space. The algorithm presented has been theoretically proved and tested in simulation with a nonlinear model of

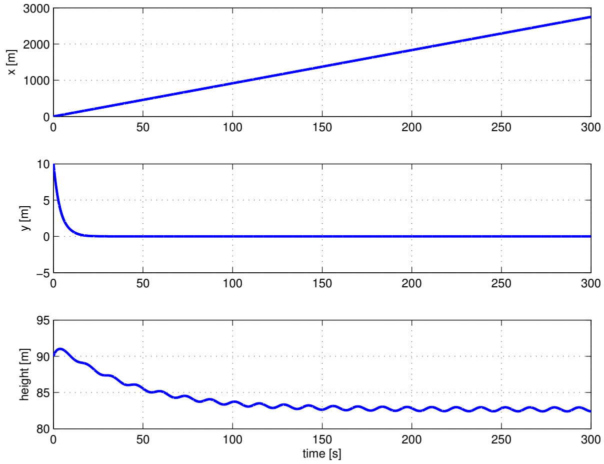

Fig. 4. Aircraft's position $(x, y, |z|)$.

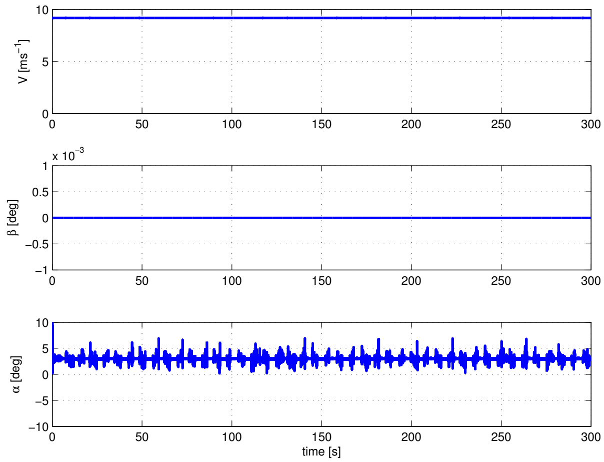

Fig. 5. Relative airspeed magnitude and direction (V, β, α).

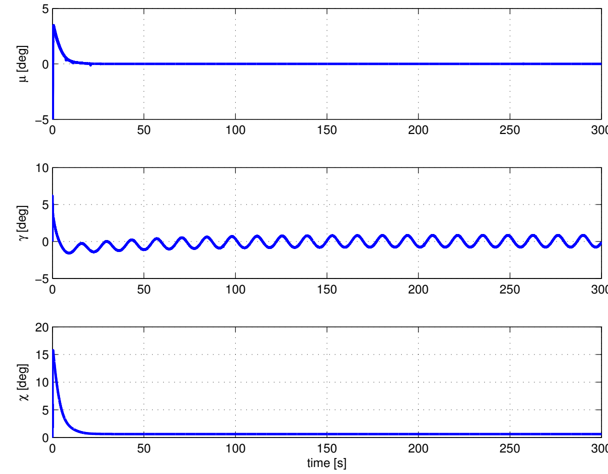

Fig. 6. Aircraft's attitude (μ, γ, χ).

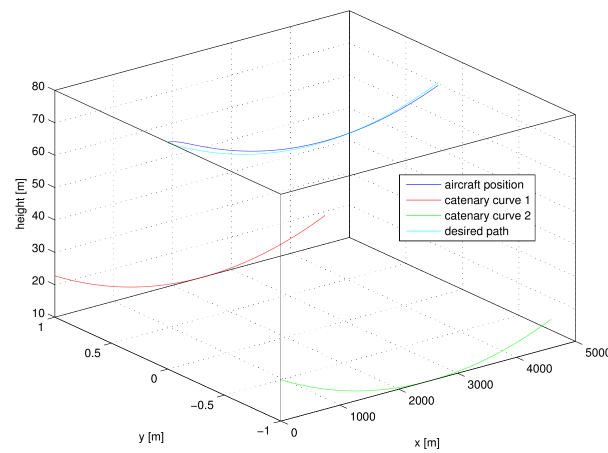

Fig. 7. 3-D path described by the aircraft following slowly varying catenaries.

an UAV. Results show that the control approach is suitable for the task and is robust to wind disturbances. Future work includes image treatment in simulation using images from a flight simulator. Also simulations with wind gust models, such as Dryden spectrum are planed. The inclusion of a pan & tilt camera to ensure that the target surface is always visible is also one of the goals for future work.

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